A Working Hypothesis on the Semantics/Accuracy Synergy

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Abstract

The main objective of this work is to assess the impact of the initial conditions regarding interpretability and accuracy on the optimizer's performance and present some guidelines in order to assist the designer. Automatic infusion of anesthesia is a task where both interpretability and accuracy of the controller are highly desirable features. Under this pretext, a description on how these goals can be measured is made and a set of evolutionary optimization schemes is set up. The results show that independently of the optimization process used, the introduction of a semantically valid individual in the initial population is a factor of success in the evolutionary optimization process.

Keywords: interpretability, accuracy, optimization, semantic integrity, fuzzy control

1 Introduction

In many domains of application the inherent expressive power of the fuzzy systems is a clear advantage that the designer can't afford to lose. This feature combined with the universal approximation property of such systems (subject to the assumption of an unbounded number of rules) makes them well suited for a wide range of applications in modelling, classification and control.

A paradigmatic application from the bio-medical field is the automatic infusion of anesthesia, more precisely the automatic neuromuscular blockade control of patients under surgery. For this kind of application the expressiveness or interpretability of the deployed system plays a key role, in what concerns overcoming the reluctance of the anaesthetist to perform his task in a different manner, by delivering a control system which actions could be easily understood and validated.

Nevertheless, given the critical field of application, the accuracy of this controller has to be pursued by all possible means. Given a set of linguistic terms and a rule base provided by the anaesthetist which impose restrictions on the number of rules it's necessary to fine tuning the linguistic terms in order to attain maximum performance.

Immediately two questions arise: *(i)* To what extent can this optimization be performed without a heavy loss of interpretability? and *(ii)* Which are the ideal initial conditions of the optimization process in order to boost-up the accuracy of the optimized controller?

In respect to the former question, faced with the lack of theoretical results on the existence or absence of a trade-off between accuracy and semantics in fuzzy modelling and control, the authors started a few years ago, an empirical study on the subject. Some previous results were progressively presented elsewhere [3, 4, 6]. Moreover in the last decade many works have been reported on the preservation of semantics during fuzzy system optimization and on the relationship between accuracy and semantics of these systems, including a comprehensive compilation of some of these results on two edited volumes [1, 2].

Regarding the latter question, this paper intends to pinpoint some clues which may enlighten its response. The taken approach consisted in setting up different evolutionary optimization schemes. These range from a simple unconstrained optimization process to optimization under constraints and to multi-objective optimization. Each optimization process was subject to different initial conditions, both in respect to accuracy, both in respect to interpretability. To obtain additional details about the experimental setups as well as the complete set of obtained results the reader is referred to [5].

In the following section we briefly define the conditions of our study. This includes the mathematical model of the patients, the control scheme as well as the applied semantic constraints. Section 3 summarizes the obtained results while in section 4 the conclusions are summarized.

2 The Conditions of the Study

2.1 Patients Model

The control of neuromuscular blockade (or relaxation) is characterized by a high degree of uncertainty associated with the dynamic behaviour of the bio-system under control. This behaviour may be modelled by a linear pharmacokinetic model relating the drug infusion rate $u(t)$ with the plasma concentration $c_p(t)$ and a non-linear model relating $c_p(t)$ with the induced pharmacodynamic response $r(t)$ [11, 12]. The resulting overall model, with atracurium as muscle relaxing agent, is described by the state equations:

$$\begin{array}{rcllll} \dot{x}_i(t) & = & -\lambda_i\, x_i(t) & + & a_i\, u(t) & \\ c_p(t) & = & \sum_{i=1}^{2} x_i(t) & & & \\ \dot{c}_{\mathrm{E}}(t) & = & -k_{\mathrm{E}0}\, c_{\mathrm{E}}(t) & + & k_{\mathrm{E}0}\, c_p(t) & \quad (1) \\ r(t) & = & \frac{100 c_{p50}^{\gamma}}{c_{p50}^{\gamma}+c_{\mathrm{E}}^{\gamma}(t)} & & & \end{array}$$

where $i = 1, 2$ and λ_i, a_i , $k_{\mathrm{E}0}$, c_{p50} and γ are patient-dependent parameters. The variable $r(t)$, normalized between 0 and 100, correspond to the level of muscular activity, with 0 corresponding to full paralysis and 100 to full muscular activity.

The observed responses in the operating theatre detain a larger time-lag and variability than those that could be inferred from the above-mentioned model. To meet this drawback an empirical model for atracurium was developed in [7]. It results from the inclusion of a first-order system, $g(s) = \frac{1/\tau}{s+1/\tau}$, in the pharmacokinetic model (in a series connection).

