

# The Importance of Learning in Fuzzy Systems

Ahmad Lotfi

Department of Mechanical and Manufacturing Engineering  
The Nottingham Trent University  
Burton Street, Nottingham, NG1 4BU UK  
e-mail: ahmad.lotfi@ntu.ac.uk

## Abstract

One of the superior capabilities of fuzzy systems is that they can use the information expressed in a linguistic pattern. Though most fuzzy systems, have been formed to emulate human decision making behaviour, the linguistic information stated by an expert may not be precise or that it is difficult for the expert to articulate the accumulated knowledge to encompass all circumstances. Hence, it is essential to provide a learning capability for fuzzy systems, namely, to generate or modify the expert rules based on experiences. In this paper, a review of techniques available for updating/learning the parameters of a fuzzy system are presented.

**Keywords:** Learning, neuro-fuzzy, adaptive, fuzzy systems

## 1 Introduction

The main advantage of a fuzzy system is its ability to utilise information expressed in linguistic form. The design of a fuzzy system is very much an art. The designer considers the knowledge accumulated and crafts the membership function(s) and/or the inference mechanism in the system. Often, these membership functions are fixed once the design process has terminated. It would be desirable to have a fuzzy system whose membership function parameters can be “adapted” or “learned” from input and output data. This will facilitate the design process much easier.

There are essentially, two different approaches in the definition of fuzzy rules. The rules are either defined using an empirical experience or applying an adaptive/learning scheme to adjust the free parameters of the fuzzy system commencing with some arbitrary initial values. The learning fuzzy systems can be implemented using parameter adjustment algorithms and in most cases, the gradient of a cost function with respect to each adjustable parameter is calculated and the parameters will be updated accordingly. There are also some derivative-free optimisation such as genetic algorithms (GAs) and random search methods. GAs have been successfully applied to generate fuzzy rules and adjust membership functions of fuzzy systems [4]. The readers are referred to [9] for a review of using GAs to tune the membership functions of fuzzy systems.

In the literature, the parameter adjustment procedure in fuzzy systems, is called variously, e.g., adaptive, tuning [1], and learning fuzzy systems [6, 10]. Nevertheless, we intend to clearly differentiate the goals of adaptation from those of learning. A model that treats each operating situation independently is limited to *adaptive or tuning* operation, whereas a system that correlates past experiences with current situations is capable of *learning*.

## 2 History

A very first introduction of learning fuzzy system was in 1979 when Procyk and Mamdani [12] proposed a Self-Organising Control (SOC) policy for fuzzy controllers which was able to develop and improve as more data become available. There

were some further studies exploring the concept of SOC [3]. The other method to aggregate the learning ability of fuzzy systems is to use the learning ability of Neural Networks (NNs). The combined system is known as a *neural-fuzzy system* [5, 6]. The advantage of this method is that we are able to use the properties and training algorithms developed for neural networks in “learning” the underlying parameters of a neural-fuzzy system.

### 3 Neural-Fuzzy Systems

A strong synergetics relationship between fuzzy system and neural network exists that has been exploited to integrate fuzzy and neural systems. There are mainly two ways that the integration of these two systems can work. Either the neural network is used to realise a fuzzy system or the fuzzy system is used to influence the dynamics of a neural network. When fuzzy concept is used to assist neural system is referred to as *fuzzy-neural system*. When neural network assists a fuzzy system to induce rules or tune the membership functions is referred to as *neural-fuzzy or neuro-fuzzy system* [11]. It is to be noted that the terminologies neural-fuzzy and fuzzy-neural are widely used interchangeably in the literature. Though, we insist on using the term neural-fuzzy for learning fuzzy systems where the neural network is used to emulate fuzzy system features, such as, rules, membership functions and inference mechanism.

One of the main advantages of learning fuzzy systems over classical learning systems and neural networks is their ability to utilise intuitive knowledge, which may be presented in a linguistic form. Though, once the membership functions and rules of the system are stored, it is desirable to preserve the linguistic information. That is, one would like to be able to use the same intuitive understanding, which was used to create the fuzzy system, to interpret its behaviour at all times in the future. In general, this ideal cannot be guaranteed in a learning fuzzy system.

A neuro-fuzzy model is built using a multilayer neural network and it has a total of five layers [8]. Nodes in layer 1 are input nodes that directly transmit input signals to the next layer. Layer 5

is the output layer. Nodes in layer 2 and 4 are “term nodes” and they act as membership functions to express the fuzzy values of linguistic variables. Each node of layer 3 is a “rule nodes” and represents a single fuzzy rule [4]. In total, there are  $K_1 \times K_2$  nodes in layer 3 where  $K_1$  and  $K_2$  are the number of individual membership functions for each input.

### 4 Parameters Learning

The initial fuzzy labels assigned to a linguistic variable are not entirely capable of incorporating the human experience into the fuzzy if-then rules. Obviously, a designer, with experience, can produce a membership function which suits a particular situation. However, this is explicitly excluded here, as the experience by the designer in hand-crafting the membership functions for one system does not in general carry over to the design of another system. A learning ability for membership functions in fuzzy systems can make the incorporation between the membership functions of fuzzy rules and human experience more effective.

