A New Approach to Fuzzy Goal Programming

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Abstract

The multiobjective problem with fuzzy goals (which may also represent fuzzy right hand sides of the constraints) is formulated as a fuzzy goal programming problem. A new approach and a new solution method are proposed.

Keywords: goal programming, multicriteria programming, fuzzy programming, fuzzy goals.

1 Introduction

In most practical decision problems more than one criterion (objective) has to be taken into account. Thus, a lot of research has been devoted to multicriteria programming, but the many existing results do not cover all the practical needs yet. It is so, because multicriteria programming solutions have to be based on the attitude of the decision makers with respect to the individual objectives and the relations among them. Another practical problem that causes much trouble in practical applications is the imprecision of the data that are usually available at the moment of decision making. This imprecision has to be modeled in a proper way and handled somehow, so that the decision maker can obtain a usable result. In our approach we make us of the fuzzy approach.

Thus, the object of this paper is the general problem where the decision maker has several objectives (goals) formulated in an imprecise (fuzzy) way. For solving this problem, we use a very popular approach to multicriteria decision making: the goal programming.

Many papers have been devoted to fuzzy goal programming already, but – as it is generally the case in multicriteria programming – there is no approach that could be considered the best one in all the practical situations. Thus, the aim of our paper is to propose a new approach that might be useful in some decision situations. The new approach will be compared to the existing ones, so that the decision maker will have an easier task while looking for a fuzzy goal programming model for his problem.

2 New approach

Let us assume that the decision maker has k_3 goals (linear objective functions $\mathbf{C}_j$ $(j=1,\dots,k_3)$ with crisp coefficients) and for each of the goals fuzzy target values D_j $(j=1,\dots,k_3)$ that are *L-R* fuzzy numbers $\left(D_j=\left(\underline{d}_j,\overline{d}_j,\alpha_j,\beta_j\right)_{L_j-R_j}\right)$ ([2]) with membership functions μ_{D_i}. The symbol *L-R* stands for shape functions. If both *L* and *R* are of the linear type, we are dealing with trapezoidal fuzzy numbers, i.e. with such whose membership function have the trapezoidal form.

The decision maker understands the target values in the following way: for goals $1,..,k_1$ the corresponding fuzzy targets should not be exceeded (if possible), for goals $k_1+1,\dots,k_2$ the corresponding fuzzy targets should be attained (if possible), for goals $k_2+1,\dots,k_3$ the corresponding fuzzy targets should be exceeded (if possible). Of course, as we are dealing with fuzzy numbers, the meaning of "exceeded", "attained" will have to be clarified. The general formulation of the problem is as follows (where $\mathbf{Ax} = \mathbf{B}$ are crisp linear constraints):

$$\begin{aligned}
&\mathbf{C}_j\mathbf{x} \hat{\le} D_j \ (j = 1,\dots,k_1) \\
&\mathbf{C}_j\mathbf{x} \hat{=} D_j \ (j = k_1+1,\dots,k_2) \qquad (1)\\
&\mathbf{C}_j\mathbf{x} \hat{\ge} D_j \ (j = k_2+1,\dots,k_3)
\end{aligned}$$

$\mathbf{Ax} = \mathbf{B}$

As usually, if it will not be possible to satisfy the wishes of the decision maker, we will try to minimise the unfavourable deviations from his fuzzy target values. For goals $1,..,k_1$ the unfavourable deviations $D_j^+, j = 1,\dots,k_1$ will occur if the corresponding fuzzy targets are exceeded, for goals $k_1+1,\dots,k_2$ – if the corresponding fuzzy targets are not attained (deviations D_j^+ and $D_j^-, j = k_1+1,\dots,k_2$), for goals $k_2+1,\dots,k_3$ – if the corresponding fuzzy targets are not exceeded (deviations $D_j^-, j = k_2+1,\dots,k_3$). A new intuitive definition of the deviations is proposed

