Automation of Human Reasoning in Economical Analysis

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Abstract

An application of so-called perception-based logical deduction in the modeling of an economic analysis given in natural language is presented. Fuzzy IF-THEN rules and theory of evaluating linguistic expressions are used, and possibilities of automated reasoning are outlined.

Keywords: Fuzzy type theory, Fuzzy IF-THEN rules, Perceptions.

1 Introduction

In this paper, we will describe an application of perception-based logical deduction (PbLD) in the modeling of economic analysis given in natural language.

In paper [11], we used the fuzzy type theory (FTT) [7], theory of evaluating linguistic expressions [10] and perception-based logical deduction (PbLD) [8] in the analysis of a detective story. In this paper, we will apply this method to a model of economical analysis of macroeconomic situation. For this purpose, we use as an example a part of free economical analysis of the Czech Savings Bank. It describes the influence of economic growth and other factors to the change of unemployment rate in the Czech Republic. It turns out that influence of these auxiliary factors (non-motivating social system, rigid labor market, high tax load) is important and decreases the expected positive effect of high economic growth.

Of course, the model of this particular economic analysis should be taken as an illustration of our methodology based on FTT and PbLD. Its main features are:

- It works inside well-developed and sound formal theory.
- It extensively uses formal theory of the, so-called, *evaluating linguistic expressions*, which are natural-language expressions describing positions on an ordered scale. Examples of evaluating linguistic expressions are *small*, *more or less big*, etc. These expressions are used incessantly by people. Therefore, sound formal model of them behaving accordingly to human intuition is very important.
- Evaluating linguistic expressions are used in fuzzy IF-THEN rules and sets of them which we call *linguistic descriptions*. The model of meaning of linguistic descriptions is constructed using formal tools of FTT. It allows us to model the role of context (possible world) in accordance with intuition using standard notions of intension and extension (see Section 2).
- Due to its formal nature, our methodology is open to various kinds of improvements in the direction of higher proportion of automated extraction of linguistic descriptions, automated deduction, etc.

Our approach relates to the, so-called, precisiated natural language (PNL) introduced by L. A. Zadeh in [15]. It aims at formalizing natural language sentences or texts using tools from the field of soft computing. E. g., the authors in [3] showed some application of PNL in the analysis of simple economic sentence.

There are also connections to *common-sense reasoning* [13], originated in large part by John McCarthy [6]. It includes formalization of reasonings performed by humans, taking into account their nonmonotonicity and other features.

2 Preliminaries

2.1 Fuzzy type theory

As stated, the main tool for the construction of a model[*)] of our economic analysis is fuzzy type theory. In this section, we will very briefly overview some of its main points.

[*)]We are using the term *model* in two meanings: as a general description of some situation, system etc., and as a formal model in the sense of mathematical logic.

A detailed explanation of FTT can be found in [7]. The classical type theory is in details described in [1].

The *Types* is a set of types constructed iteratively from the atomic types ϵ (elements) and o (truth values). $Form_\alpha$ denotes a set of formulas of type $\alpha \in Types$. If $A \in Form_\alpha$ is a formula of type $\alpha \in Types$ then we write A_α.

Formulas of type o (truth value) can be joined by the following connectives (derived formulas): $\vee$ (disjunction), $\wedge$ (conjunction), $\&$ (strong conjunction), ∇ (strong disjunction), $\Rightarrow$ (implication). General ($\forall$) and existential ($\exists$) quantifiers are defined as special formulas. For the details on their definition and semantics — see [7].

If $A \in Form_{o\alpha}$ then A represents a property of elements of the type α. By abuse of language, we will often say "A is a property" (of elements of type α) and similarly, $A_{(o\alpha)\alpha}$ is a relation (between elements of type α). We will freely write or omit the type when no misunderstanding may occur.

A theory T is a set of formulas of type o (determined by a subset of special axioms, as usual). Provability is defined as usual.

The operator

$$\imath z_\alpha A_o := \iota_{\alpha(o\alpha)}(\lambda z_\alpha A_o)$$

picks up an element of type α such that the formula A_o is true in the degree 1 for it.