For a given fuzzy system, it is obvious that altering I) The membership function of linguistic values, II) Fuzzy reasoning methods, and III) The number of rules, will affect the overall input-output mapping of the system.

Assignment of a membership function to each fuzzy value is, in general non unique. However, there has been some research for giving a systematic approach in this direction [2]. It depends, to a certain extent, on the designer in the choice of the membership functions, as well as the interpretation of the “fuzziness” of the variables concerned. Altering the membership functions has a dominant effect on the other two (fuzzy reasoning and number of rules). It can be seen that for a fixed number of rules in the rule set, changing the membership function can achieve the same input-output mapping, to a particular degree of approximation, regardless of the fuzzy reasoning method. Alternatively, for a fixed fuzzy reasoning method, we can attain to the same degree of approximation, the same input-output mapping with different number of rules and different membership functions.

In the next section the gradient-descent or back-propagation algorithm which is also used for neural networks is employed to adapt the parameters of a fuzzy systems.

## 5 Learning Rule

If the correct output, corresponding to a particular set of inputs to a fuzzy system is known, then it is possible to adjust the free parameters of a fuzzy system by means of error backpropagation. The method by which this is done is the same as the way that multilayer perceptron are trained, that is:

1. For each known input/output relation, a cost function,  $J$ , is calculated by:

$$J = \frac{1}{2}\epsilon^2$$

where  $\epsilon = y_d - y$ ,  $y_d$  is desired output, and  $y$  is the output of the fuzzy system.

2. Considering  $\sigma_{j,k}$  as the values of the free parameters of membership functions, the partial derivatives,  $\frac{\partial J}{\partial \sigma_{j,k}}$ , for each of the free parameters, is calculated.
3. The free parameters  $\sigma_{j,k}$  are then adjusted by:

$$\sigma'_{j,k} = \sigma_{j,k} - \eta \frac{\partial J}{\partial \sigma_{j,k}}$$

where  $\sigma'_{j,k}$  are the new values of the free parameters and  $\eta$  is the "learning rate" which determines the size of the adjustments made.

## 6 Interpretation Preservation

If allowed to adapt freely, the membership functions of a learning fuzzy system may lose the meaning which was initially assigned to them. They may change their relative positions such that, for example, "low" may become greater than "high", or the range of their activations may become excessively wide or narrow.

Although the adapted membership functions may have attained a new significance, after the original

meaning has been lost, it may be very difficult or otherwise undesirable to interpret this. In some cases a learning fuzzy system may have changed to such a degree that a conventional linguistic interpretation is no longer possible. In such a case, the learning fuzzy system may be viewed as a "black box" function approximator similar in function to a neural network. All these possibilities make a conventional learning fuzzy system unsuitable for many industrial applications in which maintainability and reliability are of importance, despite their likely performance superiority.

### 6.1 Constraint Learning

A constrained training algorithm is proposed, which maintains the interpretation of learning fuzzy system during training. It is a paradigm which will enable the learning fuzzy controller to adapt and optimise itself while still remaining conceptually comprehensible to a human expert. This may be achieved at the cost of a slight degradation of the performance of the learning fuzzy system, in the sense that the error function may attain a higher value than if the membership functions are allowed to adapt freely. In most cases, this trade-off is acceptable, as it is often more important to be able to interpret the behaviour of the learning fuzzy system than to achieve a lower minimum in the cost function.

The membership function assigned to a fuzzy value should not exceed certain maximum and minimum limits of fuzziness after adaptation. If the similarity between the initial membership function and the membership function during training is measured, when the similarity measure exceeds its limit, the linguistic meaning assigned to the membership function is said to be lost.

### 6.2 Constraint Learning Rule

Consider the updating rule given earlier, the free parameter values can be modified for constraint learning by updating the parameters after each iteration using the following update rule.

$$\sigma'_{j,k} = \sigma_{j,k} - \eta \frac{\partial J}{\partial \sigma_{j,k}} \times \mathfrak{R}_{\sigma_{j,k}}$$

The restriction functions,  $\mathfrak{R}_\sigma$ , is introduced to limit the freedom of updating the parameters and it is governed by the following expression.

$$\mathfrak{R}_\sigma = \frac{1}{1 + e^{-\left(\frac{\sigma - \underline{\sigma}}{\nu\sigma}\right)}} - \frac{1}{1 + e^{-\left(\frac{\sigma - \bar{\sigma}}{\nu\sigma}\right)}}$$

where  $\nu_\sigma$  is dispersion parameters of  $\sigma$ . This is introduced so that a “gentle roll off” is achieved. As the parameters approach their predefined maximum and minimum limits, smaller and smaller updates are performed. By controlling the parameter  $\nu$ , this can be controlled.

Hence, regardless of whether or not a minimum solution for the cost function,  $J$ , is achieved or not, the restriction bounds do not permit the free parameters to move beyond the defined limits. It preserves the interpretation at the possible expense of yielding a less optimal solution.

## 7 Conclusions

In this paper, a review of learning fuzzy systems are presented followed by a gradient descent method to update the parameters of a fuzzy system. It has been shown that the integrity of learning fuzzy system can be preserved by using a constrained training algorithm. There is a trade-off between obtaining the minimum of cost function and the preservation of integrity in the sense of the interpretability of the learning fuzzy systems.

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