Interpretation of Support Vector Machines by means of Fuzzy Rule-Based Systems

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Abstract

Support Vector Machines (SVM) have demonstrated their ability in solving classification problems in an optimal way with a solid mathematical background. In this paper we improve the interpretability of SVM's by showing that every SVM is exactly represented by a Fuzzy Rule Based-System, for every kernel function used. Nevertheless, this system is in some way compact in their rules and for that reason, we introduce another FRBS, called δ-FRBS, that approximates it and which is suitable to decompose its rules in simple fuzzy propositions. We show it with an example in the last section.

Keywords: Support Vector Machines, Fuzzy Rule-Based Systems.

1 Introduction

Support Vector Machines (SVM) are learning systems based on statistical learning theory which have attracted increasing attention from Learning Machine Community because of their optimal applications in classification data and noise resistance. SVMs have a beautiful high-dimensional mathematical background because offer a linear manner (in a feature space) to solve non-linear classification problems in the input space.

Many applications have been found, speaker verification and identification, face detection and text categorization among others [12, 10, 7] or fusion of techniques like Fuzzy Support Vector Machines, developed in [8] or a fuzzy modelling framework based on support vector machine given in [6] or a Fuzzy Kernel Perceptron, a new learning method obtained [4].

Considering a possible limitation of being SVM black boxes, studies about the situation have been developed before this article [9], and restrictions to the kernel functions have been imposed in some of them. For example, an excellent research is made in [5] where is shown that every translation invariant kernel is a Mercers Kernel but not every Mercers Kernel is translation invariant, leaving excluded wide used kernel functions.

In this paper we construct a simpler set of fuzzy rules from a trained SVM for every commonly used Kernel function and we provide an interpretation for every SVM by means of Fuzzy Rule-Based Systems (FRBS) which have previously demonstrated [1, 3] their ability to extract the knowledge from feedforward multi-layer neural networks.

In sections 2 and 3 we offer a couple of tutorials (SVM and FRBS respectively) that could help to the non specialist reader. In section 4 we give the novel set of rules and finally in section 5 provide useful examples for commonly used kernels functions.

2 Support Vector Machines

A Support Vector Machine in its dual form is given by

$$f(\vec{x}) = sign(\sum_{k=1}^{m} \alpha_i y_i K(\vec{x}, \vec{x_i}) + b) \qquad (1)$$

Where K is a a kernel function which satisfy the

Mercer's conditions [11], and using it is possible to make all the necessary operations in the original (input) space by using $< \phi(\vec{x_i}), \phi(\vec{x_j}) >= K(\vec{x_i}, \vec{x_j})$ because $K(\vec{x_i}, \vec{x_j})$ is a dot product in a feature space where the problem is linearly classifiable. And $\phi(\vec{x})$ is a function which maps vectors from the input space to the feature space.

We recommend [2, 11] for an extensive explanation of learning (training) algorithms for SVM.

Most used kernel functions are showed below:

$\exp -(\|\vec{x} - \vec{\omega}\|^2/2\sigma^2)$, $\exp -(\|\vec{x} - \vec{\omega}\|/2\sigma^2)$, $tanh(\alpha < \vec{x},\vec{\omega} > +\beta)$, $< \vec{x},\vec{\omega} >^p$, $(< \vec{x},\vec{\omega} > +1)^p$.

3 Fuzzy Rule-Based Systems

3.1 *TSK* Fuzzy Rule-Based Systems

Takagi-Sugeno-Kang FRBS usually present the following form:

$R_k =$ If X_1 is A_1 and X_2 is A_2 and $...X_n$ is A_n then $Y_k = p_n.X_n + p_{n-1}X_{n-1} + ... + p_1X_1 + p_0$

where X_i are the system input variables, A_i are fuzzy sets specifying their meaning, and Y is the output variable. The output Y of a FRBS with m TSK rules is computed as the weighted average of the individual rule outputs Y_i $i = 1, .., m$, in the following way:

$$Y = \frac{\sum_{i=1}^{m} Y_i g_i}{(\sum_{i=1}^{m} g_i)}$$

where $g_i = T(A_1(x_1), ..., A_n(x_n))$ is the matching between the antecedent part of the rule and the current system inputs, T is a t-norm and $\vec{x} = (x_1, x_2, ..., x_n)$.

This kind of Fuzzy Rule-Based Systems output will be used in the implementation of the FRBS that we propose in this article. The fuzzy rules will be implemented such that, just one fuzzy proposition is used in IF-part and THEN-part is only composed by p_0.

4 Support Vector Machines are Fuzzy Rule-Based Systems

The set of support vectors$\{\vec{x_1}, \vec{x_2}, ..., \vec{x_m}\}$ and α_i, y_i $(for\ 1 \leq i \leq m)$ are already fixed, defined by training.

