Two Fuzzy Probability Measures

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Abstract

The paper deals with two methods of a fuzzification of the Borel field of events and too the probability measure. The first approach generalizes the Zadeh definition of a crisp probability of fuzzy event. The second method is based on the Yager definition of a fuzzy probability of fuzzy event. The theoretical results obtained can be applied to modeling stochastic phenomena with uncertain character.

Keywords: fuzzy event, fuzzy σ-algebra, fuzzy probability measure, independent fuzzy events.

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1 Introduction

The basic notion in the theory of probability is a random event (a subset of the basic space), which may or may not occur depending on the implementation of a random experiment. We assume that, for a particular implementation, we can decide whether this event has or has not occurred. However, in practice, this requirement may not be complied with in a simple way. Such events may be suitably interpreted by fuzzy sets [11]. On the other hand, the expression of a probability value itself may be of a vague nature. These inaccurate values may also be described by a fuzzy sets and fuzzy numbers [8].

The following general symbols are employed, as needed, throughout the text: $\mathbb{R}$: the set of all real numbers; $\mathbb{N}$: the set of all natural numbers; $\oplus$, $\ominus$, $\otimes$: the extended arithmetic operations with fuzzy real numbers.

2 Zadeh-type fuzzy probability

The first fuzzification is based on Zadeh´s definition [11] of the probability of fuzzy event $\underset{\sim}{A}$:

$$P\left(\underset{\sim}{A}\right) = \int_{\mathbb{R}^m} \mu_{\underset{\sim}{A}} dP$$

where P is a probability measure, and $\underset{\sim}{A} = \left(\mathbb{R}^m, \mu_{\underset{\sim}{A}}\right)$ is a fuzzy set. In this section, this definition is generalized by means of Zadeh´s extension principle [2].

Definition 2.1. Let $\Omega \neq \emptyset$ be a universal set (***basic space***). A ***fuzzy random event*** is the fuzzy set $\underset{\sim}{A} = \left(\Omega, \mu_{\underset{\sim}{A}}\right)$ with the membership function $\mu_{\underset{\sim}{A}} : \Omega \to [0;1]$. Ω is the ***certain event*** $\left(\mu_{\underset{\sim}{A}} \equiv 1\right)$ and $\emptyset$ is the ***impossible event*** $\left(\mu_{\underset{\sim}{A}} \equiv 0\right)$. A fuzzy random event $\underset{\sim}{A}$ with a Borel measurable membership function $\mu_{\underset{\sim}{A}}$ is called a ***fuzzy event***.

Definition 2.2. A nonempty set Σ of fuzzy events $\underset{\sim}{A} = \left(\Omega, \mu_{\underset{\sim}{A}}\right)$ is called a ***fuzzy Borel field of fuzzy events*** over the universal set Ω if Σ has the following properties [4]:

1. $\mu_{\underset{\sim}{A}}(\omega) = \alpha, \forall \omega \in \Omega, \forall \alpha \in [0;1] \Rightarrow \underset{\sim}{A} \in \Sigma$.
2. $\underset{\sim}{A} \in \Sigma \Rightarrow \overline{\underset{\sim}{A}} \in \Sigma$.
3. $\underset{\sim}{A}_1, \underset{\sim}{A}_2, \ldots \in \Sigma \Rightarrow \bigcup_{i=1}^{\infty} \underset{\sim}{A}_i \in \Sigma$.
4. $\underset{\sim}{A}_1, \underset{\sim}{A}_2 \in \Sigma \Rightarrow \underset{\sim}{A}_1 \cdot \underset{\sim}{A}_2 \in \Sigma$.

Definition 2.3. Let $\Omega = \mathbb{R}^m$ where $m \in \mathbb{N}$ be a universal set, $\mathit{\Sigma}$ a crisp Borel field of events over Ω, Π a nonempty set of probability measures P on $(\Omega, \mathit{\Sigma})$, Σ a fuzzy Borel field of fuzzy events on

