From Communication Networks to Fuzzy Sets

Rudolf Seising
Section of Medical Expert and Knowledge-Based Systems,
Department of Medical Computer Sciences,
University of Vienna Medical School,
Spitalgasse 23, A-1090 Vienna, Austria
e-mail: rudolf.seising@akh-wien.ac.at

Abstract.

In 1931 and 1935, Ernst Adolph Guillemin, professor of electrical engineering at *Masachusetts Institute of Technology* (MIT), published the two volumes of his book "Communication Networks". In that time telephone-, telegraph- and radio networks used electricity in communication. In the course of this work electrical engineers become aware that higher mathematics was necessary to construct better communication technique. Since the 1920s circuit theory and electrical filter theory have been developed. In the 1930s Guillemin recognized the challenge to network theory including the design of electromagnetic wave filters and related networks. Norbert Wiener wrote in his 1949 article on *A New Concept of Communication Engineering* just as Guillemin wrote his books for "communication engineers".

In 1944 Lotfi Zadeh, *Bachelor of Science in electrical engineering* from the University of Teheran in Iran, came to the USA. In 1946 he got his *Master of Science* degree at MIT, where Guillemin was his teacher in advanced network theory. But after some time Guillemin and Zadeh had different opinions about circuits in theory and in the laboratory. Guillemins view was very idealistic: resistors, capacitors, inductors, they were all perfect elements. Zadeh argued that this is unrealistic. In his view resistors are not pure resistors, capacitors are not pure capacitors, and so forth.

In 1946 Zadeh became an instructor in electrical engineering at Columbia University in New York. In 1949 he wrote his Ph.D thesis on *Frequency Analysis of Variable Networks*. With his supervisor, John Ralph Ragazzini, he published *An Extension of Wiener's Theory of Prediction*. He was influenced by Shannons and Wieners information theory and he was interested in the theory of ideal and optimal filtering. From the mathematical point of view, this theory is essentially a study of certain types of mappings of function and sequence spaces. Many of the mathematical techniques used in this theory are commonly employed in quantum mechanics. Ideal filters are defined as filters which achieve a perfect separation of signal and noise whereas optimal filters give the "best approximation" of a signal. Zadeh noticed that "best approximations" depend on reasonable criterions; he formulated these criterions in formulated such a criterion in statistical terms.

In the 1950s Zadeh was one of the founders of *system theory*, a rising scientific discipline "to the study of systems per se, regardless of their physical structure". Engineers in that time were, in general, inadequately trained to think in abstract terms, but nevertheless, Zadeh believed that it was only a matter of time before system theory attains acceptance.

Systems are black boxes with inputs and outputs, and with a definite state in every time. Zadeh wrote a textbook on these input-output systems and he published a couple of papers on system theory. He was the founder of the so called *state space approach* of system theory. Within this approach a system of equations relates input, output and state to each other

$$\boldsymbol{s}_{t+1} = f(\boldsymbol{s}_t, \boldsymbol{u}_t),$$

$$\boldsymbol{y}_t = g(\boldsymbol{s}_t, \boldsymbol{u}_t) \qquad t = 0,1,2, \ldots$$

The state of the system at time t+1 is determined by state and input at time t and the output at any time t depends on input und state at the same time.

"What is optimal?" asked Zadeh in March 1958 in his editorial for the *IRE Transactions on Information Theory*. He wrote that communication engineers "tend, perhaps, to make a fetish of optimality. If

a system is not „best“ in one sense or another, we do not feel satisfied. Indeed, we are apt to place too much confidence in a system that is, in effect, optimal by definition.”[1]
In that time there was a shift in Zadehs scientific life. He took notice that there wouldn’t be any chance to compute all system equations of any real system. He noticed that all the new digital computers that have been constructed in the 40s and 50s couldn’t help to get exact knowledge of what happens in real world systems. To compute or to describe or to control processes in complex systems in particular in large scaled systems there are too many state equations and as a consequence: too many systems of differential equations. Computers cannot solve that much differential equations in a limited time.
Zadeh found another way to describe real world systems in 1964: If the world is not exact in terms of exact mathematics, try to construct more suitable mathematics!
In 1962, when he was still busy to establish system theory, he wrote an article called “From circuit theory to system theory” In this article he published his first idea of this shift:

> „There are some who feel that this gap reflects the fundamental inadequacy of the conventional mathematics – the mathematics of precisely-defined points, functions, sets, probability measures, etc. – for coping with the analysis of biological systems, and that to deal effectively with such systems, which are generally orders of magnitude more complex than man-made systems, we need a radically different kind of mathematics, the mathematics of *fuzzy* or cloudy quantities which are not describable in terms of probability distributions. Indeed, the need for such mathematics is becoming increasingly apparent even in the realm of inanimate systems, for in most practical cases the *a priori* data as well as the criteria by which the performance of a man-made system is judged are far from being precisely specified or having accurately-known probability distributions.“[2]

On a Symposium in the *Polytechnic Institute of Brooklyn* in April 1965, the famous paper on fuzzy sets was in press, Zadeh gave a talk called „A New View of System Theory“. Here Zadeh defined a „fuzzy system“: *S* is a fuzzy system, if the input *u(t)* or the output *y(t)* or the state *x(t)* or any combination is a fuzzy set. If for example the *input* of *S* at time *t* is specified as „much more than 5“, or „normal“, or „very very small“ then the *output* is a fuzzy set too and a system operating on fuzzy inputs is a fuzzy system. If the states of *S* are described by fuzzy attributes as light, heavy, not very heavy, very light etc. and if these states are fuzzy sets, *S* is a fuzzy system. State equations of fuzzy systems have the same form as other state equations but you have fuzzy sets instead of crisp sets.[3]

[1] Lotfi A. Zadeh: What is Optimal? *IRE Transactions on Information Theory* IT-4, 1958.
[2] Zadeh, Lotfi A.: From Circuit Theory to System Theory, in: *Proceedings of the IRE*, Vol. **50**, Number 5, May 1962, pp. 856-865: 857.
[3] Zadeh, Lotfi A.: Fuzzy sets and systems, in: Fox, Jerome (Ed.): *System Theory*. Microwave Research Institute Symposia Series XV. Brooklyn, New York: Polytechnic Press 1965.Fox, System (1965) pp. 29-37: 33 f.