Semantics. The structure of truth values in this paper is the Łukasiewicz$_\Delta$ algebra and so, the corresponding FTT is Łukasiewicz (Ł-FTT). Δ is the Baaz delta [5]. Let J be a language of Ł-FTT. A *frame* for J is a tuple $\mathcal{M} = \langle (M_\alpha, =_\alpha)_{\alpha \in Types}, \mathcal{L}_\Delta \rangle$ where $\mathcal{L}_\Delta$ is Łukasiewicz$_\Delta$ algebra of truth values, $=_\alpha$ is a fuzzy equality on M_α.

Recall that if $\beta\alpha$ is a type then the corresponding set $M_{\beta\alpha}$ contains (not necessarily all) functions $f : M_\alpha \longrightarrow M_\beta$.

Let p be an assignment of elements from $\mathcal{M}$ to variables. An interpretation $\mathcal{I}^{\mathcal{M}}$ is a function that assigns every formula A_α, $\alpha \in Types$ and every assignment p a corresponding element, that is, a function of the type α. A general model is a frame $\mathcal{M}$ such that $\mathcal{I}^{\mathcal{M}}_p(A_\alpha) \in M_\alpha$ holds true.

The following is a special formula representing a non-zero truth value:

$$\Upsilon_{oo} := \lambda z_o \cdot \neg\Delta(\neg z_o).$$

Lemma 1
If $T \vdash \Upsilon z_o \,\&(z_o \Rightarrow y_o)$ *then* $T \vdash \Upsilon y_o$.

2.2 Evaluating linguistic expressions

Evaluating linguistic expressions (or, simply, evaluating expressions) are expressions of natural language, for example, *small, medium, big, about twenty five, roughly one hundred, very short, more or less deep, not very tall, roughly warm or medium hot, quite roughly strong, roughly medium size*, and many others. They form a small but very important part of natural language and they are present in its everyday use any time. The reason is that people very often need to evaluate phenomena around them. Moreover, they often make important decisions based on them, learn how to control, and many other activities.

Because of lack of space and quite complicated formalism, we will only touch this theory and refer to the contribution [9]. All the details can be found in [10].

Important role in the theory of evaluating expressions is played by the concept of the *context*. It characterizes range of possible values for (numerical) variables and is represented by a special type ω. For simplicity, we will suppose that in each model, the context is given by a triple $\langle v_L, v_M, v_R \rangle$, where $v_L, v_M, v_R \in \mathbb{R}$ and $v_L < v_M < v_R$. The values v_L, v_M, v_R characterize minimal, middle and maximal value of the given context, respectively. On syntactical level, the context is denoted by a variable $w \in Form_\omega$ and represented by three constants $\bot_w, \dagger_w$ and $\top_w$.

In applications, we often need also more general type of context, which allows both positive as well as negative values. We will call this type of context *two-sided*. It is given by $\langle v_{NL}, v_{NM}, v_Z, v_{PM}, v_{PR} \rangle$, where $v_{NL} < v_{NM} < v_Z < v_{PM} < v_{PR}$. In this context, the sign is used before evaluating linguistic expressions, e.g. *negative small*, *positive big* (abbreviations: $-\,Sm$, $+\,Bi$), etc. There is also special evaluating linguistic expression *zero*. Contexts from the previous paragraph will be called *simple*.

Evaluating expressions are denoted by letters $\mathcal{A}, \mathcal{B}, \ldots$. *Intension* of $\mathcal{A}$ is a formula $\mathrm{Int}(\mathcal{A})$. Recall that intension means *a property* that is denoted by $\mathcal{A}$. It is important to note that intension does not depend on the context. For example, *very small* is a name of a property of being "very small" which may mean about 150 cm (and less) when speaking about people, about 3 mm when speaking about beetles, etc. The type of $\mathrm{Int}(\mathcal{A})$ is $(o\alpha)\omega$. The latter will often be denoted by φ.The formal theory of evaluating expressions is denoted by T^{Ev}. This theory provides means how the above concepts of context, intension and others can be effectively formalized.

2.3 Fuzzy IF-THEN rules and perception-based logical deduction

The perception-based logical deduction in the frame of FTT has been described in [8]. Though the method is more general, we will suppose that all considered linguistic expressions are evaluating ones.