Ω, and $\underset{\sim}{A} = \left(\Omega, \mu_{\underset{\sim}{A}}\right) \in \Sigma$ a fuzzy event. Let, further, $\underset{\sim}{P} = \left(\Pi, \mu_{\underset{\sim}{P}}\right)$ be such a fuzzy bunch on Π that $\exists P \in \Pi$ and $\mu_{\underset{\sim}{P}}(P) = 1$. Then the fuzzy bunch $\underset{\sim}{P}$ is called a ***fuzzy probability measure*** on Ω and the ***fuzzy probability of a fuzzy event*** $\underset{\sim}{A}$ is the fuzzy set $\underset{\sim}{P}(\underset{\sim}{A}) = \left([0;1], \mu_{\underset{\sim}{P}(\underset{\sim}{A})}\right)$ where

$$\mu_{\underset{\sim}{P}(\underset{\sim}{A})}(p) = \sup_{\substack{P\in\Pi \\ \int_{\Omega} \mu_{\underset{\sim}{A}} dP = p}} \mu_{\underset{\sim}{P}}(P)$$

for $\forall p \in [0;1]$. If no measure $P \in \Pi$ exists such that $\int_{\Omega} \mu_{\underset{\sim}{A}}\, dP = p$, we put $\mu_{\underset{\sim}{P}(\underset{\sim}{A})}(p) = 0$. The triplet $\left(\mathbb{R}^m, \Sigma, \underset{\sim}{P}\right)$ is called a ***fuzzy probability space*** on $\mathbb{R}^m$.

Theorem 2.1. For any fuzzy event $\underset{\sim}{A} \in \Sigma$, we have:

a) $\underset{\sim}{P}(\varnothing) = \{0\}, \underset{\sim}{P}(\Omega) = \{1\}$,

b) $\underset{\sim}{P}\left(\overline{\underset{\sim}{A}}\right) = \{1\} \ominus \underset{\sim}{P}(\underset{\sim}{A})$ for $\forall \underset{\sim}{A} \in \Sigma$,

c) $\mu_{\underset{\sim}{P}(\overline{\underset{\sim}{A}})}(p) = \mu_{\underset{\sim}{P}(\underset{\sim}{A})}(1-p)$ for $\forall p \in [0;1]$,

d) $\underset{\sim}{P}(\underset{\sim}{A})$ fuzzy number $\Rightarrow \underset{\sim}{P}\left(\overline{\underset{\sim}{A}}\right)$ fuzzy number.

Theorem 2.2. For any set of fuzzy events $\underset{\sim}{A}_i \in \Sigma$, $i = 1,\ldots,n$, we have:

a) $\underset{\sim}{P}\left(\bigcup_{i=1}^{n} \underset{\sim}{A}_i\right) = \{1\} \ominus \underset{\sim}{P}\left(\bigcap_{i=1}^{n} \overline{\underset{\sim}{A}}_i\right)$,

b) $\underset{\sim}{P}\left(\sum_{i=1}^{n} \underset{\sim}{A}_i\right) = \{1\} \ominus \underset{\sim}{P}\left(\prod_{i=1}^{n} \overline{\underset{\sim}{A}}_i\right)$.

Definition 2.4. Fuzzy events $\underset{\sim}{A}, \underset{\sim}{B} \in \Sigma$ are ***independent*** if, for $\forall P \in \Pi$,

$$\int_{\Omega} \mu_{\underset{\sim}{A}} \mu_{\underset{\sim}{B}} dP = \int_{\Omega} \mu_{\underset{\sim}{A}} dP \cdot \int_{\Omega} \mu_{\underset{\sim}{B}} dP .$$

Fuzzy events $\underset{\sim}{A}_i \in \Sigma$, $i = 1,\ldots,n$, are ***mutually independent*** if, for $\forall \{i_1,\ldots,i_k\} \subseteq \{1,\ldots,n\}$, $\forall P \in \Pi$,

$$\int_{\Omega} \prod_{j=1}^{k} \mu_{\underset{\sim}{A}_{i_j}} dP = \prod_{j=1}^{k} \int_{\Omega} \mu_{\underset{\sim}{A}_{i_j}} dP .$$

Theorem 2.3. If fuzzy events $\underset{\sim}{A}, \underset{\sim}{B} \in \Sigma$ are independent, then the fuzzy events $\underset{\sim}{A}, \overline{\underset{\sim}{B}}$; $\overline{\underset{\sim}{A}}, \underset{\sim}{B}$; $\overline{\underset{\sim}{A}}, \overline{\underset{\sim}{B}}$ are also independent.

