Planning time restricted logistic tours with fuzzy logic

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Abstract

We present a modification of the vehicle routing problem with the additional restriction that time windows are associated with each customer by defining an interval wherein the customer has to be supplied. We introduce a two phase strategy of this vehicle routing and scheduling problem with time windows using fuzzy logic to realize inference steps in fuzzy if-then rule bases.

Keywords: time window, planning, fuzzy logic.

1 Introduction

The topic of this paper is a variant of the vehicle routing problem with time windows applying a two-phase method with fuzzy logic. The vehicle routing problem (VRP) with time windows (VRPTW) is one of the most common problems in contemporary operations research. The goal of the problem is calculating an optimal route for delivery of packages to customers who have specified when they will be available to receive their packages. VRP has a historical and theoretical background in the traveling salesman problem (TSP). By given a finite number of nodes and the cost of the travel between them we must find the cheapest way to visit all the nodes and return to the starting one. There are many other types of problems found in VRP such as route selection for different fleet of vehicles or VRP with time windows. The objective of VRPTW [1] is to minimize the vehicle fleet and the sum of travel time and waiting time needed to supply all customers in their required hours. Based on the vehicle routing problem we can characterize VRPTW by the following restrictions [2]:

- A solution becomes infeasible if a customer is supplied after the upper bound of its time window.
- A vehicle arriving before the lower limit of the time window causes additional waiting time on the route.
- Each route must start and end within the time window associated with the depot.
- In the case of soft time windows, a later service does not effect the feasibility of the solution, but is penalized by adding a value to the objective function.

From a graph theoretical point of view the VRPTW may be stated as follows. Let $G=(V,A)$ be an undirected graph with node set $V = V_N \cup \{v_0\}$ and the arc set A, where $V_N = \{v_i \in V \mid i = 1,\dots,n\}$, stands for customer nodes and v_0 stands for the central depot, where all routes start and end. Each node $v_i \in V$ has an associated demand q_i, service time s_i, service time window $[e_i, l_i]$ and an ordered pair of coordinates (x_i, y_i). Based on the geographical coordinates, it is possible to calculate the distance d_{ij} between every two distinct nodes v_i and v_j, and the corresponding travel time t_{ij}. Under the time window constraints there may or may not be a transition or arc between some node pairs.

Hence, the set of arcs can be defined as follows:

$$A = \{(v_i, v_j | v_i, v_j \in V \wedge t_{0i} + s_i + t_{ij} \le l_j\} \qquad (1)$$

If the vehicle reaches the customer v_i before e_i, a waiting time occurs. The route's schedule time is the sum of the travel time, waiting time and the service time. The objective is to service all customers while minimizing the number of vehicles, travel distance, schedule time and waiting time without violating vehicles capacity constraints and the customers time window.

2 VRPTW using fuzzy logic

Many researchers had suggested solutions to VRP in different areas such operations research, artificial intelligence or decision support systems. There are different contributions like simple heuristics based on local search and sweep developed mostly in the '60 and '70. Another approach is mathematical programming based heuristic, which approximate VRP with generalized assignment and set partitioning problems. Into the category of exact optimization belong *k-tree* and *lagrangean relaxation*. In the area of artificial intelligence, genetic algorithm is used for solving VRP such as Gideon[6]. Another artificial intelligence methods are simulated annealing as generalization of the Monte Carlo method, tabu search as a meta-heuristic, ant system as a distributed meta-heuristic for solving combinatorial optimization problems or fuzzy-neural approaches for instance proposed by [7] to represent the correlation of the attributes with driver's route selection.

Here we will introduce a two-phase method using fuzzy logic. First, a strategy must evaluated to determine critical appointments kept by the customers v_i. The algorithm should avoid to fail to meet the deadlines. Then, an insertion step is applied to this first-step solution. Based on the current location and time we can calculate an interstation as intermediate node to avoid a waiting time, if the vehicle reaches the customer v_k before e_k. For instance, the vehicle can supply customer v_j before v_k on the route, if arc (v_i,v_k) having the travel distance time t_{ik} is supposed. We can distinguish the time window $[e_i,l_i]$ in four successive time periods:

- phase 1 (start phase)
  The customer can probably supply without guarantee. It is only possible to start the service.
- phase 2 (active phase)
  The active phase state that the customer can unrestricted supply with guarantee.
- phase 3 (abandonment phase)
  The customer can probably supply without guarantee. This pahse seems similar to the start phase with inverse probability values.
- phase 4 (inactive phase)
  This phase causes no supply guarantee. The possibility degree is zero.

