A New Method for Weighting Criteria Determination With Fuzzy Pairwise Comarison

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Abstract.

In this paper we present an approach for solving the weighting criteria assignment problem that is associated to many multicriteria decision analysis (MCDA) procedures. We use fuzzy pairwise comparison in a flexible method that allows us to manage with both vague and precise valuations.

Keywords: multi-criteria decision-making, fuzzy weighting criteria.

1 Introduction.

In MCDA, it is essential to determine the performance criteria, that describe the different points of view taken into account in a decision problem. Most of MCDA methods require a quantification of the importance of these criteria as input data, but only some of them deal with the determination of the corresponding weights.

The classical methods [1], [2], require exact values for the rating of a pair of criteria.Then they assume that the decision-maker has particular values or weights in his mind. But this statement is not immediately clear. Often, the valuation cannot be assessed precisely due to reveral reasons: the partial ignorance of the problem, or some non avaliable information, for instance. So, classical methods cannot be effectively handle these kind of problems.

To resolve this difficulty, some few methods for determining the weights of criteria, based on Fuzzy Set Theory, have been proposed [3], [4], [5]. For fuzzy methods, the x_{ij} value are "imprecise quantities": the decision-maker can express his preference judgements among the criteria with fuzzy terms such as "close to 10", "about 7", "approximately 4" or linguistic terms such as "very important", "not very important", "medium". These data have to be transformed into fuzzy numbers to operate with them.

Reviewing the literature on methods for determining the weights of criteria using fuzzy pairwise comparisons, we have noticed that these methods cannot be used in many problems because of:

- The fuzzy reciprocal judgement matrix $\bar{A}=[\bar{a}_{ij}]$must be positive ($\bar{a}_{ij}$: rating of criteria i with respect to criteria j).

- Data $\bar{a}_{ij}$ cannot be quoted in fuzzy form with a membership function .

- Due to the inconsistency in pairwise comparison, the equation systems that we have to solve could have serveral solutions and the convergence of the calculation is not guaranteed.

We propose a multi-expert method based on a hierarchical structure for the decision criteria definition and classification, with a pre-processing of the expert opinions to reduce the inconsistency of their binary comparisons.

2 The new method proposed.

The method that we are going to describe, follows a global procedure that is very similar to the other existing methods, as it is shown in the next figure:

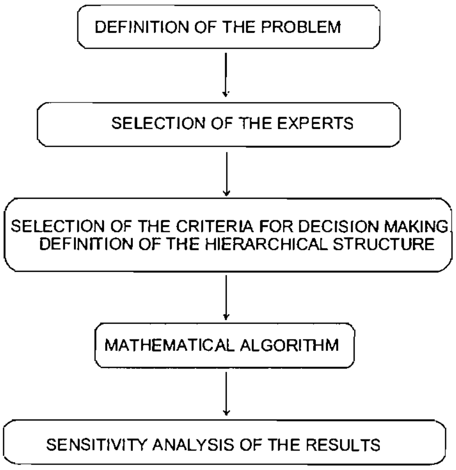

Figure 1. Global procedure.

The first task for the group of experts is to define the decision criteria and their hierarchical structure. They must reach a consensous on this subject. Then, we have the comparison groups by clustering the criteria at the same level with a common precedent (Figure 2).

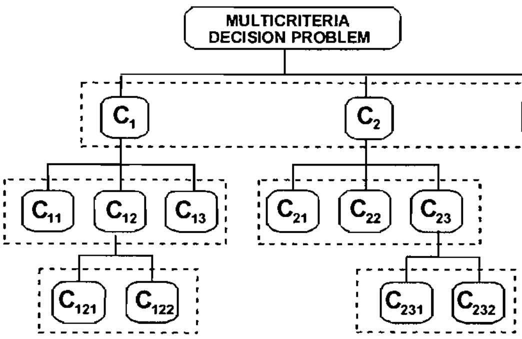

Figure 2. Hierarchical structure and comparison groups.

The next stage of the procedure deals with the opinions of the experts and the subsecuent numerical treatement to quantify them (Figure 3)

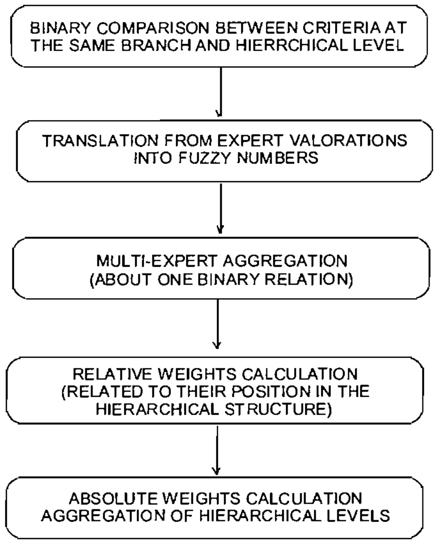

Figure 3. Mathematical algorithm

First, each expert makes the binary comparisons between all the creiteria at the same group. In this work, all the experts have expressed their opinions with judgemental statements like "More Important than" and "Much More Important than".

After that, we have classified the expert into categories, with regard to the linguistic terms that they have used. In the picture below (Table 1) we show the eight categories that we have defined. Of course, we supose that if one expert uses, for example, the statement "More Important", he has in his mind also the reciprocal one, "Less Important".

2.1 Translation from linguistic term valuations into fuzzy binary relations.

Then, *for each category*, all the Linguistic Terms have an associated membership function to quantify the binary comparisons. In Figure 4, membership functions for linguistic terms used in categories 2 and 3 are shown.