Theorem 2.4. For any set of mutually independent fuzzy events $\underset{\sim}{A}_i \in \Sigma$, $i = 1,\ldots,n$, we have:

a) $\underset{\sim}{P}\left(\prod_{i=1}^{n} \underset{\sim}{A}_i\right) = \bigotimes_{i=1}^{n} \underset{\sim}{P}(\underset{\sim}{A}_i)$,

b) $\underset{\sim}{P}\left(\sum_{i=1}^{n} \underset{\sim}{A}_i\right) = \{1\} \ominus \left(\bigotimes_{i=1}^{n} \left[\{1\} \ominus \underset{\sim}{P}(\underset{\sim}{A}_i)\right]\right)$.

3 Yager-type fuzzy probability

The second fuzzification is based on Yager´s definition [10] of the fuzzy probability of fuzzy event $\underset{\sim}{A}$:

$$\underset{\sim}{P}(\underset{\sim}{A}) = \bigcup_{\alpha\in[0;1]} \alpha\{P(A_\alpha)\}$$

where P is a probability measure and A_α is the α–cut of fuzzy set $\underset{\sim}{A}$. In this section, this definition is generalized and the necessary fuzzy structures are described [9]. Let U be a universal set, $\mathcal{F}(U)$ the system of all fuzzy sets on U.

Definition 3.1. The system of fuzzy sets $\tilde{M} \subseteq \mathcal{F}(U)$ is called a ***fuzzy set ring*** if, for $\forall \underset{\sim}{A}_i \in \tilde{M}$, $i \in \mathbb{N}$, $\tilde{M}$ has the following properties:

1. $\underset{\sim}{A}_i \cup \underset{\sim}{A}_j \in \tilde{M}$.
2. $\underset{\sim}{A}_i - \underset{\sim}{A}_j \in \tilde{M}$.
3. $\bigcup_{i\in\mathbb{N}} \underset{\sim}{A}_i \in \tilde{M}$.

Definition 3.2. The system of fuzzy sets $\tilde{M} \subseteq \mathcal{F}(U)$ is called a ***fuzzy set σ–algebra*** if, for $\forall \underset{\sim}{A}_i \in \tilde{M}$, $i \in \mathbb{N}$, $\tilde{M}$ has the following properties:

1. $\underset{\sim}{A}_i \cup \underset{\sim}{A}_j \in \tilde{M}$.
2. $U - \underset{\sim}{A}_i \in \tilde{M}$.
3. $\bigcup_{i\in\mathbb{N}} \underset{\sim}{A}_i \in \tilde{M}$.

Definition 3.3. Let a fuzzy set $\underset{\sim}{A} \in \mathcal{F}(U)$ have the finite support $supp\ \underset{\sim}{A}$. Let U be a complete ordered set with $\le$ as the ordering relation. The fuzzy set $\underset{\sim}{A}$ is called a ***quasi-convex*** if α–cut A_α has, for $\forall \alpha \in (0;1]$, the following property: if $x_1, x_2 \in A_\alpha$, $x_1 \le x_2$, $x_1 \ne x_2$, then, for $\forall y \in supp\ \underset{\sim}{A}$ and $x_1 \le y \le x_2$, we have $y \in A_\alpha$. A convex or quasi-convex fuzzy set $\underset{\sim}{A}$ is called a ***pseudo-convex*** fuzzy set.

The reason for introducing the term quasi-convex is the demand for proper definition of convexity on a discrete universe. We need this for random variables with discrete distribution laws.

Definition 3.4. The normal and pseudo-convex fuzzy set $\underset{\sim}{a} = \left(\mathbb{R}, \mu_{\underset{\sim}{a}}\right)$ is called a ***generalized fuzzy number***. The set of all generalized fuzzy numbers we denote by $\mathcal{A}^*$.

Definition 3.5. Let $\tilde{M} \subseteq \mathcal{F}(U)$ be a fuzzy set σ–ring on U. A fuzzy set function $\underset{\sim}{\lambda} : \tilde{M} \to \mathcal{A}^*$ is called a ***fuzzy measure*** on $\tilde{M}$ if $\underset{\sim}{\lambda}$ has the following properties:

1. $supp\underset{\sim}{\lambda}\left(\underset{\sim}{A}\right) \subseteq \mathbb{R}_+$ for $\forall \underset{\sim}{A} \in \tilde{M}$.
2. For $\forall \underset{\sim}{A} \in \tilde{M}$ where $i \in \mathbb{N}$
$$\underset{\sim}{\lambda}\left(\bigcup_{i\in\mathbb{N}} \underset{\sim}{A}_i\right) = \bigcup_{i\in\mathbb{N}} \underset{\sim}{\lambda}\left(\underset{\sim}{A}_i\right).$$
3. $\underset{\sim}{\lambda}(\varnothing) = \{0\}$.