A fuzzy IF-THEN rule is a linguistic expression of the form

$$\mathcal{R} := \ \mathsf{IF}\ X\ \mathsf{is}\ \mathcal{A}\ \mathsf{THEN}\ Y\ \mathsf{is}\ \mathcal{B}.$$

where $\mathcal{A}, \mathcal{B}$ are evaluating expressions. The linguistic predication 'X is $\mathcal{A}$' is called *antecedent* and 'Y is $\mathcal{B}$' is called *consequent*.

Intension of a fuzzy IF-THEN rule $\mathcal{R}$ is (represented by) a formula

$$\mathrm{Int}(\mathcal{R}) := \lambda w\, \lambda w' \cdot \lambda x\, \lambda y \cdot A_{(o\alpha)\omega} wx \Rightarrow B_{(o\beta)\omega} w' y \tag{1}$$

A *linguistic description LD* is a set of fuzzy IF-THEN rules. Its *topic* is a set of linguistic expressions $\{\mathrm{Int}(\mathcal{A}_j) \mid j = 1, \ldots, m\}$ and its focus is $\{\mathrm{Int}(\mathcal{B}_j) \mid j = 1, \ldots, m\}$, where m is a number of IF-THEN rules in the linguistic description *LD*.

In practice, we usually need more antecedent variables. In this case, the antecedent variables are usually connected by linguistic connective AND that is interpreted by a logical connective $\wedge$.

In perception-based logical deduction, we must introduce several special formulas. We will give only their informal description and refer to [8] for their precise definitions. The formula $\prec$ denotes the relation of sharpness between (intensions of) evaluating expressions. For example, if x is, at least partly, "very big" in all contexts then it is also "big" in all of them, i.e. $\mathrm{Int}(\mathit{very\ big}) \prec \mathrm{Int}(\mathit{big})$. We will also introduce formula $\mathit{Eval}_{o(\varphi\alpha\omega)}$ ($\mathit{Eval}\ wx\, \mathrm{Int}(\mathcal{A})$ expresses that an element x in context w is evaluated by $\mathcal{A}$).

One of principal paradigms of the concept of precisiated natural language is that the world knowledge, i.e. the knowledge accumulated by people during their life, is *perception based.* We will formalize the concept of perception using a formula $\mathit{Perc} \in \mathit{Form}_{(o\varphi)\alpha}$ which expresses that an intension z_φ is a *perception* of $x_\alpha \in \mathit{Form}_\alpha$.

If we consider a linguistic description *LD* then Perc^{LD} is a perception w.r.t. Topic^{LD}. For example, for a given linguistic description *LD* the formula

$$\mathit{Perc}^{LD} x \mathit{Ev}$$

means that the evaluating expression *Ev* from the topic of *LD* is a *perception* of x. More precisely, there is a context w in which x is evaluated in the best way by the sharpest *Ev* (among evaluating expressions belonging to the topic of *LD*). We will also introduce a *local perception* $\mathit{LPerc}_{o(\varphi\alpha\omega)}$ relative to a specific context w.

Lemma 2
(a) Let $\mathit{Ev}_1 \prec \mathit{Ev}_2$. *Then*

$$T^{Ev} \vdash \mathit{Perc}\ x\, \mathit{Ev}_1 \Rightarrow \mathit{Perc}\ x\, \mathit{Ev}_2.$$

(b) Let $T^{Ev} \vdash z_\varphi wx \Rightarrow z'_\varphi w' y$. *Then* $T^{Ev} \vdash \mathit{Eval}\ wxz_\varphi \Rightarrow \mathit{Eval}\ w'yz'_\varphi$.

The linguistic description *LD* characterizes a certain kind of dependence (relation) between features of objects (and, consequently, the objects themselves) using natural language. People use it when they want to describe a certain situation or process but they do not know it precisely. Therefore, the most important role of the linguistic description is to provide a conclusion about consequent objects Y when an information about antecedent objects X is given. Such an information has a character of *perception* of properties of the latter objects and so, the corresponding procedure is called *perception-based* logical deduction.

