Vehicle Routing and Scheduling with Fuzzy Time Windows and Fuzzy Goal

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Abstract

The **NP**-hard problem of vehicle routing and scheduling problem with fuzzy time windows and fuzzy goal is formulated and two heuristic algorithms for solving this problem are presented.

Keywords: vehicle routing, time windows, fuzzy numbers

1 Introduction

This paper deals with the following problem: For a given set of customers some goods is delivered by a vehicle from a central point further called depot. Every customer demands an amount of goods (or service) fixed earlier. The supply of goods is made by a fleet of K vehicles. Every vehicle takes the goods from the depot, next delivers the goods to selected customers and returns back to the depot, which constitutes a route. The routes cannot violate the capacity constraints on the vehicles and exactly one vehicle can realize the supply of goods. Some preferences concerning the time of delivery (or the start of service) for each customers are given. These preferences, in contrast to *Vehicle routing with time windows* [3, 5, 6] , are modeled by trapezoidal fuzzy numbers [9]. It means that the customer establishes certain time interval within which the start of service satisfies him entirely, i.e. the satisfaction degree equals 1. He admits the possibility of an earlier service start as well of a later one but then the satisfaction degree (represented by the membership function) is less than 1. One should assign a route to each vehicle so that the service of every customer is accomplished while the total working time of the vehicles is minimized. This goal is also represented by a trapezoidal fuzzy number. Now the problem consist of assigning a route and finding the time schedule for each vehicle so that every customer belongs to exactly one route (the service is executed by only one vehicle) while the degree of satisfaction of time windows constraints and goal constraint is maximal. The solution concept of the problem is based on Bellman - Zadeh approach [1]. We stated the considered problem as a mixed integer linear problem and present two heuristic algorithms for solving the problem. Some experimental results demonstrating the effectiveness of the algorithms are also given.

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2 Model of the problem

Let us introduce the following notation and notions: $N = \{1, 2, \ldots, n\}$ - the set of customers for which the service must be done, $N_0 = N \cup \{0\}$ (index 0 indicates the depot, i.e. the point where every route of the vehicle originates and terminates), s_i - service time or dwelling time at the point i-th (customer), $i \in N_0$ (s_0 - the duration time of operations connected to the start of the vehicle route), t_{ij} - travel time from customer i to customer j, $i, j \in N_0$, $\widetilde{TW}_i$ - trapezoidal fuzzy number (a_i, e_i, l_i, b_i) characterizing preferences of customer i about the service time (fuzzy time window). Interval $[e_i, l_j]$ is the set of those times of the customer service start that satisfy the customer to the degree 1 ($\mu_{\widetilde{TW}_i}(x) = 1, \ \text{for } x \in [e_i, l_i]$). On the other hand the customer preferences concerning earlier service starts, i.e. those from interval $[a_i, e_i]$, or later ones, i.e. those from interval $[l_i, b_i]$, are smaller and are expressed respectively by linear functions $\frac{x-e_i}{e_i-a_i}$ and $\frac{l_i-x}{b_i-l_i}$. In this paper we assume that depot has a classical time window $[e_0, l_0]$.

Let us consider a route (further on called also a tour) $T = (0, 1, \ldots, p, 0)$ of a vehicle beginning at the point 0, passing sequentially through points (customers) $1, \ldots, p$ and ending at the depot 0, and the schedule $H = (t_0, t_1, \ldots, t_p, t_{p+1})$ of route T. Value t_0 is the start time of the vehicle route, t_i is the start time of customer service i and t_{p+1} is the finish time of vehicle tour T. Numbers t_i fulfill the following constraints:

$$\begin{aligned} a_0 &\leq t_0, \\ t_i + s_i + t_{ii+1} &\leq t_{i+1}, i = 0, 1, \ldots, p, \\ t_{p+1} &\leq b_0. \end{aligned} \tag{1}$$

For a given schedule H of route T we can define the degree satisfaction of fuzzy time windows, further on denoted by dsftw(H), as follows:

$$\text{dsftw}(H) = \min_{1 \leq i \leq p} \mu_{\widetilde{TW}_i}(t_i).$$

A schedule with the maximal satisfaction degree of the fuzzy time windows will be called optimal and denoted by H^*. We have:

$$\text{dsftw}(H^*) = \max_{H \in \mathcal{H}} \text{dsftw}(H),$$

where $\mathcal{H}$ is the set of all schedules satisfying constraints (1). The optimal schedule is the solution of the following linear programming problem:

$$\begin{array}{rcl} y & \rightarrow & \max, \\ t_0 & \geq & a_0, \\ t_{p+1} & \leq & b_0, \\ a_i + (e_i - a_i) y_i & \leq t_i \leq & b_i - (b_i - l_i) y_i, \; i = 1, \ldots, p \\ y_i & \leq & y. \end{array}$$

