

Combination of qualitative and quantitative sources of knowledge for risk assessment

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Abstract

This paper focuses on representation of the reliability of system in the framework of possibility theory. Particularly, given a (probabilistic) quantitative knowledge pertaining to the time to failure of a system (risk function) and some qualitative knowledge about the degree of pessimism and optimism about the information supplied by this quantitative knowledge, one may construct a possibilistic reliability function of the system. The latter models the possibility that the system survives up to the current time. The proposal is motivated by the observation that the system may fail even if the risk function considers this failure as unlikely. Besides, research from cognitive science shows that probability values, particularly when human factor is involved in the system tend to be overestimated. This methodology involves implicitly the combination of the two pieces of knowledge into a more refined knowledge expressed in the possibility framework. Several ramifications of the proposal, particularly considering the common exponential lifetime distribution will be investigated. On the other hand in the absence of any qualitative knowledge the proposal is straightforwardly related to the notion of probability-possibility transformations investigated in the fuzzy literature.

Keywords: Fusion, risk, probability, possibility, fuzzy set theory.

1 Introduction

The need for rigorous methodologies for analysis of system dependability becomes crucial, particularly with the exponential increasing of system

complexity and the involvement of these systems in critical situations where damage may have drastic consequences on the whole society. Typically probabilistic risk assessment (PRA) [8] based on lifetime distributions is the most common methodologies for such purposes. However, with the increase involvement of human factors in systems themselves and the various types of uncertainty pervading the systems as well as their high complexity, the information supplied by PRA is sometimes insufficient for rigorous decision-making. Consider for instance a situation where PRA supplies a value of probability of accidents of 10^{-4} , does this mean that the accident occurs one every 10^4 years? Does this really imply that the accident would hardly ever occur? Can we make a sure statement that this system is safe? Does low probability really entails low possibility of the occurrence of the underlying event?

Human reliability analysis [7] shows that the meaning of probability and the conclusions that may be derived from PRA are context dependent and, any attempt to generalization may be worthless. Schackle [6] advocated the degrees of possibility, or equivalently of surprise, as an alternative and natural way to represent the risk pertaining to the given situation.

This paper attempts to construct a possibilistic knowledge concerning the reliability of a given by combining the quantitative information supplied by methodology like PRA and some qualitative knowledge given in terms of optimistic and pessimistic bounds on the outcome of PRA. Some ramifications of the proposal in reliability context will be examined

2- A framework for integrating probabilistic risk knowledge and qualitative knowledge

Let us assume that some preliminary analysis suggests that for a given system, where the interest

is its reliability, there exists some probability density function (f) characterizing the time to failure of the system. Based on some motivations rised in the introduction of this paper, it holds that the probability, particularly low probabilities does not mean systematically that the failure of the system is likewise lowly possible. An important consequence of this observation is that the (probabilistic) mean time to failure (MTF) is usually larger than the possibly mean time to failure (PMTF).

Further, this note has also been justified by Schackle [6] in his framework of potential surprize in economical context. Besides social and cognitive studies, see for instance [7], reported that people as well as experts tend, in many circumstances, to overestimate the probabilities of the events under considerations.

In order to model such information, let us assume that the expert can provide some qualitative information expressed in terms of degrees of pessimism ε and optimism ε' about the system failure as supplied by the density function f whose mean is f_m . These degrees can be expressed in terms of proportions of the standard deviation f_σ pertaining to the density function f . Besides, the pessimistic degree should be greater than the optimistic value.

Then one may express the integration of the probabilistic and the qualitative arguments through a possibility distribution π_T (T refers to the variable time to failure) constructed using the following rational assumptions.

Assumption 1

Within the time interval induced by the optimistic and pessimistic bound around the mean value f_m , it is completely possible that the system fails.

Assumption 2

Outside the interval mentioned in Assumption 1 and excluding the very low probabilities (up to some threshold δ), the possibility of the system failure varies linearly in the same direction as the density probability behaves such that the preference induced by the density probability is preserved.

Assumption 3

The possibility distribution π_T is continuous and the density function assumes no *extremum* outside the interval mentioned in Assumption 1.

Assumption 1 allows us to model the intuition that the time to failure T of the system can be completely possible even if it is unlikely to occur. While the extent of possible time values that may cause failure is determined by the pessimistic and the optimistic bounds around the mean value of the probability density function f of T . Assumption 2 expresses that the shape of the underlying distribution can be set to a linear model. While the basis for the construction of that model is based on the preservation of preference among the different time values, excluding those which are very unlikely to occur, up to the threshold γ . More formally, $f(t_1) \geq f(t_2) \Rightarrow \pi_T(t_1) \geq \pi_T(t_2)$. Assumption 3 enables the possibility distribution to be also continuous likewise the density function f . While the existence of an extremum outside the region around the mean of the density signifies that the preference structure cannot be captured by a single piece of a linear function. However, in most standards density functions, it holds that the requirement about the density function is satisfied. Throughout this paper, the proofs of the forthcoming propositions will be omitted due page limit restriction.