Let $\tilde{M}$ be a fuzzy set σ–algebra on U. A finite fuzzy measure $\underset{\sim}{P}$ such that $\underset{\sim}{P}(U) = \{1\}$ is called a ***fuzzy probability measure***.

Theorem 3.1. Let M be a set σ–algebra on U, $\alpha(M) = \left\{0 = \alpha_0 < \alpha_1 < \cdots < \alpha_n \mid \alpha_i \in (0;1)\right\}$ a finite set of real numbers with the following property: $\alpha \in \alpha(M) \Rightarrow (1-\alpha) \in \alpha(M)$. Let $\tilde{A}$ be a sequence of sets $\tilde{A} = \left\{A(i) \in M \mid A(i) \subseteq A(i-1); i \in \{1,\ldots,n\}\right\}$. Then the family of fuzzy sets
$$\tilde{M} = \left\{\underset{\sim}{A} \mid A_{\alpha_i} = A(i); A(i) \in \tilde{A}\right\}$$
generated by the system of sequences $\tilde{A}$ is a fuzzy set σ–algebra. We say that the fuzzy set σ–algebra $\tilde{M}$ is ***generated*** by the set σ–algebra M.

Now we can ask if and when we are able to generate a crisp set σ–algebra from a fuzzy set σ–algebra. One interesting class of fuzzy set σ–algebras that are able to generate a crisp σ–algebra is given by the following definition.

Definition 3.6. Let $\underset{\sim}{A} \in \mathcal{F}(U)$ be a fuzzy set. $\underset{\sim}{A}$ is called a **step fuzzy set**, if the set
$$\alpha\left(\underset{\sim}{A}\right) = \left\{\alpha \in [0;1] \mid \exists x \in U; \mu_{\underset{\sim}{A}}(x) = \alpha\right\},$$
is finite. The set $\alpha\left(\underset{\sim}{A}\right)$ is called a **set of membership degrees** of the fuzzy set $\underset{\sim}{A}$.

Definition 3.7. A family of step fuzzy sets $\tilde{M}$ is called **complete** if has the following properties:

1. The set $\alpha\left(\tilde{M}\right) = \bigcup_{\underset{\sim}{A}\in\tilde{M}} \alpha\left(\underset{\sim}{A}\right)$ is finite.
2. If $\underset{\sim}{A} \in \tilde{M}$ and $\underset{\sim}{B}$ are fuzzy sets such that, for $\forall \alpha_i \in \alpha\left(\underset{\sim}{A}\right), \alpha_i > 0$, we have $A_{\alpha_i} = B_{\alpha_{i+1}}$, then $\underset{\sim}{B} \in \tilde{M}$.
3. If $\underset{\sim}{A} \in \tilde{M}$ and $\underset{\sim}{B}$ are fuzzy sets such that, for $\forall \alpha_i \in \alpha\left(\underset{\sim}{A}\right), \alpha_i > 0$, we have $A_{\alpha_{i+1}} = B_{\alpha_i}$, then $\underset{\sim}{B} \in \tilde{M}$.

Theorem 3.2. Let $\tilde{M}$ be a complete fuzzy set σ–algebra on U. Then the set
$$M = \left\{A \subseteq U \mid \exists \underset{\sim}{A} \in \tilde{M}, \alpha \in \alpha\left(\tilde{M}\right); A = A_\alpha\right\}$$
is a set σ–algebra.

Theorem 3.3. Let $\tilde{M}$ be the fuzzy set σ–algebra generated by a set σ–algebra M, P probability measure on M. The fuzzy set function $\underset{\sim}{P} : \tilde{M} \to \mathcal{A}^*$ where
$$\mu_{\underset{\sim}{P}\left(\underset{\sim}{A}\right)}(p) = \sup_{P(A_\alpha)=p} \left\{\alpha \mid \alpha \in [0;1]\right\}$$
is a fuzzy probability measure. We say that $\underset{\sim}{P}$ is ***generated*** by P.