On the basis of the theory presented in [8], the following special inference rule of perception-based logical deduction can be introduced. Let *LD* be a linguistic description consisting of rules of the form (1) and $x \in \mathit{Form}_\alpha$, $y \in \mathit{Form}_\beta$, $w \in \mathit{Form}_{\alpha o}$, $w' \in \mathit{Form}_{\beta o}$. Then the following scheme is a valid special inference rule:

$$r_{PbLD} : \frac{\mathit{LPerc}^{LD} wx\mathit{Ev}_i^A, \quad LD}{\mathit{Eval}\ w'\ \hat{y}_i \mathit{Ev}_i^C} \tag{2}$$

where $\hat{y}_i \equiv \imath y \cdot \mathit{Ev}_i^A wx \Rightarrow \mathit{Ev}_i^C w'\, y$, $i \in \{1, \ldots, m\}$, $T \vdash \mathit{Topic}^{LD} \mathit{Ev}^A$ and $T \vdash \mathit{Focus}^{LD} \mathit{Ev}^C$.

This rule has the following interpretation: Let *LD* be a linguistic description consisting of fuzzy IF-THEN rules of the form (1). If we find a formula $\mathrm{Int}(\mathcal{A}_i) \equiv \mathit{Ev}_i^A$ of some expression from the topic Topic^{LD} and an element $\mathbf{u}^0$ in the context $\mathbf{w}^0$ such that $\mathit{Ev}_i^A \mathbf{w}^0 \mathbf{u}^0$ has a non-zero truth degree then (denoting $\mathbf{b}_i^0 \equiv \mathit{Ev}_i^A \mathbf{w}^0 \mathbf{u}^0$) we conclude that the element $\imath y \cdot \mathbf{b}_i^0 \Rightarrow \mathit{Ev}_i^C w' y$ which is *typical for the formula* $\mathbf{b}_i^0 \Rightarrow \mathit{Ev}_i^C w' y$, is evaluated by the linguistic expression $\mathit{Ev}_i^C \in \mathit{Focus}^{LD}$ in every context w'.

Note that the main result of r_{PbLD} is the element $\hat{y}_i$. Then in every model $\mathcal{M}$ we can find a specific element $\mathcal{I}_p^{\mathcal{M}}(\hat{y}_i) = v \in M_\beta$ using the operation $\mathcal{I}_p^{\mathcal{M}}(\iota_{\beta(o\beta)})$ which in fuzzy set theory is just the defuzzification function. In our case, the DEE method (Defuzzification of linguistic expressions) should be used. The detailed, less formal explanation of the perception-based logical deduction including examples is presented in [12].

Human reasoning that is based on a complex of experience, observation, logical reasoning and world knowledge is necessarily non-monotonic. Hence, our model must include also the theory of *non-monotonic reasoning* (cf. [2, 11]). We deal with a class of theories that themselves are consistent but when using them simultaneously, we may arrive at a contradiction or, at least, to a non-desirable result. Therefore, we consider a special preference relation that tells us which theory should be used in the given state (called *belief state* in [2]). At each state, we work in a special theory which in our case is determined by a linguistic description (one or more) and possibly also by some perception (recall that this is a formula representing intension of some evaluating linguistic expression).

In this paper, we are not going into details of this approach. We will suppose that our formal theories contain only those parts of the theory of evaluating expressions T^{Ev}, that are necessary for deductions based on used linguistic descriptions and perceptions.

3 Example of a model of economic analysis

We present a model of a macroeconomic situation using PbLD. We use a part from free economical analysis of Czech Savings Bank for the fourth quarter of 2006:[†)]

> Our computations show, that acceleration of economy by one per cent causes unemployment rate decrease only by 0.3 per cent. Non-motivating social system, rigid labor market and high tax rate on labor expenses are by our opinion main culprits of structurally high rate of unemployment, although economy grows at high rate.

We will present an analysis of this quotation by means of the above outlined theory.

3.1 Method

As mentioned, our theory can be classified as a part of the methodology introduced by L. A. Zadeh in his papers [14, 15] and called *Precisiated Natural Language* (PNL). The latter is an attempt at developing a unified formalism for various tasks involving natural language propositions.

