Theoretical foundation for iterative assessment of conditional confidence measures in the framework of conditional measure theoretic-approach

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Extended Abstract

In situations where some chronological order (or a time) is involved, it makes sense to speak about conditional statements in the sense that the new knowledge is obtained given an old information taken for granted. Consequently, when the new and old information are attached confidence values, we rather deal with conditional confidence measures. In a more general setting when several time increments are involved, the iterative reasoning becomes undoubtedly relevant and necessary. This paper focuses on the analysis of confidence measure obtained from such iterative process based on the measure theoretic approach and, especially, its broader structure of capacity [2]. Particularly, the notions of expectation and typicality will be highlighted in the line of this analysis as important aspects of the study. Indeed, these notions have direct incidence on the algebraical properties of the result as well as on the way by which the iterative process is handled in terms of computational aspect. Potential applications of such analysis lie in the safety critical cases where the notion of confidence plays a central role, and it holds that time is a relevant component in such case-studies [1].

Strictly speaking, it is known from the academic textbooks of statistics and/or probability that the (conditional) probability is calculated first and then the (conditional) expectation is driven. Besides, it was always stressed, in the expression like $P(A|B)$, on the condition $P(B)>0$, while it holds that conditioning on zero-event probability often occurs in practice. On the other hand, it was recognized that early pioneer works of Kolmogorov for constructing the basic foundations of recent probability theory have taken exactly the opposite direction. That is, a (conditional) expectation is first provided and, then, the (conditional) probability is driven. This is, typically, a different philosophical issue to the conditioning problem. In other words, the latter view can be translated into a current life language as "tell me what do You expect, I will tell You how to choose your probability", or in a confidence terms "tell me what do You expect, I will tell You how confident You are". Within this framework, the dilemma of conditioning with zero-probability event was partly solved using the Radon-Nikodym differentiation theorem of measures (or set functions). In this paper we will extend the second approach of conditioning to Capacity functions (or special types of capacities) introduced by Choquet [2] which generalize the standard notion of measure. This also builds a bridge to the notion of random set as pointed out by Matheron [3]. Particularly, the use of capacity allows avoiding some undesired outcomes when dealing with expectation of medium populations. For instance, if the player who gambled one hundred times and won \$10 in 99 of the 100 cases and lost 990 in on case, one may expect that in the 101 gamble, he win \$10, while the standard average (for the expectation) leads to a very different result. This motivated the use of non-additive measure as a basis for the expectation calculus. Typicality can also be viewed in connection with the expectation in the sense that typical elements of a group are those that might be expected according to some (conditional) expectation operator (or, equivalently, to some (conditional) measure). This views of typicality contrasts with some proposals in fuzzy literature where the concept of typicality leads to a singleton.

So far, Goodman et al. [5] have proposed an algebraic structure of conditional events, which equipped with logical connectives like conjunction, disjunction and complementation operations, provides a structure analogous to that of Boolean algebra for manipulating conditional events. We will examine the extent to which the expectation supplied by the aforementioned formalism agrees with the algebraic structure of [5]. Finally, the notion of iterative conditioning is performed as a generalization of the standard probability calculus in the formula like $P(A \mid B \cap C) = P[(A \mid C) \mid B]$. Special cases of belief structure [4] will also be considered.

Keywords: conditional expectation, random sets, conditional measure, capacity.

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