For that reason,

$$\sum_{k=1}^{m} \alpha_i y_i K(\vec{x}, \vec{x_i}) = \langle \overrightarrow{\alpha_i y_i}, \overrightarrow{K(\vec{x}, \vec{x_i})} \rangle = h(\vec{x})$$

being the last expression the dot product in R^m and vectors $\overrightarrow{\alpha_i y_i} = (\alpha_1 y_1, ..., \alpha_m y_m)$, $\overrightarrow{K(\vec{x}, \vec{x_i})} = (K(\vec{x}, \vec{x_1}), ..., K(\vec{x}, \vec{x_m}))$.

Theorem 1 *For every Support Vector Machine as*

$$f(\vec{x}) = sign(\textstyle\sum_{k=1}^{m} \alpha_i y_i K(\vec{x}, \vec{x_i}) + b)$$

the following Fuzzy Rule-Based System

R_1 : *If* $h(\vec{x})$ *is* $I_{(\eta,\infty)}$ *then* $Y_1 = 1$
R_2 : *If* $h(\vec{x})$ *is* $I^*_{(\eta,\infty)}$ *then* $Y_2 = -1$

where

$$I_{(\eta,\infty)}(x) = \begin{cases} 1 & if\ x \in (\eta, \infty) \\ 0 & if\ x \in (-\infty, \eta) \end{cases}$$

$$I^*_{(\eta,\infty)}(x) = 1 - I_{(\eta,\infty)}(x)$$

$\eta = |b|$ *if* $b \leq 0$, *and* $\eta = -b$ *if* $b > 0$;

with output $Y = \frac{\sum_{i=1}^{2} Y_i g_i}{(\sum_{i=2}^{m} g_i)}$

where $g_1 = I_{(\eta,+\infty)}(h(\vec{x}))$ and $g_2 = I^*_{(\eta,+\infty)}(h(\vec{x}))$;

is such that, for every $\vec{x}$ in the input space,

$$\begin{aligned} Y &= \frac{\sum_{i=1}^{2} Y_i g_i}{(\sum_{i=1}^{2} g_i)} \\ &= sign(\sum_{k=1}^{m} \alpha_i y_i K(\vec{x}, \vec{x_i}) + b) = f(\vec{x}). \end{aligned}$$

Let $\vec{e}$ be a vector belonging to input space and we evaluate it the Fuzzy Rule-Based System announced above,

If $h(\vec{e}) \in (-\infty, \eta)$ the output fired is

$$\begin{aligned} Y &= \frac{\sum_{i=1}^{2} Y_i g_i}{\sum_{i=1}^{2} g_i} = \frac{Y_2 g_2}{g_2} = Y_2 = -1 \\ &= sign(h(\vec{e}) + b) \\ &= sign(\sum_{k=1}^{m} \alpha_i y_i K(\vec{x}, \vec{x_i}) + b) = f(\vec{x}) \end{aligned}$$

If $h(\vec{e}) \in (\eta, \infty)$ the output fired is

$$\begin{aligned} Y &= \frac{\sum_{i=1}^{2} Y_i g_i}{(\sum_{i=1}^{2} g_i} = \frac{Y_1 g_1}{g_1} = Y_1 = 1 \\ &= sign(h(\vec{e}) + b) \\ &= sign(\sum_{k=1}^{m} \alpha_i y_i K(\vec{x}, \vec{x_i}) + b) = f(\vec{x}) \end{aligned}$$

$$Y = \frac{\sum_{i=1}^{2} Y_i g_i}{(\sum_{i=2}^{m} g_i)} = \begin{cases} 1 & \text{if } h(\vec{e}) \in (\eta, \infty) \\ -1 & \text{if } h(\vec{e}) \in (-\infty, \eta) \end{cases}$$

Even thought we have found a FRBS that fires the same output that a SVM for every Kernel function, interpretability has not been improved.

For that reason, in order to improve interpretability of SVM by means a FRBS we will construct another FRBS that approximates to the later given, and it will let us extract knowledge in an easier way:

Theorem 2 *For every Support Vector Machine as*

$$f(\vec{x}) = sign(\textstyle\sum_{k=1}^{m} \alpha_i y_i K(\vec{x}, \vec{x_i}) + b)$$

the following Fuzzy Rule-Based System

R_1 : *If* $h(\vec{x})$ *is* $S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}$ *then* $Y_1 = 1$
R_2 : *If* $h(\vec{x})$ *is* $S^{*}_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}$ *then* $Y_2 = -1$

where

$S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x)$=*Sigmoid function with* $(\eta-\frac{\delta}{2}, \eta+\frac{\delta}{2})$ *(Fig.1),*

$S^{*}_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x) = 1 - S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x)$,

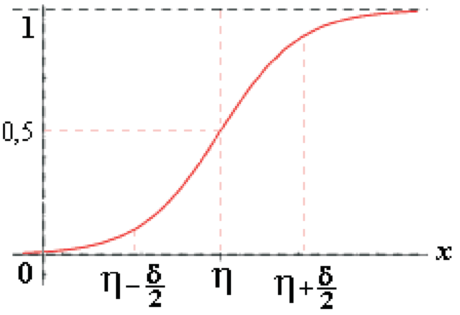
$\eta = |b|$ if $b \leq 0$, and $\eta = -b$ if $b > 0$;

with output $Y = \frac{\sum_{i=1}^{2} Y_i g_i}{(\sum_{i=2}^{m} g_i)}$ where

$g_1 = S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(h(\vec{x}))$ and $g_2 = S^{*}_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(h(\vec{x}))$

is such that, when $\delta \longrightarrow 0$ for every $\vec{x}$ in the input space, $Y \longrightarrow f(\vec{x})$

Let $\vec{e}$ be a vector belonging to input space and we evaluate it the Fuzzy Rule-Based System announced above,



$$S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})} = \frac{1}{1 + e^{(-\lambda_\delta(x-\eta))}}$$

Figure 1: Sigmoid function with $(\eta - \frac{\delta}{2}, \eta + \frac{\delta}{2})$.