2.2 Controller Setup

In an attempt to provide a bridge between the engineer and the anaesthetist we choose to optimize a Mamdani fuzzy proportional-derivative (FPD) controller. This choice was due not only to its inherent simplicity and expressiveness but also to the good overall behaviour, for this problem, when matched against a set of other fuzzy controllers both adaptive and non-adaptive [4].

The controller setup is as follows: For each fuzzy universe of discourse, five linguistic terms were assumed ranging from *Medium Negative* to *Medium Positive* for Error and Error Change and from *Zero* to *Very Big Positive* for Control action. Each one was represented by a normal triangular membership function. The exact meanings of these terms are to be determinate by the optimization process. The rule base, as presented in Table 1, was previously elicited and remains unchanged during optimization.

Table 1: Rule base for Mamdani PD-type controller

		$\Delta e(k)$				
		MN	SN	ZE	SP	MP
$e(k)$	MN	VBP	BP	MP	MP	SP
	SN	BP	MP	MP	SP	ZE
	ZE	MP	MP	SP	ZE	ZE
	SP	MP	SP	ZE	ZE	ZE
	MP	ZE	ZE	ZE	ZE	ZE

2.3 Measuring Accuracy

In order to evaluate the precision of the control system a measure to gauge both the tracking error and control effort was used and is defined by:

$$Q = \sum_{i=1}^{N} \tfrac{1}{2}[(ref_i - y_i)^2 + \rho(u_i - u_{i-1})^2] \quad (2)$$

where ref_i, y_i, u_i, denote respectively the target value for the relaxation, the relaxation level and the control signal in the sampling instant i.

2.4 Measuring Semantics

The essence of semantic integrity of granular (and fuzzy, in particular) interfaces is essentially captured through the series of following fundamental and intuitively appealing requirements, cf. e.g. [8, 9, 10]: a) natural zero positioning b) limited overlap between neighbouring fuzzy sets (distinguishability) c) coverage of the universe of discourse, and d) unimodality of fuzzy sets forming the interface.

To evaluate these requirements, that together represent what is meant by interpretability, we used the three indices below.

$$
\begin{aligned}
&J_1 = K_1 \|c_{ZE}\|^2 step(\|c_{ZE}\|) \\
&J_2 = K_2 \times \textstyle\sum_x [(M_p(x) - 1)^2 step(M_p(x) - 1)] \\
&J_3 = K_3 \times \textstyle\sum_x [(M_p(x) - 0.2)^2 step(0.2 - M_p(x))]
\end{aligned}
\tag{3}
$$

where K_1, K_2 and K_3 are constants which permits the tuning of the relative weight of each J_i when aggregated into one compound objective function. The function *step* is the standard unit step function.

The first index J_1 is intended to promote the natural localization of the linguistic term Zero (c_{ZE} denotes the center of the membership function of the linguistic term Zero) as required in *a)* above. Index J_2 penalizes membership functions with a poor distinguishability level, according to *b)*. For *c)*, J_3 is used implying that a low level of coverage of the universe of discourse is highly penalized. The employment of Normal membership functions satisfies *d)*.

The sigma-count operator, $M_p(A)$ gives a measure of the cardinality of a fuzzy set A, as follows:

$$M_p(A) = \sqrt[p]{a_1^p + \ldots + a_n^p} \tag{4}$$

where a_i $(i = 1, \ldots, n)$ are the degrees of membership defining a fuzzy set A, p being a positive integer. In the reported experiments we used $p = 1$. The objective function for interpretability is therefore stated as the aggregation of the three previous indices:

$$J = \sum_i J_i \tag{5}$$

3 Selected Results

The applied optimization schemes range from a simple unconstrained optimization process to optimization under constraints and to multi-objective optimization. The initial conditions and parameters of the EA's are given below.

The maximum number of generations was 80. Each population's individual was represented by an array of 15 real-valued ordered pairs (3 UoD's $\times$ 5 m.f.'s). Each pair consisting of a value for the center position and a value for the base of a symmetrical triangular membership function. The applied selection scheme was stochastic sampling with replacement. An uniform intermediate (randomized arithmetic recombination) crossover operator with $0,9$ probability was used. The chosen probability of mutation was $0,04$. The control task was executed over the training set (100 models) along 600 samples (200 minutes in a real environment). Moreover the semantic constraints' constants, found after some preliminary tests, were $K_1 = 10000$, $K_2 = 50$ and $K_3 = 100$.

We set up four different sets of experiments. For the first set the same randomly generated population with 100 individuals was used in all optimization runs.

In the second set of experiments the first individual was replaced by coding a group of membership functions satisfying the requirements for natural zero positioning, distinguishability and coverage of the universe of discourse. This solution detained a compound 2621 average accuracy value, see (2), and a null (optimum) value for interpretability, see (5). Fig. 1 shows the codified symmetrical triangular membership functions for this particular individual.

