Fuzzy-Evidence Approach to Uncertainty Modelling and Reliability

Bohdan S. Butkiewicz
Warsaw University of Technology, Institute of Electronic Systems
ul. Nowowiejska 15/19, 00-665 Warsaw
e-mail bb@ise.pw.edu.pl

Abstract

Conventional approach to mathematical description of experiments applies probability theory. Here, another approach is used to express uncertainty based on probability, fuzzy sets, and evidence theory. The paper presents a new reliability model of an element working in different environmental and stress conditions. Fuzzy and probability descriptions are combined together using evidence theory to describe failure rate of an element. The method presented in the paper is more general, not limited to reliability problems only.

Keywords: : uncertainty modelling, fuzzy, probability, reliability, entropy, possibility, evidence.

1 Introduction

Probability apparatus is in common use in the conventional approach to reliability. The knowledge of degradation processes is steaming from the results of testing as well as from our general experience. The results of testing undergo an easy mathematical processing and it seems unquestionable. However, the testing is carried out in different conditions being fairly far from expected environmental and stress conditions. Thus, results of testing are not always valid. The knowledge and experience, gathered in e.g. MIL 217 [6], can yield useful solutions to practical problems. Yet, it has nothing common with probability theory and the relevant predictions are of the type of ”fortune-telling”. In the handbooks we have exact formulas transforming e.g. basic failure density to that of our working conditions, however, the results constitute an approximation only. Such transformation can be expressed in terms of fuzzy logic.
Conventionally, two methods are used to solve the problem. First, very simple and popular, to calculate the failure density of the element from a test or estimate failure rate based on our experience, and apply transformation of the failure rate using [6]. Second, to calculate the confidence interval for failure rate from a test and find interval for the failure rate in exploitation conditions, applying transformations for upper and lower limits. Both solutions are not exact. Moreover, second is not correct because uncertainty of transformation is neglected, so it gives too narrow interval. We have two different models in real situation: probabilistic, obtained usually from the accelerated test, and fuzzy for transformation of environmental and stresses conditions. In the paper we try to join both models with Dempster Shafer evidence theory [2][5].

2 Probabilistic model

Suppose that a reliability test for elements was carried out. N identical elements were tested during a time t. d failures were observed. Assumption of the same environmental and stress conditions for all elements leads to estimation of failure rate in the form of

$$\hat{\lambda}_b = \frac{d}{N t} \tag{1}$$

and to binomial probability distribution of failures

$$Pr\{\nu(t) = k\} = \binom{N}{k} p^k (1-p)^{N-k} \tag{2}$$

where $\nu(t)$ denote random variable of k failures observed during time t. Hence, the failure rate during test can be estimated as in (1).

3 Transformation of environmental conditions

Consider now a transformation of the failure rate observed in stress and environmental conditions during the test to the rate expected in stress and conditions of exploitation. It is a function with some parameters, which depends on the type of the element. Typical transformation for failure rate has the form of

$$\lambda \;=\; \lambda_0[1+(\frac{S}{S_b})^n]\exp(\frac{E_a}{kN_T}T_a) \qquad (3)$$

where λ_0 - predicted value of base failure rate during exploitation, S - stress during exploitation, S_b - stress constant, n - power constant depending on the type of element, T_a - exploitation temperature in $^\circ K$, N_T - temperature constant, E_a - activation energy, k - Boltzman constant. Here, a new constant $K_a = E_a/kN_T$ is put for simplicity. Of course, the formula (3) may be used for test conditions also.

4 Fuzzy model of failure process

Uncertainty and lack of knowledge can be expressed in the terms of fuzzy logic. There are some different approaches. Author and technology expert proposed one of them [7]. It is possible to build this model, where all physical processes of degradation are taken into consideration. Physical processes are superposed in time. Accelerated tests were carried out. Four physical processes were distinguished. Some fuzzy rules were composed, which show when any process is active, and fuzzy model of degradation process was built. In any time, the weight of each process in total failure rate of the element can be found from rules, and total failure rate was found as weighted mean using Sugeno method [11]. The weights of the model vary with time. Fuzzy logic allows introducing our knowledge about physical process of failures by the choice of membership functions. Moreover, some approximate relations, obtained for other similar elements, can be used in the case of lack of experimental data. This idea is more natural then probabilistic. However, large knowledge about the degradation process is necessary to build the model. In lack of this knowledge another solution described below is proposed.

5 Fuzzy transformation of environmental conditions

The results of the reliability test must be transformed to exploitation conditions. It was always a real problem. In transformation (3), or a similar one, there are some constants. None of them are known exactly, so we have large uncertainty of the result obtained. This uncertainty can be easily expressed in terms of fuzzy numbers. In [1] the author of this paper proposed to replace the constants by fuzzy numbers. Thus, fuzzy numbers $\boldsymbol{\lambda_0}$, $\mathbf{n}$, $\mathbf{K_a}$ are introduced in equation (3).

