User-tailored fuzzy relations between intervals

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Abstract

An interactive procedure for generating a user-tailored fuzzy preference relation on intervals is proposed. This procedure allows the decision maker to choose a preference relation which suits best his intuition. The procedure is easy to use (it makes use of graphical representation) and general enough to comprise, as special cases, the intervals proposed in the literature.

Keywords: Interval, fuzzy preference.

1 Introduction

In the literature many different ways of comparing intervals are proposed. Some of them are crisp relations (they suggest an unequivocal answer to the question whether one interval is dominated by another one). Those proposed in [2] and used in many applications, like [1], belong to this class. However, as Sengupta et al. ([7]) point out, they may not be applicable to all pairs of intervals. Therefore other relations are proposed, most of them fuzzy - where "a fuzzy preference relation between intervals" means for us a relation which allows for situations when one interval is preferred to another one to a degree smaller than 1[1]. Relations proposed in [3],[4],[5],[7],[8] belong to this class. There are so many of them, because none of them is really universal and can be applied in all the situations. Sengupta et al. claim that their A-index is such a universal relation, but in our opinion it is not. We would like to suggest an interactive approach in which the decision maker himself would construct the relation which suits him best. This approach is universal in the sense that it might also lead, as a special case, to the construction of the A-index as well as of some other relations already known.

2 Initial information

In the whole paper we assume that the decision maker prefers the "smaller" interval, i.e. we consider "minimization" problems.

Definition 1 ([7]): Let $A = [a_1, a_2]$ and $B = [b_1, b_2]$ be two closed intervals, *m(A)* and *m(B)* their middle points, *w(A)* and *w(B)* the halves if their lengths. We say that A is dominated by B to the degree *A-Index(A≤B)*, where

$$\text{A - Index(A} \le \text{B)} = \frac{m(B) - m(A)}{w(B) + w(A)}$$

This index indicates a positive degree of the dominance of B over A when the middle point of A is less than the middle point of B. If both middle points are equal, the A-index indicates the degree of dominance 0. For this case the authors develop the idea further, by saying that the narrower interval is dominated by the wider one. They adopt here the idea of from [2], according to which an interval with less spread (uncertainty) is generally more attractive to the decision maker, especially when the middle points are equal. And for the case when the index gives a positive value (but less than 1), they differentiate between two subcases:

a) $w(A) \le w(B)$
b) $w(A) > w(B)$

In case a) they assume that A would be chosen anyway (it is to some extent smaller and at the same time not wider than B). In case b) they introduce a fuzzy preference relation between A and B, based on the A-index.

We do not quote here the other fuzzy relations defined in the literature, but in fact their vari-

[1] Some authors understand the term "fuzzy preference relation" in a more strict sense (e.g. [9])

ety may make the decision maker perplex at the choice. That is why we will propose an interactive way of selection the relation by the decision maker himself.

3 Interactive construction of preference relation by the decision maker

As it is often not clear which preference relation best expresses the intuition of the decision maker, we think it would be useful in practice to make use of graphical representation. What is more, we feel that the approach of differentiating cases a) and b) mentioned above is right.

Figure 1 corresponds to case a) (the figure corresponding to case b) would be analogous) and represents a part of the series of situations that might be presented to the decision maker in a graphical form. There might be some more in-between situations needed. A follow-up would be necessary only if the decision maker was rather indulgent and thought that in situation S5 the interval A is still to some positive degree dominated by interval B.