Proposition 1

Assume there exist t_ε and $t_{\varepsilon'}$ (positively valued) such that $t_\varepsilon = f_m - \varepsilon \cdot f_\sigma$ and $t_{\varepsilon'} = f_m + \varepsilon' \cdot f_\sigma$. If there exists two distinct t_γ, t'_γ such that $f(t_\gamma) = f(t'_\gamma) = \gamma$,

$$\text{and } t_\gamma \leq t'_\gamma. \text{ Let } a_1 = \frac{f(t_\varepsilon) - \gamma}{t_\varepsilon - t_\gamma} \text{ and } a_2 = -\frac{f(t_{\varepsilon'}) - \gamma}{t'_\gamma - t_{\varepsilon'}}$$

Then under assumptions 1 to 3, the possibility distribution π_T is provided by

$$\pi_T(t) = \begin{cases} 1 & \text{if } t_\varepsilon \leq t \leq t_{\varepsilon'} \\ \max(0, a_1(t - t_\varepsilon) + 1), & \text{if } t < t_\varepsilon \\ \max(0, a_2(t - t_{\varepsilon'}) + 1), & \text{if } t > t_{\varepsilon'} \end{cases}$$

If there exists only one t_γ ($=t'_\gamma$) such that $f(t_\gamma) = \gamma$, then

$$\pi_T(t) = \begin{cases} 1 & \text{for } t \leq (\geq) t_\gamma, \text{ if } t_\gamma > (<) t_{\varepsilon'}, \\ \max(0, a_2(t - t_\gamma) + \gamma), & \text{for } t < t_\varepsilon, \text{ if } t_\gamma < t_{\varepsilon'} \\ \max(0, a_1(t - t_\gamma) + \gamma), & \text{for } t > t_{\varepsilon'}, \text{ if } t_\gamma > t_{\varepsilon'}. \end{cases}$$

The second part of the preceding permits us to take account for the case where the density function is monotone (either increasing or decreasing) over the

whole interval $[0, \infty]$. In that case, the information of optimism (or pessimism) value will be ignored.

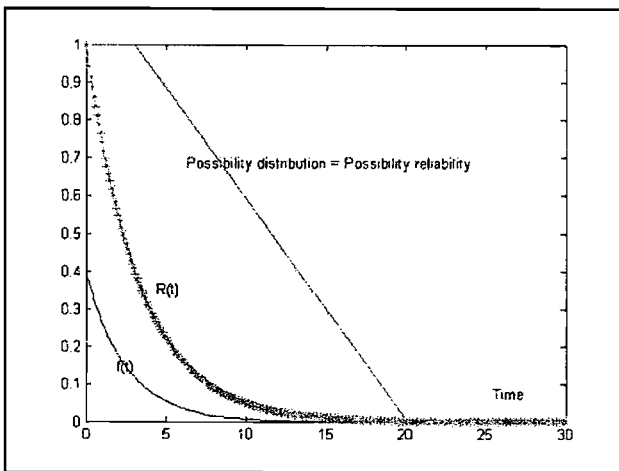
Example

To illustrate the above construction, let us consider a typical example where the risk assessment provides an exponential density distribution $f (f(t) = \lambda e^{-\lambda t})$, which is very common in reliability analysis, with a parameter, say, $\lambda = 0.4$. Assume the degrees of optimism and pessimism about the (mean) time of failure supplied by further evidence are respectively 0.2 and 0.5 (as proportion of the standard deviation $\sigma = \frac{1}{\lambda}$ of the exponential distribution). We also

assume that probabilities below $\frac{P_{max}}{10}$ are useless (P_{max} stands here for the greatest probability in the range $[0, \infty]$). Figure 1 shows the associated possibility distribution π_T .

Interestingly is the reliability function also denoted survivor, which in probabilistic framework is given by $R(t)=P(T>t)=\int_t^{\infty} f(u)du$. Its counterpart in the possibility framework is expressed via a possibility measure: $R_p(t) = \Pi(T > t) = \max_{u>t} \pi_T(u)$

It is also drawn both probabilistic and possibilistic reliability functions.



Example of possibilistic reliability driven from a probability density function of the time to failure.