Theorem If $\boldsymbol{\lambda_0}$, $\mathbf{n}$, $\mathbf{K_a}$ are fuzzy numbers with membership functions $\mu_{\lambda o}$, μ_n, μ_{Ka}, then failure rate defined as

$$\boldsymbol{\Lambda} \;=\; \boldsymbol{\lambda_0}\,[1+(\frac{S}{S_b})]^{\mathbf{n}}\exp(\mathbf{K_a}T_a) \qquad (4)$$

is a fuzzy number with the membership function

$$\mu_{\boldsymbol{\Lambda}}(\Lambda) \;=\; \sup_{\lambda_0,\,n\in\mathrm{R}} \min\{\mu_{\lambda_0}(\lambda_0),\,\mu_{\mathbf{n}}(n),$$
$$\mu_{\mathbf{K_a}}[(T_a)^{-1}\ln(\frac{\Lambda}{\lambda_0}[1+(\frac{S}{S_b})^n]^{-1})]\} \qquad (5)$$

The theorem may be easily proved directly from Zadeh's extension principle [13].

If the membership functions for the parameters $\boldsymbol{\lambda_0}$, $\mathbf{n}$, $\mathbf{K_a}$ have triangular shapes, then these memberships give, in an easy way, a center value, and lower and upper limits for possible values of the failure rate.

6 Probability-possibility transformation

The probability model in (2) describes the test results. Stress and environmental conditions transformation is a fuzzy function. How to join both descriptions together? The problem can be solved applying probability-possibility transformation [14] [3] [4]. Sudkamp [10] showed that there is no transformation that preserves basic second order properties, but we are interested in information arising from the test. Klir [4] proposed a transformation, preserving uncertainty in the form of

$$E \;=\; -\sum_{A\in\mathrm{F}} m(\mathbf{A})\log_2 Pl(\mathbf{A}) \qquad (6)$$

introduced before by Yager [12] where $m(\mathbf{A})$ is degree of evidence, so called basic probability assignment [8], $Pl(\mathbf{A})$ is plausibility measure, and F is focal set of fuzzy set $\mathbf{A}$. Probability measures are a special case of plausibility measures if focal sets of $m(\mathbf{A})$ contain only singletons A_k, i.e. only crisp sets. Then $m(A_k) = p(A_k)$, where $p(A_k)$ denotes the probability assigned to the set A_k, and plausibility measure is equal $Pl(A_k) = p(A_k)$. Thus, the measure $E(m)$ becomes conventional entropy - Shannon measure of information [9]

$$H \; = \; - \; \sum_k p\,(A_k) \log_2 p\,(A_k) \qquad (7)$$

which expresses uncertainty related with $\{p(A_k)\}$. Entropy satisfy all Lebegue axioms of measure theory and some others related with properties of information. It can be treated as expected value of uncertainty associated with distribution $p(A_k)$. Moreover, the measure of nonspecifity V in probability assignment $m(\mathbf{A})$ defined as

$$V \; = \; \sum_{\mathbf{A} \in \mathrm{F}(m)} m(\mathbf{A}) \log_2 |\mathbf{A}| \qquad (8)$$

where $|\mathbf{A}|$ denotes the cardinality of the set $\mathbf{A}$, become, in the crisp case, generalization of Hartley measure of information (see [9])

$$I \; = \; \log_2 |\mathbf{A}| \qquad (9)$$

The set A_k can be viewed as the α-cut of the fuzzy set $\mathbf{A}$ defined on space X by membership function $\mu_{\mathbf{A}}(x)$. Nonspecifity can be rewritten in a simple form

$$V \; = \; \sum_k (\mu_k - \mu_{k-1}) \log_2 |A_k| \qquad (10)$$

where μ_k are ordered monotonically $0 \leq \mu_1 \leq \mu_2 \leq \ldots \leq \mu_k \leq \ldots \leq 1$. Resuming, the nonspecifity is possibilistic measure of uncertainty, and Shannon entropy is probabilistic measure of uncertainty.

Now, let us transform probability distribution into possibility distribution preserving this uncertainty putting

$$\mu_k \; = \; \frac{Pr\{\nu(t) \, = \, k\}}{\sup_k Pr\{\nu(t) \, = \, k\}} \qquad (11)$$

Set A_k can be considered as α-cut on the level μ_k and contains integers x_l satisfying

$$A_k = \{x_l \in [0, N] \, | \, \mu(x_l) \geq \mu_k\} \qquad (12)$$

The values A_k and memberships μ_k define fuzzy set - number of possible failures $\mathbf{A}$. Now, it is necessary to fulfill the condition of equal uncertainty $H(\nu) = V(\mathbf{D})$ where $\mathbf{D} = c\mathbf{A}$ is a fuzzy number with similar membership shape as $\mathbf{A}$, $\mu_D(D_k) = \mu_A(D_k/c)$, but modified α-cuts $D_k = cA_k$. Entropy of binomial distribution may be estimated as

$$\hat{H}(\nu) \; = \; -\binom{N}{k} \hat{p}^{\,k} (1-\hat{p})^{N-k} \ln[\binom{N}{k} \hat{p}^{\,k} \cdot \cdot (1-\hat{p})^{N-k}] \qquad (13)$$

Uncertainty of the number $\mathbf{D}$ equals

$$V(\mathbf{D}) = \sum_{k=1}^{N} [\mu_k(D_k) - \mu_{k-1}(D_{k-1})] \log_2 |D_k| \qquad (14)$$

Comparing right sides of (13) and (14) the value of constant c can be calculated numerically by iterations to satisfy equality.