Definition 3.8. Let (U, M, P) be a probability space, $\tilde{M}$ the fuzzy set σ–algebra generated by the set σ–algebra M, $\underset{\sim}{P}$ the fuzzy probability measure on $\tilde{M}$ generated by the probability measure P. The triplet $\left(U, \tilde{M}, \underset{\sim}{P}\right)$ is a ***fuzzy probability space***, and $\left(U, \tilde{M}, \underset{\sim}{P}\right)$ is ***generated*** by the probability space (U, M, P). A fuzzy set $\underset{\sim}{A} \in \tilde{M}$ is called a ***fuzzy random event***. A fuzzy number $\underset{\sim}{P}\left(\underset{\sim}{A}\right)$ is called a ***fuzzy probability of the fuzzy random event*** $\underset{\sim}{A}$.

Definition 3.9. Let $\underset{\sim}{A}$ be a fuzzy set on a universal set U, $\mu_{\underset{\sim}{A}}$ the membership function. A function $\mu^*_{\underset{\sim}{A}} : [0;1] \to U$ (if it exists) where $\mu^*_{\underset{\sim}{A}}(\alpha) = x$, iff $\mu_{\underset{\sim}{A}}(x) = \alpha$, is called a ***quasi-inverse membership function*** of the fuzzy set $\underset{\sim}{A}$.

Theorem 3.4. For $\forall \underset{\sim}{A}, \underset{\sim}{B} \in \tilde{M}$, $\underset{\sim}{A} \subseteq \underset{\sim}{B}$, $\forall \alpha \in [0;1]$, we have $\mu^*_{\underset{\sim}{P}\left(\underset{\sim}{A}\right)}(\alpha) \leq \mu^*_{\underset{\sim}{P}\left(\underset{\sim}{B}\right)}(\alpha)$.

Theorem 3.5. For $\forall \underset{\sim}{A} \in \tilde{M}$, $\forall \alpha \in [0;1]$, we have:

a) $\mu^*_{\underset{\sim}{P}\left(\underset{\sim}{A}\right)}(\alpha) = P(A_\alpha)$,

b) $\mu_{\underset{\sim}{P}(\underset{\sim}{A})}(p) = \sup\limits_{\mu^*_{\underset{\sim}{P}(\underset{\sim}{A})}(\alpha)=p} \alpha$.

Definition 3.10. Fuzzy random events $\underset{\sim}{A}, \underset{\sim}{B} \in \tilde{M}$ are ***independent*** if random events A_α, B_β are independent for $\forall \alpha, \beta \in [0;1]$. Fuzzy random events $\underset{\sim}{A}_i \in \tilde{M}$, $i = 1, \ldots, n$, are ***mutually independent*** if random events $A_{i\alpha_i}$ are mutually independent for $\forall \alpha_i \in$ $\in [0;1]$, $i = 1, \ldots, n$.

Theorem 3.6. If fuzzy random events $\underset{\sim}{A}, \underset{\sim}{B} \in \tilde{M}$ where $\tilde{M}$ is a complete fuzzy set σ–algebra are independent, then the fuzzy random events $\underset{\sim}{A}, \overline{\underset{\sim}{B}}$; $\overline{\underset{\sim}{A}}, \underset{\sim}{B}$; $\overline{\underset{\sim}{A}}, \overline{\underset{\sim}{B}}$ are also independent.

Theorem 3.7. For any two independent fuzzy random events $\underset{\sim}{A}, \underset{\sim}{B} \in \tilde{M}$ and for any set of mutually independent fuzzy random events $\underset{\sim}{A}_i \in \tilde{M}$, $i = 1, \ldots, n$, we have:

a) $\underset{\sim}{P}(\underset{\sim}{A} \cap \underset{\sim}{B}) = \underset{\sim}{P}(\underset{\sim}{A}) \otimes \underset{\sim}{P}(\underset{\sim}{B})$,

b) $\underset{\sim}{P}\left(\bigcap\limits_{i=1}^{n} \underset{\sim}{A}_i\right) = \bigotimes\limits_{i=1}^{n} \underset{\sim}{P}(\underset{\sim}{A}_i)$.