Use trapezoid fuzzy sets for example labeled *TW* to model this time frame in the following sense:

- $TW(t_{active}) = 1$
- $TW(t_{inactive}) = 0$
- Let $t_{start1} < t_{start2}$ and $t_{abandon1} < t_{abandon2}$, then also $0 < t_{start1} < t_{start2} < 1$ *and*
  $1 > t_{abandon1} > t_{abandon2} > 0$

The planning tour algorithm follows a sequential approach by planning each tour itself. A combination of the nearest neighbourhood heuristic in calculating the next node or location based on the current location and time and a specific version of the insertion algorithm by inserting new intermediate nodes on the route to avoid a waiting time is obvious. Figure 1 demonstrate the application flow. The first question is how to calculate a critical important appointment starting at depot v_o. It is important to keep all appointments on the recommended route. Different strategies to detect the urgency keep an appointment can be stated. Based on the time window separated in four successive periods strategies can be stated as follows. Our approach to supply all customers in keeping critical appointments within the service time window emphasize prefer appointments only reachable within the abandonment period (phase 3) from the current location. We can calculate the adapted location and time parameters by adding the travel time, waiting time and service time. On the other hand postpone appointments with a greater and obviously larger active period. Keep the conditions by smaller boundaries. Prefer customers located in the neighbourhood having a limited distance value. Prefer also customers having a shorter active phase starting soon to avoid to fail to meet the deadline. We transform the strategies realized in the first algorithm phase by using if-then statements. Later we discuss transformation steps to receive fuzzy if-then rule bases which perform the discussed strategies.

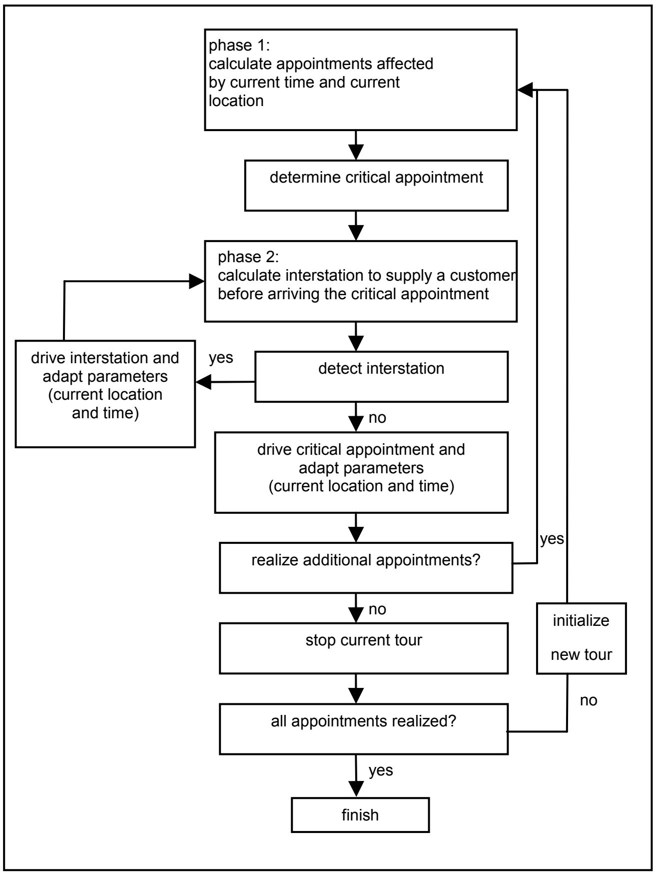

Figure 1: planning tour algorithm

The modeling steps within the first phase described explicitly above can be stated as follows:

- (1.1) IF *time window on arrival within abandonment stage* THEN *highest priority*
- (1.2) IF *time window on arrival active* AND *time on arrival within the active phase* THEN *very high priority*
- (1.3) IF *customer distance nearby* THEN *less very high priority*
- (1.4) IF *time windows starts soon* AND *time window short* THEN *high priority*
- (1.5) IF *time window on arrival within start phase* THEN *middle priority*
- (1.6) IF *time window late active* THEN *low priority*

The planning tour algorithm consists as two-phase method. The second phase calculate intermediate nodes to improve the efficiency.