Values S, S_b, n, K_a, T_a are constant during the test. Thus, uncertainty of $\mathbf{D}$ is related only with value $\mathbf{\Lambda}$ and $\lambda_{\mathbf{0}}$, because binomial distribution (2) depends on d and N only. Therefore

$$\mathbf{\Lambda} = \mathbf{D}/N\,t \quad \lambda_{\mathbf{0}} = \mathbf{D}/N\,t_1 \qquad (15)$$

where

$$t_1 \; = \; t\,[1 + (\frac{S}{S_b})^n] \exp(K_a\,T_a) \qquad (16)$$

is recalculated value of the time t resulting from the model (3). The fuzzy number $\lambda_{\mathbf{0}}$ has membership function

$$\mu_{\lambda_{\mathbf{0}}}(\lambda_0) = \mu_{\mathbf{D}}(\lambda_0 N t_1) = \mu_{\mathbf{A}}(\lambda_0 N t_1/c) \qquad (17)$$

Applying fuzzy transformation of the test conditions on exploitation conditions the fuzzy failure rate $\mathbf{\Lambda}$ in exploitation conditions may be found

$$\mathbf{\Lambda} \; = \; \lambda_{\mathbf{0}}\,[1 + (\frac{S}{S_b})]^{\mathbf{n}} \exp(\mathbf{K_a} T_a) \qquad (18)$$

where membership function is described by (5).

Example. Consider a sample of N=100 elements tested in the time t=500h. Two failures (d=2) were observed. Suppose the fuzzy model is described by (4) and fuzzy numbers $\mathbf{n}$, $\mathbf{K_a}$ have triangular membership functions with parameters $\mathbf{K_a} = (4 \cdot 10^{-3}; 5 \cdot 10^{-3}; 6 \cdot 10^{-3})$, $\mathbf{n} = (2.1; 3.0; 3.9)$. Let the value S_b be known and constant S_b=0.4. The membership functions, Fig. 1 and Fig. 2, are calculated from (17) and (5) in exploitation conditions: temperature T_a=308^0K and electrical stress S=0.5.

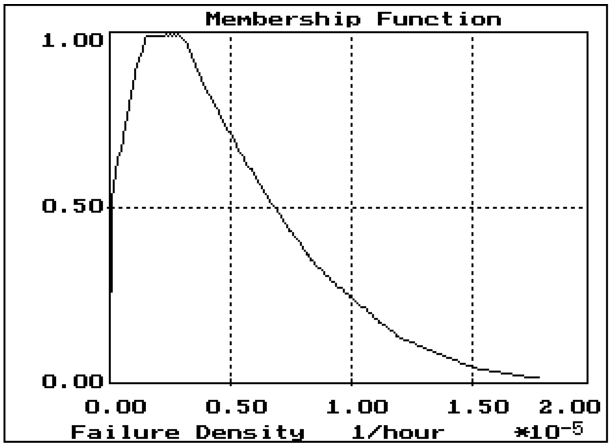

Figure 1: Membership function for λ_0

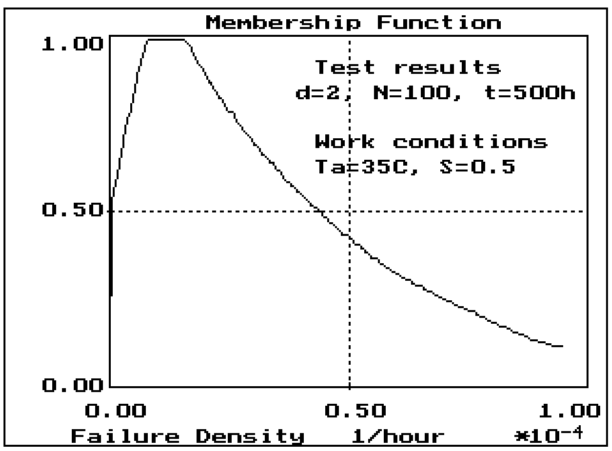

Figure 2: Membership for $\boldsymbol{\Lambda}$ in exploitation conditions

7 Conclusion

Presented method is general and may be applied not only to reliability problems. The method may be used to join probabilistic description with fuzzy description when human experience and some empirical knowledge must complete measurement data. The evidence theory supported by information theory may solve such problems.

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