4 Examples

Example 4.1. (*Zadeh-type fuzzy probability*)

We have a fuzzy probability (fuzzy bunch) $\underset{\sim}{P}$ on the universal set $\Pi = \{P_1, P_2, P_3, P_4\}$ with the membership function:

P_i	P_1	P_2	P_3	P_4
$\mu_{\underset{\sim}{P}}(P_i)$	0.6	0.8	1.0	0.5

where probability measures P_i for $i = 1,2,3,4$ on the basic space $\Omega = \{1,2,3,4,5\}$ are given:

ω_j P_i	1	2	3	4	5
P_1	0.20	0.20	0.20	0.20	0.20
P_2	0.30	0.25	0.20	0.15	0.10
P_3	0.10	0.15	0.20	0.25	0.30
P_4	0.28	0.18	0.08	0.18	0.28

Let fuzzy events $\underset{\sim}{A}_k$ for $k = 1,2,3$ have the membership functions:

ω_j $\underset{\sim}{A}_k$	1	2	3	4	5
$\underset{\sim}{A}_1$	1.0	0.5	0.0	0.0	0.0
$\underset{\sim}{A}_2$	0.0	0.5	1.0	0.5	0.0
$\underset{\sim}{A}_3$	0.0	0.0	0.0	0.5	1.0

The calculated fuzzy probabilities of fuzzy events are:

i	$\mu_{\underset{\sim}{P}}(P_i)$	$P_i(\underset{\sim}{A}_1)$	$P_i(\underset{\sim}{A}_2)$	$P_i(\underset{\sim}{A}_3)$
1	0.6	0.300	0.400	0.300
2	0.8	0.425	0.400	0.175
3	1.0	0.175	0.400	0.425
4	0.5	0.370	0.260	0.370

A plot of the membership function of fuzzy probability $\underset{\sim}{P}(\underset{\sim}{A}_1)$ is shown in Fig. 4.1.

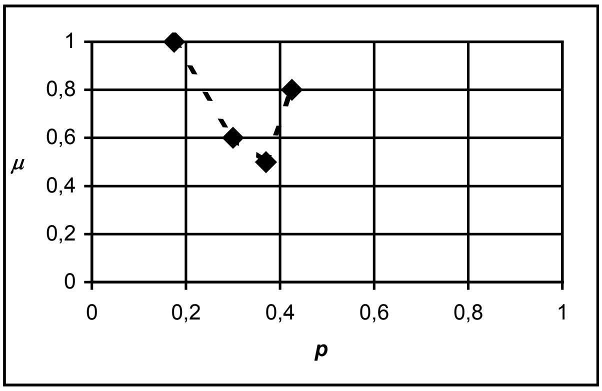

Fig. 4.1

A plot of the membership function of fuzzy probability $\underset{\sim}{P}(\underset{\sim}{A}_1)$ is shown in Fig. 4.2.

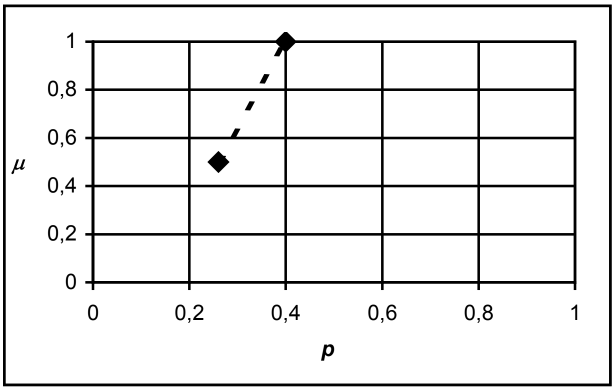

Fig. 4.2

A plot of the membership function of fuzzy probability $\underset{\sim}{P}\left(\underset{\sim}{A}_1\right)$ is shown in Fig. 4.3.

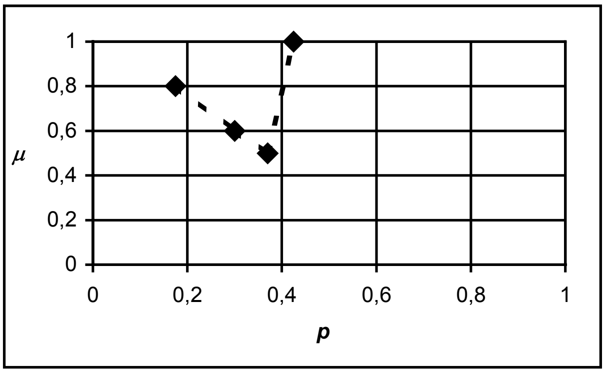

Fig. 4.3

Example 4.2. (*Yager-type fuzzy probability*)