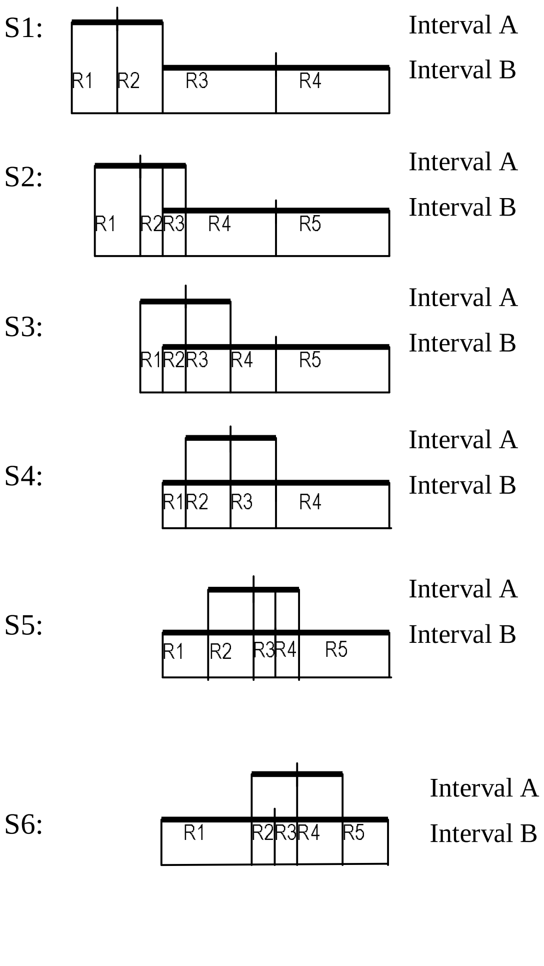

Figure 1: Set of situations presented to the decision maker for case a)

The vertical lines correspond to the end- and middle points of the intervals.

Confronted with the above set of figures, the decision maker will be asked to answer two questions:

I) in which situation he thinks interval A stops being dominated by interval B to the degree 1.

II) in which situation he thinks interval A starts being dominated by interval B to the degree 0.

According to the answer given, a user-tailored preference relation would be constructed. Let us consider the following special case (other special cases can be generated in an analogous way)

i) answer to I): S2, answer to II): S4

Having received this answer, we are justified to assume that region R3 in S2 is not quite accepted by the decision maker and that region R1 in S4 is completely unaccepted by him (these regions are a clear worsening of the corresponding situation with respect to the immediately preceding one). We can then ask the decision maker whether these regions are a problem independently of their relative size or whether there is a threshold over which they lose their acceptance in his eyes. Let us assume that the answer is “there is a threshold up to which I can accept them”. We can then ask whether this threshold is relative to the width of B or of A (or maybe to the sum of both widths). Let us assume that the answer is: with respect to the width of B (equal to $2\,w(B)$). In this case we ask the decision maker to indicate two numbers:

- β_1 - a number from interval (0,1) such that $\beta_1 \cdot 2w(B)$ is the maximal width of R3 in S2 for which A is considered to be dominated by B to degree 1
- β_2 - a number from interval (0,1) such that $\beta_2 \cdot 2w(B)$ is the minimal width of R1 in S4 for which A is considered to be dominated by B to degree 0.

Additional information about upper bounds on $\beta_1 \cdot 2w(B)$ and $\beta_2 \cdot 2w(B)$ would be necessary. We can also assume some reasonable bounds without asking the decision maker, i.e. as equal to *w(A)*. On the basis of this information, we can construct fuzzy preference relation $Pr(A \leq B; S2, S4, \beta_1, \beta_2, 2, 2)$, where parameters *S2* and *S4* refer to the answers given by the decision maker, β_1, β_2 to the parameters and 2 to the fact that both thresholds are defined in relation to the second interval in the expression *A≤B*.

Definition 2: Let λ be any number for interval (0,1). Then:

$Pr(A \leq B; S2, S4, \beta_1, \beta_2, 2, 2) \geq \lambda$

$$\Leftrightarrow \begin{array}{l} a_1 \leq b_1 + \beta_2 \cdot 2w(B) + \\ + \lambda \cdot (\beta_1 \cdot 2w(B) - 2w(A) - \beta_2 \cdot 2w(B)) \end{array}$$

It is easy to check that for λ=1 we deal with situation S2 (for the corresponding threshold size of R3) and for λ=0 - with situation S4 (for the corresponding threshold size of R1).

In an analogous way other user-tailored relations could be generated. Let us notice that this generation scheme includes, as a special case, the A-index proposed in [7]. This index corresponds to relation $Pr(A \leq B; S2, S6, 0, 0, 2, 2)$, where relation $Pr(A \leq B; S2, S6, \beta_1, \beta_2, 2, 2) \geq \lambda$ is defined as follows (we assume that according to the decision maker the troublesome regions - over a certain threshold - are R3 in S2 and R3 in S6 - the last set corresponds to those values in A which are smaller than the middle point of A, but greater than the middle point of B – this is the first of the series of situations presented in which such points exist):

Definition 3: Let λ be any number from interval (0,1). Then:

$Pr(A \leq B; S2, S6, \beta_1, \beta_2, 2, 2) \geq \lambda$

$$\Leftrightarrow$$

$$a_1 \leq b_1 + w(B) + \beta_2 \cdot 2w(B) - w(A)$$
$$+ \lambda \cdot (\beta_1 \cdot 2w(B) - w(A) - w(B) - \beta_2 \cdot 2w(B))$$

Also some other relations defined in the literature are comprised in the scheme proposed. Other relations, e.g. those from [3] an [4], which take into account not only sets which more or less clearly contradict the relation considered (A≤B), like R3 in situation S2 (the relation a≤b is not fulfilled for all couples $a \in A, b \in B$), but also those which clearly act in the favourite direction (like R4 and R5 in S2, where the relation is fulfilled for all such couples) could be comprised in an extended interactive generation scheme, analogous to the one presented here - so that the decision maker would have even more freedom in his choice and could decide that the wider the sets for which the relation a≤b for all the corresponding couples is fulfilled, the higher the preference of A over B is.