Let $t_0^*, \ldots, t_{p+1}^*, y^*$ be the optimal solution of the above problem. It constitutes the optimal schedule H^* for the route T, i.e. such times of successive customers service start that the least degree satisfaction of the fuzzy time window is maximal. Now we introduce some additional data and variables.

K - the number of available vehicles, $M = \{1, \ldots, K\}$,
Q_k - capacity of the vehicle k, $k \in M$,
q_i - demand of customer i, $i \in N$.
$\mu_{\widetilde{G}}(x)$ - goal satisfaction function $(0, 0, g_1, g_2)$ - expresses the decision maker preferences concerning the goal. The total realization time of all routes x that belongs to the interval $[0, g_1]$ satisfies the decision maker to degree 1. On the other hand, a longer time x satisfies him less and the corresponding satisfaction is expressed by function $\mu_{\widetilde{G}}(x) = \frac{g_2-x}{g_2-g_1}$ for $x \in [g_1, g_2]$. The problem considered, that of finding K routes for vehicle in the situation of the existence of fuzzy time windows $(\widetilde{TW}_i)$ and a fuzzy goal $\widetilde{G}$, can be formulated as a mixed integer linear problem as follows:

$$x_{ijk} = \begin{cases} 1 & \text{if customer } j \text{ follows customer } i \\ & \text{in the sequence visited by vehicle } k, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{customer } i \text{ is serviced by vehicle } k, \\ 0 & \text{otherwise.} \end{cases}$$

t_i - start time of the customer service i,
S_k - start time of the vehicle route k,
F_k - finish time of the route of vehicle k,
x_i, y_i - auxiliary variables used for evaluation the degree satisfaction of fuzzy time windows and fuzzy goal respectively $i \in N_0$.

In Figure 1 the integer linear programming model for the considered problem is presented. The model is extension of the formulation given in [2]. The objective function (2) maximizes the degree of satisfaction of all the constraints concerning customers time windows and the total realization time of all tours. Constraint (3) ensure that the vehicle arriving at a point has to leave this point. Constraint (4) states that every vehicle starts and finishes its tour in the depot. Constraint (5) ensures that the service of the customer is done by exactly one vehicle and constraint (6) connects variables x_{ijk} and y_{ik}. The total demand of customers constituting the route of the vehicle cannot violate the vehicle capacity, what states constraint (7). Constraints (8-9) ensure that every tour begins and ends within the time window of the depot. Constraints (10-12) connect variables t_i and x_{ijk} and eliminate all subtours. The degree of satisfaction of time windows constraints of customers is computed by constraints (13-15). Similar meaning for the total working time of the vehicle have constraints (16-19). In this model T denotes a sufficiently large number.

$$y \rightarrow \max, \tag{2}$$
$$\sum_{i\in N_0} x_{irk} - \sum_{j\in N_0} x_{rjk} = 0, \text{ for } r \in N, k \in M, \tag{3}$$
$$\sum_{i\in N} x_{i0k} = \sum_{j\in N} x_{0jk} = 1, \text{ for } k \in M, \tag{4}$$
$$\sum_{k\in M} y_{ik} = 1, \text{ for } i \in N, \tag{5}$$
$$\sum_{j\in N_0} x_{ijk} = y_{ik}, \text{ for } i \in N, k \in M, \tag{6}$$
$$\sum_{i\in N_0}\sum_{j\in N_0} q_i \cdot x_{ijk} \leq Q_k, \text{ for } k \in M, \tag{7}$$
$$S_k \geq e_0, \text{ for } k \in M, \tag{8}$$
$$F_k \leq l_0, \text{ for } k \in M, \tag{9}$$
$$t_i + s_i + t_{ij} - (1 - x_{ijk}) \cdot T \leq t_j, \text{ for } i, j \in N, k \in M, \tag{10}$$
$$S_k + s_0 + t_{0j} - (1 - x_{0jk}) \cdot T \leq t_j, \text{ for } j \in N, k \in M, \tag{11}$$
$$t_i + s_i + t_{i0} - (1 - x_{i0k}) \cdot T \leq F_k, \text{ for } i \in N, k \in M, \tag{12}$$
$$a_i + (e_i - a_i) \cdot y_i \leq t_i, \text{ for } i \in N, \tag{13}$$
$$b_i - (b_i - l_i) \cdot y_i \geq t_i, \text{ for } i \in N, \tag{14}$$
$$y \leq y_i, \text{ for } i \in N, \tag{15}$$
$$\sum_{k\in M} (F_k - S_k) = x_0, \tag{16}$$
$$x_0 \geq 0, \tag{17}$$
$$g_2 - (g_2 - g_1) \cdot y_0 \geq x_0, \tag{18}$$
$$y \leq y_0, \tag{19}$$
$$t_i, y_i, y, y_0, S_k, F_k \geq 0, \text{ for } i \in N, k \in M, \tag{20}$$
$$y_i, y_0, y \leq 1, \text{ for } i \in N. \tag{21}$$