For the third experimental setup the first individual was replaced by a codified solution obtained via unconstrained single objective (accuracy) optimization. As can be seen in figure 2, this solution exhibited several drawbacks including one singular fuzzy set, poor distinguishable membership functions and a misplaced linguistic term for Zero. It detained a somewhat better average accuracy Q value (2614) than the previous human-defined interpretable one and a compound 3760

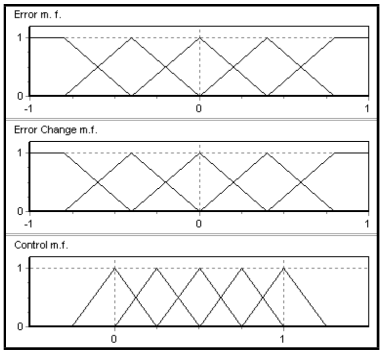

Figure 1: Membership functions with a null (optimum) value for interpretability.

value for interpretability index J.

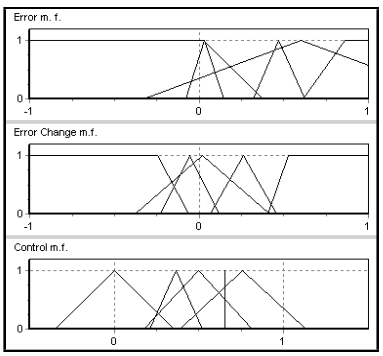

Figure 2: Optimized membership functions using accuracy as single objective

Finally, experiments were also conducted by replacing the two first individuals by the two previously described solutions in the initial population.

3.1 Single Objective Optimization

For this setup each objective, accuracy as stated in (2) and interpretability as stated in (5), was considered independently of the remaining one. An elitist approach was used by keeping the two best individuals of a generation for the next generation. The fitness function f, for the minimization problem was $f = 1/(1 + \Delta)$, with Δ subject to no constraints assuming a normalized accuracy value or a normalized interpretability value depending on wether the setting was an accuracy-guided optimization or an interpretability-guided one.

Table 2 presents the average, the standard deviation and the best Q accuracy value for the accuracy-guided optimization. The rows show the values for the four initialization schemes: randomly generated population ($Non\, init.$), inclusion of interpretable solution in the initial population ($Int.\, solution$), inclusion of individual with accuracy-optimized membership functions ($Acc.\, solution$) and inclusion of the two former individuals ($Both\, solutions$). Additionally, although without any sort of influence in the optimization process, the corresponding interpretability values are also show in the same table.

Table 2: Statistical figures for the accuracy-guided optimization.

	Accuracy (Q)			**Interpretability (J)**		
	Average	*Std. Dev.*	*Best*	*Average*	*Std. Dev.*	*Best*
Non init.	2515,7	55,6	2483,5	6432,9	**1553,1**	3759,6
Int. solution	**2476,0**	6,1	**2471,7**	6165,7	2329,1	2192,5
Acc. solution	2480,1	**4,0**	2475,5	10390,1	4020,4	6027,0
Both solutions	2479,6	5,2	2473,5	**6120,2**	2715,5	**2030,9**

From table 2 it is clear that this process is highly dependent on the initial population. Good initial candidates, either from the interpretability or from the accuracy point of view, are an important factor to the success of this process. Both the average and the best values in respect to accuracy were obtained with the inclusion of the interpretable solution in the initial population.

In table 3 the average, standard deviation and best J interpretability values of the interpretability-guided process are presented. Moreover the corresponding accuracy values are also displayed in the same table. For this optimization scheme only two different initialization methods were employed: randomly generated population ($Non\, init.$) and inclusion of individual with accuracy-optimized membership functions ($Acc.\, solution$). The initialization setups which include the interpretable solution were discarded (since from the interpretability's viewpoint that's an optimal solution).

Table 3: Statistical figures for the interpretability-guided optimization.

	Interpretability (J)			**Accuracy (Q)**		
	Average	*Std. Dev.*	*Best*	*Average*	*Std. Dev.*	*Best*
Non init.	14,3	16,4	**0,3**	3642,7	592,6	**2839,4**
Acc. solution	**8,7**	**8,7**	**0,3**	**3590,0**	**520,7**	2977,8

As can be seen in table 3 the inclusion of the accurate solution originate better results, not only in respect to accuracy as expected, but also in respect to interpretability.

Resembling the previous optimization scheme, we can see that good results were obtained for the pursued objective. Nevertheless there is no correspondence whatsoever for the neglected objective.

3.2 Optimization under Constraints

In this setup, and as before, an elitist approach was employed (the two best individuals of a generation were kept for the next generation). The fitness function f, for the minimization problem was $f = 1/(1 + \Delta)$ where $\Delta = Q + J$, with the normalized cost values Q and J given respectively by (2) and (5) accordingly to the penalty function method.

