Application of Aggregation Operators in Regime-Switching Models for Exchange Rates

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Abstract

A new method of construction of regime-switching models based on utilization of aggregation operators has been recently indicated in [2] (without any appropriate investigation of its practical usability). The goal of this paper is to provide a comprehensive exposition of the new method of model construction and mainly to explore their usability in practical modeling (using the time series of exchange rates of the Slovak Crown to Euro). We conclude that the new models exhibit superior in-sample as well as out-of-sample fitting performance in comparison with models based on more standard approaches to time series analysis.

Keywords: Time series models, local trends, periodic components, ARMA models, regime-switching models, aggregation operators, in-sample and out-of-sample fitting.

1 Structure of the model

Regime-switching models have been proposed and utilized by many authors investigating financial time series. A readable exposition is presented in [5], where the classes of STAR (Smooth Transition Autoregressive) and LSTAR (Logistic STAR) models have been introduced.

The following generalization GSTAR of 2-regimes LSTAR models was proposed in [1] for a time series y_t:

$$y_t = \Phi_1(B)\, y_t\, (1 - H(q_t)) + \Phi_2(B)\, y_t\, H(q_t) \qquad (1)$$

where

$\Phi_1(B)$, $\Phi_2(B)$ are autoregressive polynomials in the shift operator B,

H is a so-called transition function, i. e., a non-decreasing surjective map of the values of a so-called threshold variable q_t to the interval [0, 1].

Similarly as in case of LSTAR models we assume that $q_t = y_{t-d}$ for a suitable delay d and H has the form

$$H(q_t) = h(\gamma\,(q_t - c)) + \frac{1}{2} \qquad (2)$$

where

$$h: R \to [-\frac{1}{2}, \frac{1}{2}]$$

is a non-decreasing shape function, $c = h^{-1}(0)$ is the threshold constant and γ is the smoothness parameter.

Examples of possible shape functions are:

a) logistic function (shifted to h(0) = 0):

$$h(x) = \frac{1}{1+e^{-x}} - \frac{1}{2} \qquad (3)$$

b) cubic spline function (see [1])

$$h(x) = \begin{cases} -\frac{1}{2} & x < -3 \\ \frac{1}{4}x - \frac{1}{108}x^3 & -3 \le x \le 3 \\ \frac{1}{2} & x > 3 \end{cases}.$$

A method of construction of regime-switching models, based on combinations of shape functions with aggregation operators (for details on aggregation operators see, e. g. [3]) has been indicated in [2]. The idea is to find an optimal form of transition between two possible regimes based on information contained in variables $y_{t-1}, \ldots, y_{t-d}$. Two possible suggested approaches are:

<u>Case i</u>.

Transform values $y_{t-1},\ldots, y_{t-d}$ by a fixed transition transformation $f\colon R \to [0, 1]$ into values $u_i = f(y_{t-i})$, $i = 1, \ldots, d$ and then apply a continuous d-dimensional aggregation operator $\mathfrak{A}\colon [0, 1]^d \to [0, 1]$, i. e.

$$F(y_{t-1}, \ldots, y_{t-d}) = \mathfrak{A}(u_1, \ldots, u_d) = \mathfrak{A}(f(y_{t-1}), \ldots, f(y_{t-d})).$$

<u>Case ii.</u>

To apply first some continuous aggregation operator $\mathfrak{B}\colon R^d \to R$ to the observed values y_{t-i}, $i = 1, \ldots, d$ and transform the resulting output by means of a transition function $g\colon R \to [0, 1]$, i. e.,

$$F(y_{t-1}, \ldots, y_{t-d}) = g(\mathfrak{B}(y_{t-1}, \ldots, y_{t-d})).$$

Typical continuous aggregation operators on the real line (the map $[0, 1]^d$ onto $[0, 1]$) are

- ✓ Arithmetic mean $AV(x_1, \ldots, x_d) = \frac{1}{d}\sum_{i=1}^{d} x_i$;
- ✓ Weighted means $W(x_1, \ldots, x_d) = \sum_{i=1}^{d} w_i x_i$, where $w_i \in [0, 1]$, $\sum_{i=1}^{d} w_i = 1$;
- ✓ OWA operators $W'(x_1, \ldots, x_d) = \sum_{i=1}^{d} w_i x'_i$, where x'_i is a non-decreasing permutation of $x_1, \ldots, x_d$, i. e., $x'_1 \le x'_2 \le \cdots \le x'_d$.

In class of OWA operators we can find also MIN and MAX operators, corresponding to extremal cases $w_1 = 1$ and $w_i = 0$ otherwise, eventually $w_d = 1$ and $w_i = 0$ otherwise. In our contribution we use also the self-dual continuous idempotent aggregation operator

$$\text{MAX-MIN} = \frac{\text{MAX}}{\text{MAX} + 1 - \text{MIN}}$$

(self-dual aggregation operators are discussed for example in [3]).

2 Modeling exchange rates time series.

The time series of exchange rates of Slovak Crown to EURO (ECU before January 1, 1999) for the period January 1, 1993 – March 10, 2005 has been investigated. Its graphical representation is shown in Fig. 1.