Figure 1: Model of the problem considered.

3 Tours construction heuristics

The model presented in Figure 1 has $O(Kn^2)$ binary variables, where K is the number of vehicles and n is the number of customers. It is impossible to solve this model exactly for realistic size problems. Only small ones could be solved with commercial packages like CPLEX, so two heuristics have been worked out for the formulated problem. They have been implemented in the logic programming language IF/PROLOG with a constraint technology package.

3.1 Representation and operations on tours

We will be denote by $[\ ], [e_1, e_2, \ldots, e_n], L_1||L_2$ and $\textsc{Reverse}(L)$ respectively the empty list, the list of elements $e_1, e_2, \ldots, e_n$, concatenation of two lists and the reverse of list.

The tours are represented by the list of indices of the customers who are served on this tour. The list starts and ends with number 0, which represents depot.

The heuristics algorithms make use of the following two functions:

- $\textsc{Insert}(i, T)$ – creates a new tour by inserting customer i into tour T,
- $\textsc{Merge}(T_1, T_2)$ – creates a new tour by merging two tours T_1 and T_2.

In computational experiments we used the simplest merging. If $T_1 = [0]||L_1||[0]$ and $T_2 = [0]||L_2||[0]$ are two tours, then $\textsc{Merge}(T_1, T_2) = [0]||L||[0]$, where L is one of the following eight lists:

$$\begin{aligned}
&L_1||L_2,\\
&L_2||L_1,\\
&L_1||\textsc{Reverse}(L_2),\\
&\textsc{Reverse}(L_2)||L_1,\\
&\textsc{Reverse}(L_1)||L_2,\\
&L_2||\textsc{Reverse}(L_1),\\
&\textsc{Reverse}(L_1)||\textsc{Reverse}(L_2),\\
&\textsc{Reverse}(L_2)||\textsc{Reverse}(L_1).
\end{aligned}$$

It is possible to investigate other, more complex and more general, operations for merging tours.

3.2 Calculation degree of temporal constraints satisfaction

Let S be the set of tours and let $P(S)$ be the set of customers which are served on tours from set S.

Let $\mathcal{T}(S)$ be the set of all possible schedules for tours from the set S. For the schedule $t \in \mathcal{T}(S)$, t_i denotes the start time of serving customer i and $\textsc{Total}(t)$ is the total time of realization of all tours in S.

We will denote by $\textsc{Sat}_{\widetilde{G}}(S)$ the maximal degree of satisfaction of all temporal constraints, which depends on fuzzy time windows $\widetilde{TW}_i$, for every customer $i \in P(S)$, and on the satisfaction of the goal $\widetilde{G}$ by the value of $\textsc{Total}(S)$:

$$\textsc{Sat}_{\widetilde{G}}(S) = \sup_{t\in\mathcal{T}(S)} \min\{\min_{i\in P(S)} \mu_{\widetilde{TW}_i}(t_i),\ \mu_{\widetilde{G}}(\textsc{Total}(t))\}. \tag{22}$$

It is worth noticing that (22) describes only the degree of satisfaction of all temporal constraints by the partial solution and not the feasibility of it, which depends on the number of customers served and the number of vehicles used.

The value of (22) may be computed with the package for fuzzy constraints [4].

3.3 Construction by insertion

The algorithm starts with the set of tours S_0 which consists of K empty tours $[0,0]$.