4 Example of practical application of the approach

Here we will present one of several domains where the presented approach can be applied and in fact is often applied, although in an informal way.

Let *B* denote the investment budget of a company for the coming period and let *A* (representing various magnitudes $A_1, \ldots, A_n$) stand for the expected expenses corresponding to each of *n* investment projects that are considered for realisation. Both the budget and the expenses are not known precisely yet and are thus given in the form of intervals. The aim is to evaluate the projects (in the scale [0,1] - 0 will stand for "decisively reject", 1 for "decisively accept", the in between values would mean a partially positive evaluation). Here we consider the evaluation only according to one criteria - the possibility of satisfying the budgetary constraint. The same projects would be evaluated also according to other criteria, e.g. their harmony with the strategic goals of the company.

If we used the fuzzy approaches to comparing intervals proposed so far in the literature, there would be "crisp" points delimitating the full acceptance and the full rejection, corresponding to the end- or middle points of the intervals. E.g. full acceptance might mean situation S1 from Figure 1 (possibly with interval A shifted more to the left): the pessimistic scenario of the expenses is less or equal the pessimistic scenario of the budget. On the other hand, full rejection may correspond to situation S4 with region R1 empty (possibly with interval A shifted more to the right): when the optimistic scenario of the expenses is greater or equal the pessimistic scenario of the budget.

Any other preference relation from the literature would correspond in its threshold points (full acceptance and full rejection) to the pessimistic, optimistic or average scenarios of the project expenses and budget.

Let us assume now that the manager of the company wants to know which investment projects are fully able to meet the budgetary constraint, i.e. which projects will meet the budgetary constraint even in case of the worst scenarios of both project expenses and budget. It is only from among those projects that he wants to select those which are important from the strategic point of view.

If the manager has formulated his condition in such a way, the only projects that would be presented to him for further consideration are those which correspond to situation S1 (possibly with *A* shifted more to the left). If a project corresponded to situation S2, even with a very small region *R3*, the manager will never learn about such a project - even if it was very interesting from other points of view.

Such an approach may be relevant in case of a hypothetical company which cannot accept any risk at all. However, such companies do hardly exist in practise. The normal methodology in project management (e.g. [6]) accepts some level of risk, trying to reduce or eliminate its impact by setting aside an adequate financial reserve. This reserve may be a certain percentage of the budget or of the width of interval B - the latter case would correspond to relation $Pr(A \le B; S2, S4, \beta_1, \beta_1, 2, 2)$, where β_1 would correspond to the reserve. Projects evaluated as preferred to the degree one (according to relation $Pr(A \le B; S2, S4, \beta_1, \beta_1, 2, 2)$) to the budget B are then such that would certainly meet the budgetary constraint, but using the reserve if necessary. This class may be considerably wider than that of projects fulfilling to the degree one the original fuzzy preference relation, without taking into account the reserve.

The managers can use the approach having already set the reserve or trying to find out how big the reserve should be in order to allow the company to carry out projects which are important from the strategic point of view (than the decision maker would determine the reserve trying out several values of β_1 and seeing which impact on the choice of projects they have). The reserve works in the same way in the other extreme situation, in which we compare the optimistic scenario of the expenses and the pessimistic scenario of the budget (S4): the optimistic expenses can be a bit greater than the pessimistic budget and the project can still be acceptable to some extent, if this "bit" corresponds to the risk reserve.

Conclusions

We propose an interactive procedure of generating fuzzy preference relation on intervals which would be a user-tailored one. The relation is generated on the basis of the answers of the decision maker given to a few simple questions, based on a graphical representation of various situations two intervals can be in with respect to each other. In such a way also relations known from the literature could be generated, but they constitute only a special case of the proposed general scheme. Using the proposed approach, the decision maker can modify his preference relation, being conscious of his choices. In our opinion he is often not conscious of the choice made when he is simply forced to choose from among one of the relations published in the literature. The results of this paper can be applied anywhere where the problem of comparing interval numbers occurs – and this the case of many management decisions, which concern costs and budgets from the future - not known exactly at the moment.

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