Figure 1. The exchange rates of Slovak Crown to Euro for the period January 1993 – March 2005

The data from 12 years (1999 – 2004) has been used for model building. Subsequent data from year 2005 served for testing the quality of prediction. The best models in the following classes has been fitted:

a) <u>Linear models in the class ARMA(p, q)</u> applied to residuals obtained after a local trend and periodic components have been removed. A local trend fitted the dramatic increase in local mean level in October 1998 and its partial reverse in 2004. Significant periodic components corresponded approximately to 2 years and 4 years (election cycle period). The best model for residuals was found in the class ARMA(2, 1). This model will be denoted as Ma.

b) <u>LSTAR models</u>.

The first step was testing against linearity that yielded rejection of AR-class hypothesis. The optimal results (minimal p-value) have been obtained for the delay parameter values equal to $d = 1$ or $d = 2$. Based on information criteria of the type AIC, SIC and HQC (for details see [7]) for LSTAR models the optimal model of the type LSTAR(1, 2) has been selected both for $d = 1$ and $d = 2$. (The corresponding values of threshold parameters of the transition function has been $c = 40{,}1$ for $d = 1$ and $c = 40{,}3$ for $d = 2$.) These models will be denoted as Mb1 and Mb2.

c) STAR models with threshold variable of aggregated type (case ii).

Following the optimality of delay parameters 1 and 2 for LSTAR models, we applied aggregation operators MIN, MAX and AV on variables y_{t-1} , y_{t-2}. In all cases the best models were of the type LSTAR(1, 2). (The optimal values of the threshold constant c were 40,3 for MIN and 40,1 for MAX and AV.) These models will be denoted as Mc1, Mc2 and Mc3.

d) STAR models with aggregated transition function values (case i) applied on values $f(y_{t-1})$, $f(y_{t-2})$ with the logistic type function (3).

As aggregation operators were used MAX-MIN and the arithmetic mean AV. The best models were in the class (2, 2) for MAX-MIN (with the $c = 39{,}1$) and in the class (1, 2) for AV (with $c = 39{,}5$). These models will be denoted as Md1 and Md2.

Finally we compared the quality of out-of-sample predictions for the above mentioned selected 8 models Ma, Mb1, Mb2, Mc1, Mc2, Mc3, Md1, Md2. The quality comparison between all pairs of these models was based on the Diebold – Mariano (DM) test of equal forecast accuracy [4]. The results are presented in the Table 1. The element in the intersection of the i-th row and j-th column of this table is equal to

a) 1
b) -1
c) 0

if the prediction quality of the model in the row i in comparison with the model in the column j is

a) significantly better
b) significantly worse
c) is not significantly different.

The following Table 2 shows the number of comparisons in which individual models have significantly better prediction quality than competing models.

Table 1: Results of DM tests

	Ma	Mb1	Mb2	Mc1	Mc2	Mc3	Md1	Md2
Ma	x	1	1	1	-1	-1	-1	-1
Mb1	-1	x	0	-1	-1	-1	-1	-1
Mb2	-1	0	x	-1	-1	-1	-1	-1
Mc1	-1	1	1	x	-1	-1	-1	-1
Mc2	1	1	1	1	x	1	-1	-1
Mc3	1	1	1	1	-1	x	1	0
Md1	1	1	1	1	-1	-1	x	-1
Md2	1	1	1	1	1	1	1	x

Table 2: Summary test results for individual models

Ma	Mb1	Mb2	Mc1	Mc2	Mc3	Md1	Md2
3	0	0	2	5	5	4	7

We see that the best prediction quality has been achieved by the model Md2:

$$f(x) = \frac{1}{1 + e^{-\frac{(x-39,5)}{0,5}}} - \frac{1}{2}$$

$$G(y_{t-1}, y_{t-2}) = \frac{f(y_{t-1}) + f(y_{t-2})}{2}$$

$$y_t = (0{,}45 + 0{,}99\, y_{t-1})\,(1 - G(y_{t-1}, y_{t-2})) +$$
$$+ (0{,}36 + 1{,}23\, y_{t-1} - 0{,}29\, y_{t-2})\, G(y_{t-1}, y_{t-2})$$

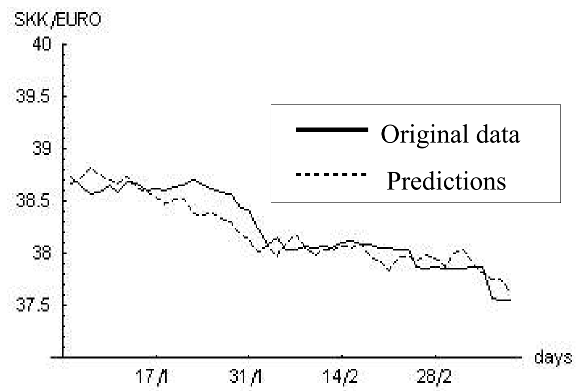

Figure 2. Original data and predictions for 48 days with model Md1

Residuals (see Fig. 3) exhibit stationarity (on the basis of the KPSS test [6] with the significance level

$\alpha =$ 0,05) that can be fitted by the normal distribution N(0; 0,012).

Figure 3. Residuals obtained after the model Md1

3 Conclusion.

In our practical modeling study, the best quality has been achieved for the model of the new class d), (that has not been practically applied yet) where the values of the transition function are obtained as the outputs of an aggregation operator with inputs given by the images (in a common shape function) of a fixed number of delayed values of the modeled time series. We believe that these encouraging modeling results will also inspire deeper theoretical investigations of the new class of models.

Remark.

The modeled original data of the exchange rate SKK/EURO has been downloaded from the web page of the Slovak National Bank http://www.nbs.sk

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