Table 4 presents the average, the standard deviation and the best value for each objective. As before, each row displays the results for each initialization scheme.

Table 4: Set characterization of the multi-criteria results

	Accuracy (Q)			**Interpretability (J)**		
	Average	*Std. Dev.*	*Best*	*Average*	*Std. Dev.*	*Best*
Non init.	2578,8	73,8	**2515,8**	36,6	20,9	8,7
Int. solution	2529,1	11,0	2520,6	3,6	1,4	2,5
Acc. solution	2618,1	77,4	2530,5	23,3	11,2	11,4
Both solutions	**2528,6**	**7,6**	2517,1	**2,3**	**1,0**	**0,8**

The best average values both in respect to accuracy and to interpretability were obtained with the inclusion of both the interpretable and the accurate solution in the initial population. Very close mean values were obtained by only including the interpretable solution in the initial population. For this choice of parameters, when interpretability is clearly an objective, the inclusion of the previously unconstrained optimized solution seems to be pointless even when matched against a totally randomized population.

3.3 Multi-objective Optimization

In the reported experiments a Pareto-based fitness assignment similar to the one used in SPEA2 algorithm [13] was applied. For each individual is calculated his strength, i.e., the number of solutions it dominates according to the optimization objectives Accuracy and Interpretability as stated in (2) and (5). The raw fitness of the individual is given by the sum of strengths of its dominators in the population and in the archive. A density estimation measure was added to this value in order to favour the more sparsely distributed solutions.

The archive was composed of 30 elements which participated in the selection process. The updating mechanism of this archive relied on information about dominance and also on density inside a dynamic niche radius proportional to the Euclidean distance between nondominated boundary solutions. The boundary solutions were always kept.

Fig.3 shows the superposition of the resulting Pareto-front approximations obtained by the different initialization schemes. In fig.3 the solution A is the one which detains the minimum Euclidean distance to the origin.

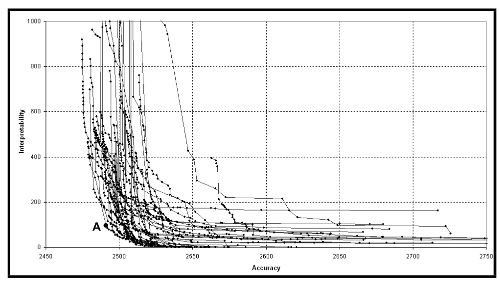

Figure 3: Pareto front approximations

Table 5 presents the average, the standard deviation and the best value for each objective under the four initialization schemes. These indices were calculated over the set of the most accurate solutions (boundary-accurate solutions) obtained in the optimization runs.

Table 5: Set characterization of the boundary-accurate solutions

	Accuracy (Q)			**Interpretability (J)**		
	Average	*Std. Dev.*	*Best*	*Average*	*Std. Dev.*	*Best*
Non init.	2507,5	20,7	2492,3	**505,4**	180,7	**217,6**
Int. solution	**2483,0**	**2,6**	2478,7	560,2	**152,5**	410,5
Acc. solution	2497,7	7,4	2480,0	2452,3	1786,3	369,9
Both solutions	2485,8	6,2	**2474,8**	1515,8	1428,9	317,5

Interesting enough, we see that the major contributions to an ideal Pareto-front came from the cases where the interpretable individual was a member of the initial population. As a matter of fact the depicted solution A (see fig.3) belongs to a Pareto front approximation obtained under such condition. The resulting optimized membership functions for solution A holding a compound interpretability value of 95.1 and an accuracy value of 2491.1, can be seen in fig. 4.

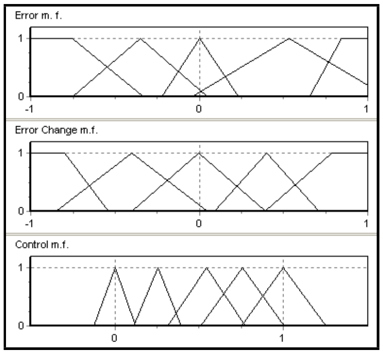

Figure 4: Membership functions for the best solution (under Euclidean distance criterion)

The analysis of the figure 3 and table 5 reveal that the best average results in terms of accuracy are consistently obtained when the interpretable solution is incorporated into the initial population.

4 Conclusions

This work was focused on the impact of the initial conditions's interpretability and accuracy on the optimizer's performance.

The results show that the introduction of a semantically valid individual in the initial population is a factor of success for the optimization process. This holds independently of the optimization process and even if the optimization process accounts only for accuracy. This suggest that a semantically valid individual is also a good initial candidate from the accuracy perspective.

As a consequence, the following working hypothesis is formulated: An initial semantically valid candidate promotes the performance of the evolutionary optimizer.

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