Two main premises of PNL are the following:

(a) Much of the world knowledge is perception based.

(b) Perception based information is intrinsically fuzzy.

The PNL methodology requires presence of the, so-called, *World Knowledge Database* (WKDB) which contains all the necessary information including perception based propositions describing the knowledge acquired by direct human experience and which can be used in the deduction process. A *multiagent, modular deduction database* (MDE) contains various rules of deduction.

Our version of PNL incorporates logical machinery. The translation from natural language to fuzzy IF-THEN rules is so far done manually. The formal frame is Łukasiewicz fuzzy type theory (Ł-FTT) and theories of evaluating linguistic expressions and fuzzy IF-THEN rules as described in the previous sections. Moreover, we apply some principles of non-monotonic reasoning. However, we are not going into details of the role of it in this paper.

We model the above natural-language macroeconomic analysis using two linguistic descriptions with hierarchic structure. We will use an intermediate variable *strength of auxiliary factors* for the overall influence of the non-motivating social system, rigid labor market and high tax rate. Hence, we will introduce linguistic description LD_{Aux} with three antecedent variables and one consequent variable, the above mentioned *strength of auxiliary factors*. This variable is then used as antecedent variable to the second linguistic description LD_{Un}, with the second antecedent variable *rate of economy acceleration*. The consequent variable in this linguistic description is *rate of unemployment change* – the result.

3.2 World knowledge

(i) $\vartheta \in$ *Types* represents a general feature of objects that can be characterized using grades. In the model, a set of this type can be, e.g., a subset of the real numbers.

(ii) $\beta \in$ *Types* represent objects of type (national) state.[†)]

(iii) $\gamma \in Type$ represents general abstract objects.

(iv) $Ss \in Form_{\vartheta\beta}, Lm \in Form_{\vartheta\beta}, Tl \in Form_{\vartheta\beta}, Uc \in Form_{\vartheta\beta}, Ea \in Form_{\vartheta\beta}, Af \in Form_{\vartheta\gamma}$, are special formulas characterizing state of social system, rigidity of labor market, level of tax load, unemployment change, economy acceleration (of objects of type β) and auxiliary factors (of object of type γ), respectively.

(v) In correspondence with the previous item, we also need to consider contexts $w_{Ss}, w_{Lm}, w_{Tl}, w_{Uc}, w_{Ea}, w_{Af} \in Form_{\vartheta o}$.

(vi) Special constant concerning the story, namely: $\mathbf{c}_{CR} \in Form_\beta$ representing in our case *the Czech Republic*. However, linguistic description below should be valid also for other countries.

3.3 Formalization and reasoning

(i) *Contexts.* Context can be either *simple* (items (a), (b), (c), (e)) or *two-sided* (items (d), (f)), see Section 2.2. Two-sided contexts are used for variables which can take both positive and negative values.

   (a) Motivation of social system: $w_{Ss} = \langle 0, 0.5, 1\rangle$ (abstract degrees).
   (b) Rigidity of labor market: $w_{Lm} = \langle 0, 0.5, 1\rangle$ (abstract degrees).
   (c) Strength of tax load: $w_{Tl} = \langle 0, 15, 40\rangle$ (%).
   (d) Rate of unemployment change: $w_{Uc} = \langle -1, -0.5, 0, 0.5, 1\rangle$ (%).
   (e) Strength of auxiliary factors: $w_{Af} = \langle 0, 0.5, 1\rangle$ (abstract degrees).
   (f) Rate of economy acceleration: $w_{Ea} = \langle -6, -3, 0, 5, 10\rangle$ (%).

[†)] http://www.csas.cz/banka/content/inet/internet/cs/treasury_ie.xml, in Czech

[†)] This type could also represent other territorial objects, e.g., regions, European Union etc.

(ii) *Linguistic descriptions*

(a) Linguistic description LD_{Aux} for the influence of auxiliary factors:[‡)]

$$\text{IF } X_{Ss} \text{ is } Sm \text{ AND } X_{Lm} \text{ is } Bi \text{ AND } X_{Tl} \text{ is } Bi \text{ THEN } X_{Af} \text{ is } Bi \quad (3)$$

. . . .