A human decision maker prefer keeping an appointment “on the way” for instance from his home location to the office bying a journal at the kiosk. Let *A* denote the current location, *b* and *c* the distance between the routes from *A* to *B* and the distance from *A* to *C* illustrated in detail in figure 2 below.

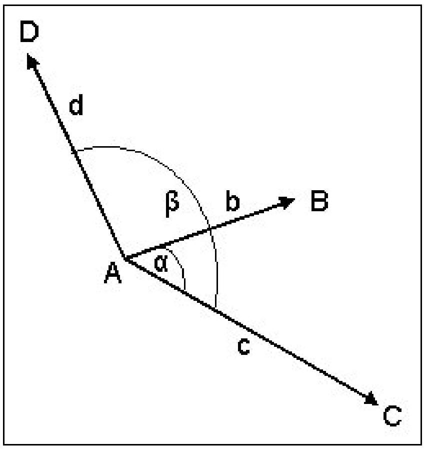

Figure 2: determination of intermediate nodes

The following criteria can be used for the evaluation step:

- The measurable angle α between the routes *b* and *c* should not exceed a greater value like 80 degree to avoid driving in different directions from *A* to *C* taking in between node *B*.
- The length of distance *b* should be shorter than the length of distance *c*.
- The additional driving time to keep an appointment at node *B* is limited to avoid obtaining greater distance values accumulated in the objective function.

All three denoted criteria above are necessary to accomplish the second algorithm step. Missing the limited angle restriction, distance or driving time assumptions could violate to keep all appointments punctual. For instance, the calculated location *D* violate the first criteria consequentially taking out as intermediate node. In real geographical information systems which store additional data like bridge information as hindrance the algorithm can enhanced a criteria comparing the number of same used streets on the routes *A-B-C* and *A-C*. In this case the database should store additional information to detect the shortest distance from the intermediate node *B* to the direct route *A-C*. Another approach finding out additional potential intermediate nodes to keep appointments are spatial or temporal isolated appointments.

Figure 3 demonstrate this scenario. Cluster the nodes $C_1,\ldots,C_5$ observe node *D* as temporal isolated one. According to detect spatial isolated nodes obtain *B* as illustrated in the figure 3 below.

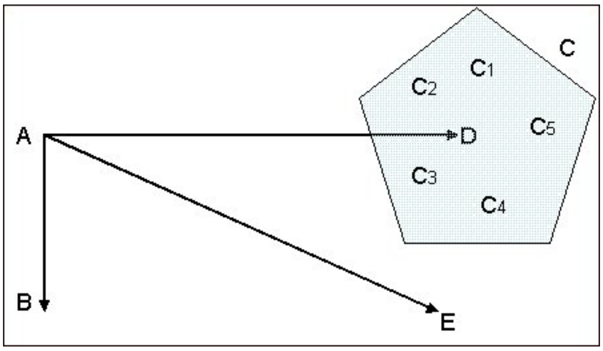

Figure 3: spatial and temporal isolated customers

Find out critical appointments by passing through the strategies described concerning the first algorithm phase. The following second phase proof the availability of additional customers possible to supply in between. Modelling the strategies realizing phase two can be stated as follows:

- (2.1) IF *customer on the way to an critical appointment* AND *time window active* THEN *highest priority*
- (2.2) IF *customer spatial isolated* AND *time window active* THEN *very high priority*
- (2.3) IF *customer temporal isolated* AND *time window active* THEN *high priority*

Modelling the steps applying fuzzy if-then rules follows. As representative scenario for the first phase we demonstrate the modelling steps regarding the statement (1.1): IF *time window on arrival within abandonment stage* THEN *highest priority*.

Every customer within the supply chain should be delivered in view of the urgency of the matter. The priority range supplying customers can be established with the range [0,100]. A high priority effect a direct supply process.