In the following iterations $i = 1,2,\dots,n$, there is created a new set of tours S_i by insertion one of the customers who have not been served yet into one of the K tours, from the set S_{i-1}, on one of possible positions. The selection of customer, tour and position is made in the such a way that the value of $\mathrm{SAT}_{\widetilde{G}}(S_i)$ is maximized.

The last set S_n is the solution of the considered problem.

3.4 Construction by merging

Algorithm starts with the set of tours S_0 which consists of n tours $[0,i,0]$, for every customer $i = 1,2,\dots,n$.

In the following iterations $i = 1,2,\dots,n-K$, there is created a new set of tours S_i by replacing two tours from the set S_{i-1} with their merge. The selection of the two tours is made in such a way that the value of $\mathrm{SAT}_{\widetilde{G}}(S_i)$ is maximized.

The last set S_{n-K} is the solution of the considered problem.

4 Computational example

Computations were made on the data set of $n = 10$ customers and $K = 3$ vehicles. They were made on a Pentium II 350MHz computer. Solutions were compared with the optimal ones founded on Pentium III 1GHz by the CPLEX solving the integer linear programming model.

In Figure 2 the course of proposed algorithms is presented in the case when trapezoidal fuzzy goal $\widetilde{G} = (0,0,100,400)$ was considered. The declining curve which starts at point $(0,1)$ corresponds to the course of construction by insertion customers into tours and the second curve corresponds to the course of construction by merging tours.

The algorithm merging tours finds the solution S in 44.72 seconds and $\mathrm{SAT}_{\widetilde{G}}(S) = 40/69 \approx 0.58$. The algorithm inserting customers on tours finds the solution S in 16.01 seconds and $\mathrm{SAT}_{\widetilde{G}}(S) = 96/185 \approx 0.52$. The optimal solution of model in Figure 1, which was found by CPLEX in 149 seconds is 0.6087, indicates that the relative error of inserting algorithm was 14.75% and the relative error of merging algorithm was 4.76%.

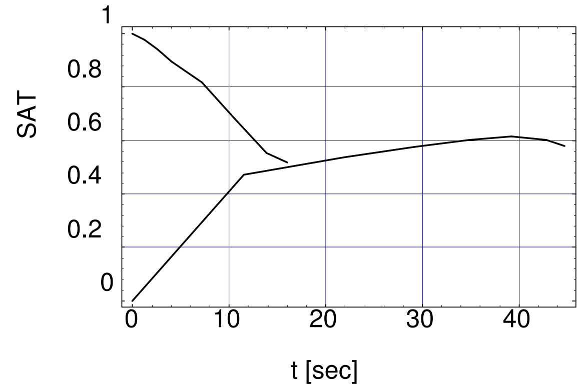

Figure 2: Case $\widetilde{G} = (0,0,100,400)$.

In Figure 3 the course of proposed algorithms is presented in the case when the trapezoidal fuzzy goal $\widetilde{G} = (0,0,100,300)$ was considered.

The algorithm merging tours finds the solution S in 40.72 seconds and $\mathrm{SAT}_{\widetilde{G}}(S) = 3/7 \approx 0.43$. The algorithm inserting customers on tours finds the solution S in 18.86 seconds and $\mathrm{SAT}_{\widetilde{G}}(S) = 88/235 \approx 0.37$. The optimal solution, which was found by CPLEX in 315 seconds is 0.4571, indicates that the relative error of inserting algorithm was 18.07% and the relative error of merging algorithm was 6.24%.

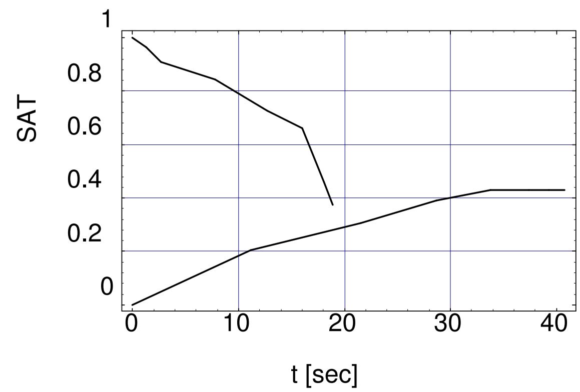

Figure 3: Case $\widetilde{G} = (0,0,100,300)$.

5 Conclusions

The computational example confirms the usefulness of two very simple operations INSERT and MERGE for small problems. For realistic size problems developing better operations (especially MERG) is necessary.

Further studies of more complex operation for merging will continue. It is important to find a compromise between the exponential number of possible results of merging tours and their quality.

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