The corresponding intensions of such rules are

$$\lambda w_{Ss}\, \lambda w_{Lm}\, \lambda w_{Tl}\, \lambda w_{Af} \cdot \lambda x_{1,\vartheta}\, \lambda x_{2,\vartheta}\, \lambda x_{3,\vartheta}\, \lambda y_\vartheta \cdot (Ev_1^A w_{Ss}\, x_{1,\vartheta}) \wedge (Ev_2^A w_{Lm}\, x_{2,\vartheta}) \wedge (Ev_3^A w_{Tl}\, x_{3,\vartheta}) \Rightarrow Ev^C w_{Af}\, y_\vartheta. \quad (4)$$

(b) Linguistic description LD_{Un} for the computation of unemployment change.

$$\text{IF } X_{Ea} \text{ is } +Bi \text{ AND } X_{Af} \text{ is } Bi \text{ THEN } X_{Uc} \text{ is } -Sm \quad (5)$$

. . . .

The corresponding intensions of such rules are

$$\lambda w_{Ea}\, \lambda w_{Af}\, \lambda w_{Uc} \cdot \lambda x_{1,\vartheta}\, \lambda x_{2,\vartheta}\, \lambda y_\vartheta \cdot (Ev_1^A w_{Ea}\, x_{1,\vartheta}) \wedge (Ev_2^A w_{Af}\, x_{2,\vartheta}) \Rightarrow Ev^C w_{Uc}\, y_\vartheta. \quad (6)$$

(iii) Furthermore, we must construct a specific *model*. This will be based on a frame

$$\mathcal{M} = \langle (M_\alpha, =_\alpha)_{\alpha \in Types}, \mathcal{L}_\Delta \rangle.$$

where $M_o = [0,1]$ and $M_\alpha = \mathbb{R}$ for $\alpha = \vartheta$.

(iv) *Perceptions*

(P1) Social system is non-motivating. We can say that motivation of social system is *very small*. Formally,

$$\mathcal{I}_p^{\mathcal{M}}(LPerc\; w_{Ss}(Ss\mathbf{c}_{CR})\; Ve\, Sm) = 1. \quad (7)$$

(P2) Labor market is rigid. It means that rigidity of labor market is *big*. Formally,

$$\mathcal{I}_p^{\mathcal{M}}(LPerc\; w_{Lm}(Lm\mathbf{c}_{CR})\; Bi) = 1. \quad (8)$$

(P3) Tax load is high. Formally,

$$\mathcal{I}_p^{\mathcal{M}}(LPerc\; w_{Tl}(Ss\mathbf{c}_{CR})\; Bi) = 1. \quad (9)$$

(P4) Economy acceleration is positive big. Formally,

$$\mathcal{I}_p^{\mathcal{M}}(LPerc\; w_{Ea}(Ss\mathbf{c}_{CR}) + Bi) = 1. \quad (10)$$

(v) *Deductions* We use a formal theory T^{Uc} for the reasoning about our example. It includes appropriate parts of theory of evaluating expressions T^{Ev}, intensions of linguistic descriptions LD_{Aux} and LD_{Un} and perceptions (P1)–(P4).

$$T^{Uc} = \{LPerc\; w_{Ss}(Ss\mathbf{c}_{CR})\; Ve\, Sm,\; LPerc\; w_{Lm}(Lm\mathbf{c}_{CR})\; Bi,\; LPerc\; w_{Tl}(Ss\mathbf{c}_{CR})\; Bi,\; LPerc\; w_{Ea}(Ss\mathbf{c}_{CR}) + Bi, (4), (6)\} \cup T^{Ev} \quad (11)$$

Below, we give the main ideas of proofs.

(a)

$$T^{Uc} \vdash LPerc^{LD_{Aux}} w_{Ss}(Ss\mathbf{c}_{CR})\; Sm \quad (12)$$

It means that *small* is the perception of variable *motivation of social system* with respect to linguistic description LD_{Aux}.