We can exemplary determine linguistic variables and linguistic terms to model the first statement (1.1). The procedure is to appraise all customers reachable only within the abandonment stage based on their current location and time and afterwards calculate their priorities descending from 100. Choosing appropriate candidates supply in the first step adduce the criteria *accuracy* and *distance* from the current location. To determine a quality measure for the different possible customers supply on the route prefer customers reachable only within the abandonment period (phase 3) very accurate not far away from the current location. The fuzzy if-then rule base assign quality measures [0,10] for each customer available only within the third abandonment period:

- IF *accuracy = very accurate* AND *distance=short* THEN *quality = very_good*
- IF *accuracy = very accurate* AND *distance = normal* THEN *quality = good*
- IF *accuracy = very accurate* AND *distance = long* THEN *quality = satisfying*
- IF *accuracy = middle* AND *distance = short* THEN *quality = good*
- IF *accuracy = middle* AND *distance= normal* THEN *quality= satisfying*
- IF *accuracy = middle* AND *distance = long* THEN *quality = bad*
- IF *accuracy = late* AND *distance = short* THEN *quality = satisfying*
- IF *accuracy = late* AND *distance = normal* THEN *quality = bad*
- IF *accuracy = late* AND *distance = long* THEN *quality = very_bad*

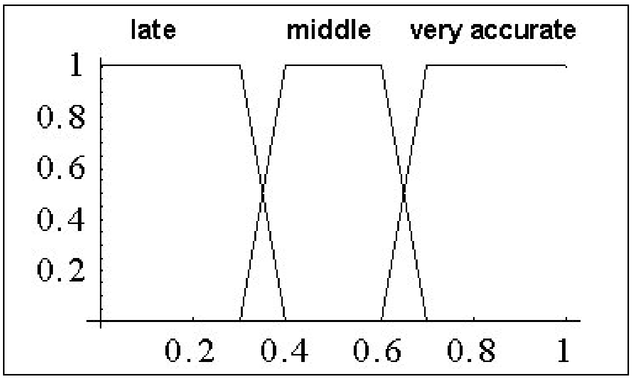

Figure 4: linguistic variable accuracy

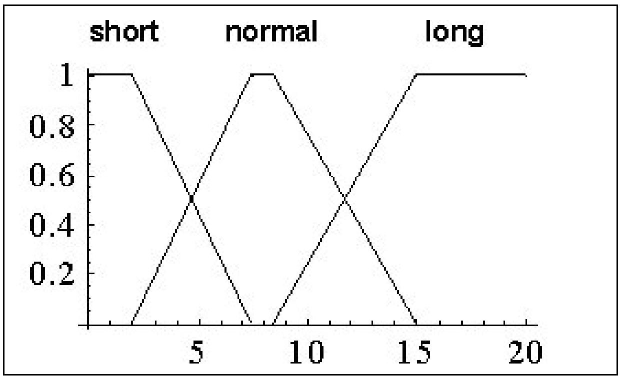

Figure 5: linguistic variable distance

After realizing the inference step we obtain a quality measure value calculated for each customer only reachable within the abandonment period. The restrictions given are a lower bound quality degree of 7 by taking the best four customers into account. The linguistic terms for the quality measure are shown below.

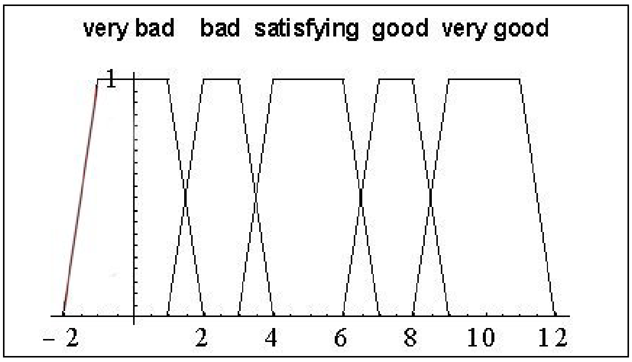

Figure 6: linguistic variable quality measure

A route is valid, if all determined customers are latest reachable by the end of the abandonment period and the sum of the customer demands q_i is less than the total vehicle capacity Q. Analog we can observe the total route quality measure include the *total route accuracy* and *route duration time* in modelling fuzzy if-then rules. The if-then statement can be stated as follows:

IF *all customers supply accurate* AND *driving time is short* THEN *route quality measure is very good*.

The algorithm prefer routes with a shorter total route duration achieve to keep all appointments very accurate. The route with the greatest route quality degree was chosen at the end of this evaluation step. In the second phase we can model the statements (2.1)-(2.3). As representative take the statement (2.1) using the linguistic variables *angle* and *additional driving time* illustrated in the figures 7 and 8.