$$D_j^+ = D_j^+(\mathbf{C}_j\mathbf{x}, D_j) = \begin{cases} 0 \text{ if } \mathbf{C}_j\mathbf{x} \le \overline{d}_j, \\ \int_{\overline{d}_j}^{\mathbf{C}_j\mathbf{x}} (1 - \mu_{D_j}(z))dz \text{ elsewhere,} \end{cases}$$

$$D_j^- = D_j^-(\mathbf{C}_j\mathbf{x}, D_j) = \begin{cases} 0 \text{ if } \mathbf{C}_j\mathbf{x} \ge \underline{d}_j, \\ \int_{\mathbf{C}_j\mathbf{x}}^{\underline{d}_j} (1 - \mu_{D_j}(z))dz \text{ elsewhere.} \end{cases}$$

The following pictures will illustrate the above definition:

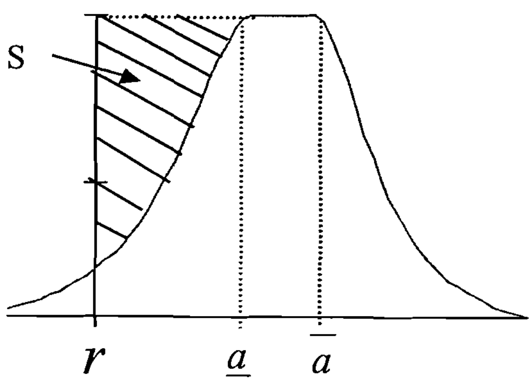

Figure 1: Graphical illustration of the "in plus" and "in minus" deviations of a crisp value r from a fuzzy value A – case 1 ($D^-(r,A) = S,\ D^+(r,A) = 0$)

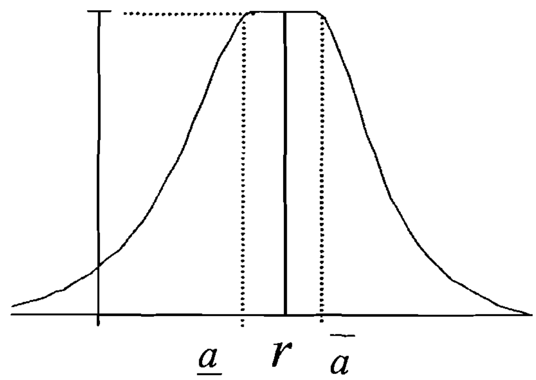

Figure 2: Graphical illustration of the "in plus" and "in minus" deviations of a crisp value r from a fuzzy value A – case 2 ($D^-(r,A) = 0,\ D^+(r,A) = 0$)

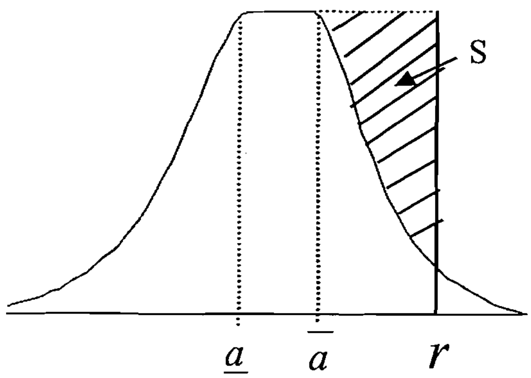

Figure 3: Graphical illustration of the "in plus" and "in minus" deviations of a crisp value r from a fuzzy value A – case 3 ($D^-(r,A) = 0,\ D^+(r,A) = S$)

In our approach D_j^+, D_j^- are crisp numbers, although they are denoted with capital letters.

Fuzzy goal programming problem (1) will become:

$$\sum_{j=1}^{k_1} w_j D_j^+ + \sum_{j=k_1+1}^{k_2} \left(w_j D_j^+ + w_j^{'} D_j^-\right) + \sum_{j=k_2+2}^{k_3} w_j D_j^- \to \min$$

$$\mathbf{Ax} = \mathbf{B} \qquad (2)$$

where $w_j\ (j = 1,\dots,n_3)$ and $w_j^{'}\ (j = k_1+1,\dots,k_2)$ are positive weights, expressing the importance of each goal to the decision maker or the relation of the order of magnitude of the values of individual goals.