Note that for the special case of the exponential density function as well as for all distributions,

which are strictly decreasing within the time interval $[0, \infty]$, it is worth mentioning the following notes:

- The possibilistic reliability coincides with the possibility distribution, i.e., $R_p(t) = \pi_T(t)$. However, in probabilistic framework the only case where the reliability function coincides with the density function describing the time to failure for a family of exponential distributions is when one forces the parameter λ to be equal to one.
- As a consequence of the preceding, reasoning in terms of the failure rate, it follows that the possibilistic failure rate (PFR) $z_p(t) = 1$. This is easily justified considering the intuition behind the failure rate. Indeed, it expresses that the possibility that the unit will fail in the time interval $[t, t+\Delta t]$, when we know the unit is functioning at time t , per sufficiently small time increment Δt , is maximum. In other words, even if the unit is functioning at the current time, it is completely possible that it will fail after a very short interval.
- By restricting the domain of the search to the fully possible alternatives, it is easily checked that as soon as $\varepsilon > \varepsilon'$, the possibilistic mean time to failure is always less than the probabilistic mean time to failure. This is in agreement with the expectation risen in the introduction of this paper.

About further alternatives

Strictly speaking, turning probabilistic knowledge into possibilistic knowledge has already been investigated in the framework of probability/possibility transformations, see for instance [1, 3-5]. However, in all the proposals it is assumed that the strict preference assumption $f(t_1) > f(t_2) \Rightarrow \pi_T(t_1) > \pi_T(t_2)$ and $f(t_1) = f(t_2) \Rightarrow \pi_T(t_1) = \pi_T(t_2)$ is satisfied over the entire region of the universe of discourse, i.e., here over the time domain $[0, \infty]$. However, it can be easily proved that due to this assumption, the underlying possibility will only refer to the most probable time to failure as the most possible value for the time to failure. In other words, this will not supply any additional information regarding the possibility that the system fails even being unlikely, which violates our expectation in this course.

On the other hand, the linear assumption induces a more effective computational aspect, while it keeps track of the preference preservation property only in a weak sense. That is, the following holds for the distribution π_T :

Proposition 2

The possibility distribution π_T fulfills the preference requirement in the following way:

- i) $f(t_1) > f(t_2)$ entail either $\pi_T(t_1) = \pi_T(t_2)$ or $\pi_T(t_1) > \pi_T(t_2)$, and
- ii) $f(t_1) = f(t_2) \Rightarrow \pi_T(t_1) = \pi_T(t_2)$

Particularly, the conclusion part of i) depends on either t_1 and t_2 belong to the region mentioned in Assumption 1 or not.

3. Absence of qualitative knowledge

Let us consider again, for its common use in reliability analysis, exponential distributions. We also consider the pruning procedure (for low probabilities) mentioned in the previous example.

Assume we do not have any additional information about the degree of pessimism (or optimism). Then it is easy to see that the resulting possibility distribution π'_T , for an exponential distribution of a parameter λ , takes a value one within the interval $\left[0, \frac{1}{\lambda}\right]$ and then decreases linearly with a slope approximately $\frac{\lambda^2}{2}$.

Now considering the PMTF as a result of the whole distribution instead of restricting to the fully specific cases. In this spirit, a rationale for PMTF would be to choose the expected value of the whole possibility distribution. Then using the result pointed out by Heilpern [2] about the expectation of a fuzzy entity, one gets the following result

Proposition 3

The PMTF underlying π'_T constructed from an exponential failure density of a parameter λ is smaller than the MTF if and only if $\lambda \geq 1$.

Strictly speaking, omitting the qualitative knowledge about the degrees of pessimism and optimism

recovers the framework of probability-possibility transformations.

In the limit case, where $\lambda = 0$ (or tends towards zero), which entails MTF tending towards infinity, we get the possibility distribution π'_T as well as the possibilistic reliability R_p that takes one everywhere, i.e., complete ignorance case.

Conclusion

In the context of reliability, we have proposed a new construction of possibility distribution underlying a time to failure of a given system, given the quantitative (probabilistic) knowledge of system failure as well as some qualitative evidence about the trust on the former knowledge. Some reliability concepts of the underlying construction have been examined.

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References

- [1] D. Dubois, H. Prade and S. Sandri, On possibility-probability transformations, in: *Proc. of Fourth IFSA Conference, Mathematics*, Brussels, 50-53, 1991.
- [2] S. Heilpern, The expected value of a fuzzy number, *Fuzzy Sets and Systems*, 47, 81-86, 1992.
- [3] G. J. Klir and M. J. Wierman *Uncertainty-Based Information*, Physica-Verlag, 1998
- [4] V. G. Lasserre, L. Maurris and L. Foulloy, A simple probability-possibility transformation for measuring error representation: A truncated triangular transformation, in *Proc. of IFSA Conference*, Prague, 1997.
- [5] M. Oussalah, On the possibility/probability transformations: A comparative analysis, *International Journal of General Systems*, 29(5), 2000, pp. 671-718.
- [6] G. L. S. Shackle, *Decision, Order and Time in Human Affairs*. Cambridge: Cambridge University Press, 1961
- [7] M. Smithson, *Ignorance and Uncertainty*, Springer-Verlag, 1989.
- [8] A. Villemeur, *Reliability, Availability, Maintainability and Safety Assessment. Methods and Techniques*, John Wiley & Sons, 1991.