PROOF: Note that there is evaluating expression *Ve Sm* in perception (P1) and evaluating expression *Sm* in IF-THEN rule (3). Suppose that in linguistic description LD_{Aux} is no other rule which has in the variable X_{ss} evaluating expression⟨linguistic hedge⟩⟨atomic expression⟩, where ⟨atomic expression⟩ is *small*. Then (12) follows from the definition of *LPerc*. □

(b)

$$T^{Uc} \vdash LPerc^{LD_{Aux}} w_{Lm}(Lm\mathbf{c}_{CR})\; Bi\,. \quad (13)$$

It means that *big* is the perception of variable *rigidity of labor market* with respect to linguistic description LD_{Aux}.

PROOF: In (b) and (c), evaluating expression in perception (P2) and (P3) is equal to evaluating expression in corresponding variables of (3), respectively. Proofs are analogous to (a). □

(c)

$$T^{Uc} \vdash LPerc^{LD_{Aux}} w_{Tl}(Tl\mathbf{c}_{CR})\; Bi\,. \quad (14)$$

It means that *big* is the perception of variable *tax load* with respect to linguistic description LD_{Aux}.

(d)

$$T^{Uc} \vdash Eval\, w_{Af}(Af\mathbf{c}_{AF})\; Bi\,. \quad (15)$$

This deduction means that $\mathbf{c}_{AF}$ is evaluated by evaluating expression *big*. It represents the result

‡)Only one rule based on natural-language macroeconomic analysis is given here. However, more similar rules can be present.

of deduction over linguistic description LD_{Aux} – *strength of auxiliary factors* is *big*.

PROOF: Follows from r_{PbLD} and (4), where $\mathbf{c}_{AF}$ is a new constant representing the result of this deduction. □

(e)

$$T^{Uc} \vdash LPerc^{LD_{Un}} w_{Af}(Af\mathbf{c}_{AF})\, Bi \qquad (16)$$

It means that *big* is the perception of variable *strength of auxiliary factors* with respect to linguistic description LD_{Un}.

PROOF: Suppose that in linguistic description LD_{Un} is no other rule which has in the variable X_{af} evaluating expression *big*. Then (16) follows from the definition of *LPerc*. □

(f)

$$T^{Uc} \vdash Eval\, w_{Uc}(Uc\mathbf{c}_{UC})(-\, Sm) \qquad (17)$$

This deduction means that $\mathbf{c}_{UC}$ is evaluated by evaluating expression *negative small*. It represents the result of deduction over linguistic description LD_{Un} – *rate of unemployment change* is *negative small*.

PROOF: Follows from r_{PbLD} and (6), where $\mathbf{c}_{UC}$ is a new constant. □

Hence, we can conclude that change of unemployment is *negative small*.

3.4 Remarks

- In our example, there was one difference between evaluating expression from perception and evaluating expression in the corresponding variable in the linguistic description (namely *very small* in (P1) and *small* in (3)). Overall result (change of unemployment is *negative small*) corresponds to the evaluating expression in the consequent of IF-THEN rule (5). Hence, the result could seemingly be obtained without complex formal apparatus. However, more complicated situations (much more variables, linguistic descriptions etc.) are usually not transparent so much. Then, our formal apparatus is not only useful but necessary.

- An implementation of the theory of evaluating expressions and perception-based logical deduction is available as a part of the software system LFLC (Linguistic Fuzzy Logic Controller, demo version available at `http://irafm.osu.cz`) [4]. This system is designed in such a way that the user is not supposed to be aware of the complicated formal systems presented in previous sections. If he/she provides contexts, linguistic descriptions in the form of IF-THEN rules and perceptions, then deductions from subsection 3.3 are performed automatically. Hence, the method described in this paper is accessible also for users without advanced knowledge of fuzzy logic.

4 Conclusions

In this paper, a model of concrete macroeconomic analysis using perception-based logical deduction was presented. In the further research, we will concentrate on:

- Modeling of more complicated economic analyses.
- Methods for automated or semi-automated extraction of linguistic descriptions and deductions.
- Incorporation of further parts of theory of nonmonotonic reasoning.

Acknowledgement

This work was supported by project MSM 6198898701 of the MŠMT ČR.

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