Enhancement of Mamdani Fuzzy Controller

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Abstract

We propose an enhancement of Mamdani fuzzy controllers. We tested it on a simple control task and verified that it outperforms the traditional approach.

Keywords: Triangular norm, fuzzy relational equation, fuzzy control, fuzzy interpolation.

1 Introduction

The concept of approximate reasoning as it was conceived by Zadeh [10] provides a framework which allows to model and process vague linguistic information. The idea is to model linguistic terms by fuzzy sets, their (logical) relationship by fuzzy relations and their composition by the so-called *compositional rule of inference* [10]. As an important field of applications we refer to control processes for which linguistic information of a human expert about the required input-output behaviour of the controller is available (for an introduction see, e.g., [2]).

Let X and Y denote the input and the output space, respectively. The expert's knowledge can be expressed by means of a rule base, $\Theta = (X_i, Y_i)_{i=1}^{n}$, of if-then rules having the form

$$\textbf{if } x \in X_i \textbf{ then } y \in Y_i, \qquad i \in \{1, \ldots, n\},$$

where $X_i \in \mathcal{F}(X)$, $Y_i \in \mathcal{F}(Y)$. (For a set Z, we denote by $\mathcal{F}(Z)$ the set of all fuzzy subsets of Z.) The rule base Θ can be represented by a fuzzy relation $R \in \mathcal{F}(X \times Y)$. Applying the compositional rule of inference to a fuzzy input $X^* \in \mathcal{F}(X)$ we obtain a fuzzy output $Y^* \in \mathcal{F}(Y)$:

$$Y^* = X^* \circ_T R, \tag{1}$$

i.e.,

$$\forall y \in Y : Y^*(y) = \sup_{x \in X} T(X^*(x), R(x, y)), \tag{2}$$

where T is a t-norm modelling a fuzzy conjunction [3]. In particular, Mamdani (or Mamdani–Assilian) inference [4] results from formula (1) for

$$R(x, y) = \max_{i \le n} T(X_i(x), Y_i(y)). \tag{3}$$

We look for a *fuzzy interpolation* $Int_\Theta : \mathcal{F}(X) \to \mathcal{F}(Y)$ satisfying the following axioms (recall that a fuzzy set is called *normal* if it attains the value 1 at some point):

[Int1] If the input coincides with one of the premises, then the resulting output coincides with the corresponding consequent, i.e., $\forall i \in \{1, \ldots, n\} : Int_\Theta(X_i) = Y_i$.

[Int2] For each normal input $X^* \in \mathcal{F}(X)$, the output $Int_\Theta(X^*)$ is not contained in all consequents, i.e., there is an index $i \in \{1, \ldots, n\}$ with $Int_\Theta(X^*) \not\leq Y_i$.

[Int3] The output $Int_\Theta(X^*)$ belongs to the convex hull of $(Y_i)_{i \in F}$, where $F = \{i : (\exists x \in X : X_i(x) > 0,\ X^*(x) > 0)\}$.

In [5, 6, 7], we investigated under which conditions Mamdani inference meets the axioms [Int1]—[Int3]. It turns out that these natural axioms of a fuzzy interpolation lead to rather restrictive conditions.

2 Generalization of Mamdani controller

In this section we describe a modification of Mamdani controller. First, we change the membership degrees of the premises by an increasing bijection $\rho\colon [0,1] \to [0,1]$. Further, we change the membership degrees of the consequents by an increasing bijection $\sigma\colon [0,1] \to [c,1]$, where $0 \leq c < 1$. Finally, we use the *degrees of conditional firing*

$$\mathcal{C}_{T,i}(X^*) = \frac{\mathcal{D}_T(X^* \circ \rho, X_i \circ \rho)}{\max_{j \leq n} \mathcal{D}_T(X^* \circ \rho, X_j \circ \rho)}, \quad (4)$$

where $\mathcal{D}_T(A,B) = \sup_{x \in X} T(A(x), B(x))$. We obtain a *controller with conditionally firing rules (CFR controller)*. Its inference rule $Con_\Theta\colon \mathcal{F}(X) \to \mathcal{F}(Y)$ is

$$Con_\Theta(X^*)(y) = \sigma^{[-1]}\Big(\max_{i \leq n} T(\mathcal{C}_{T,i}(X^*), \sigma(Y_i(y)))\Big), \quad (5)$$

where $\sigma^{[-1]}$ is the *pseudoinverse* of σ,

$$\sigma^{[-1]}(t) = \begin{cases} \sigma^{-1}(t) & \text{if } t \geq c, \\ 0 & \text{otherwise.} \end{cases}$$

This rule allows us to satisfy the axioms [Int1]–[Int3] under conditions that are quite weak and easy to verify:

Theorem 2.1 *Let $\Theta = (X_i, Y_i)_{i=1}^n$ be a rule base with normal premises and let $\rho\colon [0,1] \to [0,1]$ be any automorphism satisfying the conditions:*

[Con1] $\inf_{x \in X} \max_{i \leq n} X_i(x) > 0$ *;*

[Con2] *there is a constant $c < 1$ with $\mathcal{D}_T(X_i \circ \rho, X_j \circ \rho) \leq c$ whenever $i \neq j$;*

[Con3] *for each $i \in \{1, \ldots, n\}$, there is an element $y_i \in Y$ satisfying $Y_i(y_i) > \min_{j \neq i} Y_j(y_i)$.*

Then for any isomorphism $\sigma\colon [0,1] \to [c,1]$ the mapping Con_Θ from (5) satisfies the axioms [Int1]–[Int3].

The numerical complexity is of the same order as for the Mamdani inference.To keep it as low as possible we may choose for example $\sigma(t) = (1-c) \cdot t + c$ and $\rho(t) = t^r$, $r \in N$.

3 Test of performance

Until now, we presented theoretical arguments for CFR controller. To test how it works in practice, we made simulated experiments on one of the classical tasks of control theory—balancing a ball on a plate. The aim is to achieve and stabilize a desired position of a ball by changing the angle of a plate on which it lies. The situation was simulated using Matlab/Simulink. The model included also the static friction that has to be overcome to start movement. We made a series of tests:

1. Nonzero initial position and zero initial velocity.
2. Zero initial position and nonzero initial velocity.
3. Zero initial position and zero initial velocity, but under a presence of noise.

We tested Mamdani and CFR controller. The quality of control was evaluated using several criteria, including the number of extremes, assymptotic value, quadratic control surface (=integral of the square of error), time of reaching the desired position with a given tolerance, etc.

In the first test, CFR controller outperformed Mamdani controller in all criteria. Its control was much more smooth, avoiding rapid changes of the control variable.

In the second test we observed small oscilations of CFR controller around the desired value. We found out that this phenomenon was due to a higher sensitivity together with a coarse discretization of time. For a finer time scale CFR controller again reached better parameters.

In the third test the situation was similar: CFR controller performed worse for a coarse time discretization and better for a finer one. As the time was far longer than necessary for the hardware, this condition can be easily satisfied.

Our experiments verify that the performance of a fuzzy controller can be enhanced without changing the rules, just by choosing a different principle of implementation.

4 Conclusion

We presented arguments which show the disadvantages of the use of Mamdani controllers. These problems are inevitable when we restrict attention to compositional inference rules. Generalizing the inference rule, one can obtain satisfactory behaviour. This was demonstrated by a typical control task; our CFR controller achieved better properties without any change of the fuzzy rule base.

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