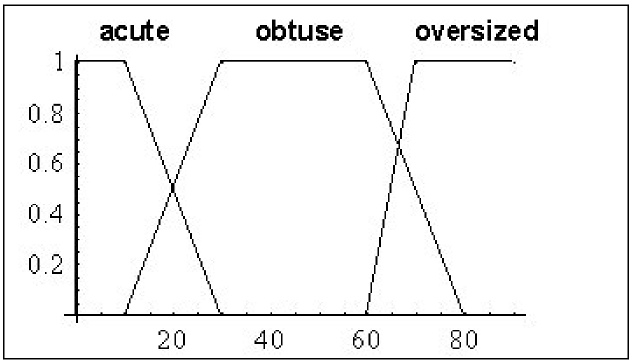

Figure 7: linguistic variable angle

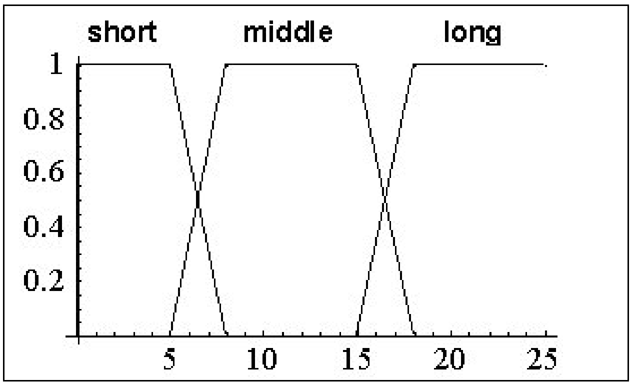

Figure 8: linguistic variable additional driving time

To determine if a customer is convenient as intermediate node we evaluate the following if-then rule base stated as follows:

- IF *angle = acute* AND *add_time=short* THEN *suitability = very_good*
- IF *angle = acute* AND *add_time=middle* THEN *suitability = good*
- IF *angle = acute* AND *add_time=long* THEN *suitability = middle*
- IF *angle = obtuse* AND *add_time=short* THEN *suitability = good*
- IF *angle = obtuse* AND *add_time=middle* THEN *suitability = middle*
- IF *angle = obtuse* AND *add_time=long* THEN *suitability = bad*
- IF *angle = oversized* AND *add_time=short* THEN *suitability = middle*
- IF *angle=oversized* AND *add_time=middle* THEN *suitability = bad*
- IF *angle = oversized* AND *add_time=long* THEN *suitability = very bad*

3 Results and conclusion

VRPTW is probably one of the most benchmarked optimization problems. In 1997 Solomon proposed a set of problems [5] which were to become the reference VRPTW benchmarks. These 100 customer problems split in six classes *R1*, *R2*, *C1*, *C2*, *RC1* and *RC2* differ in random, clustered and mixed scenarios. The following two tables representative for *C* and *RC*-scenarios summarize the fuzzy algorithm results and nearest neighbourhood heuristic.

Table 1: C1 and C2 results

Scenario	Fuzzy algorithm		NN	
	# HGV	driving time (+waiting time)	# HGV	driving time (+waiting time)
C101	10 / 880 (+14)		10 / 855 (+17)	
C106	10 / 1123 (+ 0)		10 / 978 (+ 0)	
C107	10 / 950 (+ 1)		10 / 948 (+ 0)	
C108	10 / 1012 (+ 0)		11 / 1140 (+ 236)	
C201	3 / 622 (+ 0)		3 / 590 (+ 0)	
C207	4 / 737 (+ 171)		4 / 1140 (+ 1021)	
C208	4 / 745 (+ 0)		4 / 922 (+ 438)	

Table 2: RC1 and RC2 results

Scenario	Fuzzy algorithm		NN	
	# HGV	driving time (+waiting time)	# HGV	driving time (+waiting time)
RC101	19 / 2223 (+71)		20 / 2541 (+ 0)	
RC106	14 / 1881 (+ 18)		13 / 1746 (+ 9)	
RC107	13 / 1782 (+ 4)		13 / 1625 (+ 0)	
RC108	14 / 1668 (+ 0)		13 / 1652 (+ 0)	
RC201	10 / 2090 (+ 1295)		5 / 2362 (+ 120)	
RC206	4 / 1566 (+ 466)		4 / 1768 (+ 52)	
RC207	6 / 1880 (+ 501)		4 / 1931 (+ 17)	

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