Category	1	2	3	4	5	6	7	8
Linguistic term								
With the same importance	✓	✓	✓	✓		✓	✓	✓
Only Slightly More Important				✓	✓	✓	✓	✓
More important		✓	✓	✓	✓	✓	✓	✓
Significantly More Important							✓	✓
Much More Important			✓		✓	✓	✓	✓
Very Much More Important								✓

Table 1. Expert classification into categories

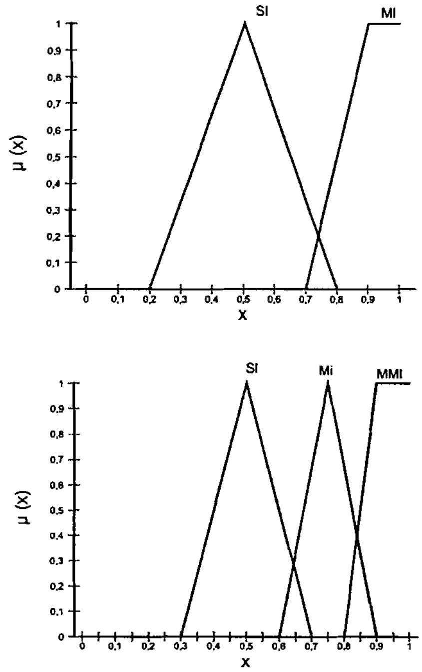

Figure 4. Membership functions associated to Categories 2 and 3.

At this stage of the method, the preferences between every pair of critera (i,j) pertaining to the same group, are defined by the corresponding fuzzy relations ($\bar{a}_{ij,pij}$) for each one of the "p_{ij}"experts that gave their opinion. Then, the fuzzy judgement matrix $\bar{A}$ is defined as follows, where the reciprocal fuzzy relation $\bar{a}_{ji,pij}$ is obtained from $\bar{a}_{ij,pij}$ as the fuzzy standard complement (we consider that it is a fuzzy relation complementary to the unit, R_{CU})[6].

$$\bar{A} = \begin{pmatrix} - & \begin{matrix}\bar{a}_{121}\\ \vdots \\ \bar{a}_{12p_{12}}\end{matrix} & \cdots & \begin{matrix}\bar{a}_{1n1}\\ \vdots \\ \bar{a}_{1np_{1n}}\end{matrix} \\ \begin{matrix}\bar{a}_{211}\\ \vdots \\ \bar{a}_{21p_{21}}\end{matrix} & - & \cdots & \begin{matrix}\bar{a}_{2n1}\\ \vdots \\ a_{2np_{2n}}\end{matrix} \\ \cdots & \cdots & - & \cdots \\ \begin{matrix}\bar{a}_{n11}\\ \vdots \\ \bar{a}_{n1p_{n1}}\end{matrix} & \begin{matrix}\bar{a}_{n21}\\ \vdots \\ \bar{a}_{n2p_{n2}}\end{matrix} & \cdots & - \end{pmatrix}$$

We need also to define the preference relationship $\bar{a}_{ii}$. But this relation has only a single value, $\bar{a}_{ii}$=(0.5,1), because there is no uncertainty when we state that one criteria has the same importance that itself.

2.2 Coherence of the binary relations.

A set of preference relations is transitive (or coherent) if:

$$(\bar{a}_{ik}+ \bar{a}_{kj})/2 = \bar{a}_{ij} \ , \forall \ i,j,k$$

But usually, the expert opinions form a set of weakly transitive binary relations. To reduce the incoherence of this set, we have calculated, for each of the "p" experts that gave their opinion about the preference relation $\bar{a}_{ij}$, the pairs of coefficients α and β, by minimizing the following equations.

$$\alpha\ \bar{a}_{ik,p} + \beta\ \bar{a}_{kj,p} = \bar{a}_{ij,p}$$
$$\alpha + \beta = 1$$

Then if there are "n" criteria inside the same group, we obtain "n-2" pairs of coefficients for each expert. By choosing the optimum pair of values, it is possible to estimate a new set of preference relations $\hat{a}_{ij,p}$. To make easier these calculation we use the First Decomposition Theorem.

2.3 Multiexpert aggregation.

We use the arithmetic-mean aggregation rule to judge the overall preference relations.

$$\hat{a}_{ij} = \frac{\sum_{k=1}^{p_{ij}} \hat{a}_{ijp_{ij}}}{p_{ij}}$$

2.4 Relative weights calculation.

We use the geometric-mean aggregation rule to obtaint tha weights of the criteria, related to their position into the hierarchical structure.

$$w_i = \left(\prod_{j=1}^{n} \hat{a}_{ij}\right)^{1/n}$$

We define this operation via the Decomposition Theorems, and it is tacitly assumed that the deviations from original shapes may be ignored.

2.5 Absolute weights calculation.

To calculate the absolute weights of the criteria that are located at the end of one branch, we multiply the relative weights of the whole branch considered.

$$W_{ijk} = w_{ijk} \cdot w_{ij} \cdot w_i$$

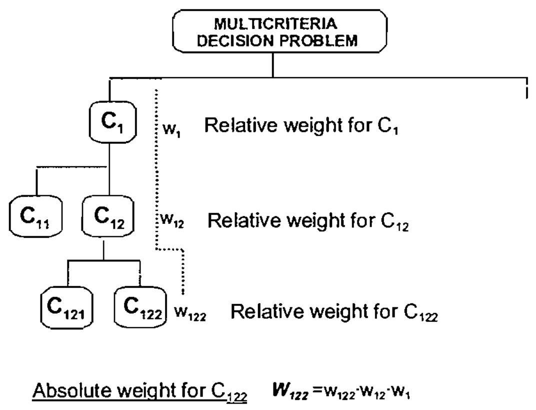

Figure 5. Example of absolute weight calculation

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