Suppose a fair coin is flipped ten times. Let A_i be the random event "head faces up i-times", $i = 0,...,10$. The random event A_i has the probability

$$P\left(A_i\right) = \binom{10}{i} 2^{-10}$$

from the binomial distribution Bi(10;1/2):

A_i	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
P	2^{-10}	$10(2^{-10})$	$45(2^{-10})$	$120(2^{-10})$	$210(2^{-10})$	$252(2^{-10})$	$210(2^{-10})$	$120(2^{-10})$	$45(2^{-10})$	$10(2^{-10})$	2^{-10}

Let fuzzy random events:

$\underset{\sim}{B}$: "head faces up seldom",
$\underset{\sim}{C}$: "head faces up sometimes",
$\underset{\sim}{D}$: "head faces up many times",

have the membership functions on the universal set $U = \left\{A_0,...,A_{10}\right\}$:

A_i μ	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
$\mu_{\underset{\sim}{B}}$	1	1	0.9	0.7	0.4	0	0	0	0	0	0
$\mu_{\underset{\sim}{C}}$	1	1	1	0.9	0.6	0.2	0	0	0	0	0
$\mu_{\underset{\sim}{D}}$	0	0	0	0	0	0	0	0.3	0.8	1	1

The calculated fuzzy probabilities $\underset{\sim}{P}\left(\underset{\sim}{B}\right)$, $\underset{\sim}{P}\left(\underset{\sim}{C}\right)$, and $\underset{\sim}{P}\left(\underset{\sim}{D}\right)$ have the membership functions:

p	$11(2^{-10})$	$56(2^{-10})$	$176(2^{-10})$	$386(2^{-10})$
$\mu_{\underset{\sim}{P}(\underset{\sim}{B})}$	1	0.9	0.7	0.4
p	$56(2^{-10})$	$176(2^{-10})$	$386(2^{-10})$	$638(2^{-10})$
$\mu_{\underset{\sim}{P}(\underset{\sim}{C})}$	1	0.9	0.6	0.2
p	$11(2^{-10})$	$56(2^{-10})$	$176(2^{-10})$	
$\mu_{\underset{\sim}{P}(\underset{\sim}{D})}$	1	0.8	0.3	

The membership functions of the fuzzy probabilities $\underset{\sim}{P}\left(\underset{\sim}{B}\right)$, $\underset{\sim}{P}\left(\underset{\sim}{C}\right)$ and $\underset{\sim}{P}\left(\underset{\sim}{D}\right)$ are plotted in Fig. 4.4.

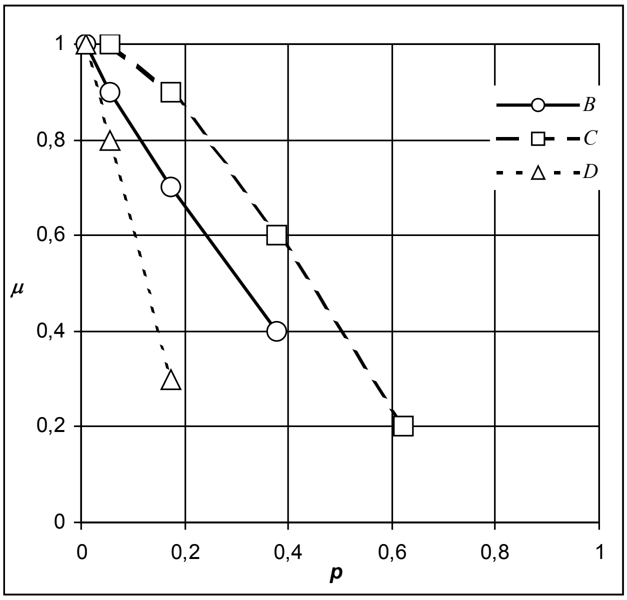

Fig. 4.4

5 Conclusion

We have presented two different fuzzy probability models. As compared to the second model, the first one can also deal with possible uncertainty precarious information about the observed probability distribution. The first fuzzification is based on the expected value of the membership function of a fuzzy event with respect to the fuzzy bunch of probability measures [2]. The second fuzzification, on the contrary, assumes only one probability measure and its concept is nearer the classical theory of probability. The first model is relatively flexible and has already been implemented on a PC to calculate reliability [3, 4]. The second model is intensively examined [9] and we expect its major application to reliability calculation as well.

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