If we assume that the target values D_j are trapezoidal fuzzy numbers, then (2) becomes (because of a special definition of the deviations) a crisp programming problem with linear constraints and a convex objective function that is piecewise linear and piecewise quadratic. Therefore the problem can be solved by means of well-known algorithms.

3 Example

The example presented here in based on [1].

Let x_1 and x_2 denote, respectively, the number of shares of company A and company B to be purchased. One share of company A costs 25\$, one share of company B costs 50\$. The amount of available funds is equal to 80.000\$ and this is a rigid limit. The risk index is equal to 0,5 (company A) and 0,25 (company B) and the annual return from one A share is 3 \$ and from one B share 5 \$. The acceptable level of risk is fuzzy and equals to $(600,700,100,200)_{L-L}$ (where L is the linear shape function). So is the satisfying total annual return – it amounts to $(9000,10000,2000,1000)_{L-L}$. This problem can be formulated in the following way:

$$\begin{aligned} &0{,}5x_1 + 0{,}25x_2 \,\hat{\leq}\, (600,700,100,200)_{L-L} \\ &3x_1 + 5x_2 \,\hat{\geq}\, (9000,10000,2000,1000)_{L-L} \\ &25x_1 + 50x_2 \leq 80.000 \\ &x_1, x_2 \geq 0 \end{aligned} \qquad (3)$$

Let us assume that the weights in the objective function of (2) are equal to 1. Then the objective function in the example is the sum of the two following expressions:

$$D_1^+ = \begin{cases} 0 \text{ if } 0{,}5x_1 + 0{,}25x_2 \leq 700 \\ \dfrac{(0{,}5x_1 + 0{,}25x_2 - 700)^2}{400} \\ \text{if } 700 < 0{,}5x_1 + 0{,}25x_2 \leq 900 \\ 0{,}5x_1 + 0{,}25x_2 - 800 \text{ if } 0{,}5x_1 + 0{,}25x_2 > 900 \end{cases}$$

$$D_2^- = \begin{cases} 0 \text{ if } 3x_1 + 5x_2 \geq 9000 \\ \dfrac{(9000 - 3x_1 - 5x_2)^2}{4000} \\ \text{if } 7000 \leq 3x_1 + 5x_2 < 9000 \\ 8000 - 3x_1 - 5x_2 \text{ if } 3x_1 + 5x_2 < 7000 \end{cases}$$

and we have the following constraints:

$$25x_1 + 50x_2 \leq 80.000$$

$$x_1, x_2 \geq 0$$

It is possible to solve the problem using only quadratic and linear programming methods. It is enough to decompose the problem into 9 sub-problems, corresponding to all the combinations of the three pieces of each component. For example, the first (trivial) sub-problem would have the following form:

$$0 + 0 \to \min$$

$$0{,}5x_1 + 0{,}25x_2 \leq 700$$

$$3x_1 + 5x_2 \geq 9000$$

$$25x_1 + 50x_2 \leq 80.000$$

$$x_1, x_2 \geq 0$$

Some of the sub-problems are infeasible (e.g. the above one), and the solution corresponding to the minimal objective function value among the solutions of the feasible sub-problems gives us the solution of our problem. This solution is:

$$x_1 = 2000,\ x_2 = 600,\ \text{Objective function} = 350$$

This solution gives us the number of shares the decision maker should purchase in order to minimize the negative deviations from his goals (which were "an acceptable level of risk" and "a satisfying total annual return"). The value of the objective function represents the total minimal value of the negative deviations from the goals.

References

[1] Anderson D.R., Sweeney D.J., Williams T.A. (1994), An Introduction to Management Science, West Publishing Company;

[2] Zadeh L.A. (1965) Fuzzy Sets. Information and Control 8, 338-353 .