R_1 : If $h(\vec{e})$ is $S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}$ then $Y_1 = 1$
R_2 : If $h(\vec{e})$ is $S^{*}_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}$ then $Y_2 = -1$

If $h(\vec{e}) \in \mathbf{R}$ the output fired is

$$\begin{aligned} Y = \frac{\sum_{i=1}^{2} Y_i g_i}{(\sum_{i=1}^{2} g_i} &= \frac{1 \cdot g_1 - 1 \cdot g_2}{g_1 + g_2} = \frac{g_1 - g_2}{g_1 + g_2} \\ (g_2 = 1 - g_1) & \\ = \frac{g_1 - (1 - g_1)}{g_1 + (1 - g_1)} &= \frac{g_1 - 1 + g_1}{1} = 2 \cdot g_1 - 1 \\ &= 2 \cdot S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(h(\vec{e})) - 1. \end{aligned}$$

As we have seen, our Fuzzy Rule-Based System fires the following output:

$$Y = 2 \cdot S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(h(\vec{e})) - 1.$$

We also find that $0 = 2 \cdot S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(h(\vec{e})) - 1$ is possible only for $S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(h(\vec{x})) = \frac{1}{2}$, idem est, the boundary is fixed in η.

Thus, when $\delta \longrightarrow 0$ (Fig.2), it means $\gamma_\delta \longrightarrow \infty$

$Y = 2 \cdot S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(h(\vec{e})) - 1 = 2 \cdot \left(\frac{1}{1+e^{(-\lambda_\delta(x))}}\right) - 1$
(with $\gamma_\delta \longrightarrow \infty$)

$$= \begin{cases} 1 & \text{if } h(\vec{e}) \in (\eta, \infty) \\ -1 & \text{if } h(\vec{e}) \in (-\infty, \eta) \end{cases}.$$

Definition 1 : A Fuzzy Rule-Based System with the features in Theorem 2 is called δ-FRBS.

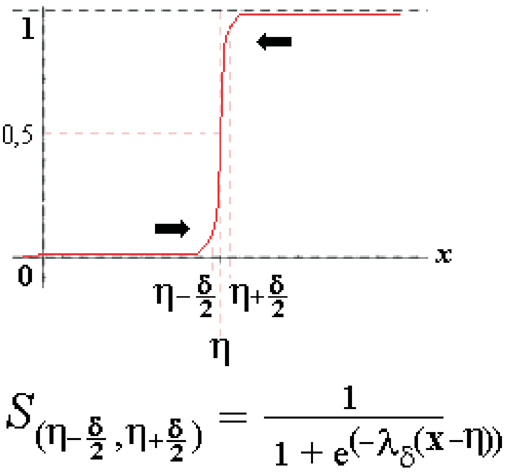


Figure 2: Sigmoid function when $\delta \longrightarrow 0$.

5 Improving the interpretability

As next step, to improve the interpretability of δ-FRBS, we invoke some results from [1, 3]. We announce them but, the respective demonstration will not be given here.

5.1 Previous results

Proposition 1 : Let $f : X \longrightarrow Y$ be a bijective function and let $\oplus$ be a binary operation defined in the domain of f, X. Then there is one and only one operation, $\otimes$, defined in the range of f, Y, verifying $f(x_1 \oplus x_2) = f(x_1) \otimes f(x_2)$.

Definition 2 : Let f be a bijective function and let $\oplus$ be an operation defined in the domain of f. The operation $\otimes$ whose existence is proven in the preceding proposition is called the f-dual of $\oplus$.

Now, let us consider the operation $+$ in $\mathbf{R}$ and the sigmoid function $S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}$. The latter is a bijective function from $\mathbf{R}$ to (0, 1). Thus we have the following,

Lemma 2 : The $S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}$-dual of $+$ is $\otimes$, defined as $*(a_1,...,a_n) = \frac{a_1 \cdots a_n}{a_1 \cdots a_n + (1-a_1)\cdots(1-a_n)\cdot e^{-(n-1)\cdot\eta\cdot\lambda_\delta}}$

Lemma 3 : The $S^*_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}$-dual of $+$ is $\otimes$, defined as $*(a_1,...,a_n) = \frac{a_1 \cdots a_n}{a_1 \cdots a_n + (1-a_1)\cdots(1-a_n)\cdot e^{(n-1)\cdot\eta\cdot\lambda_\delta}}$

5.2 Applying f-duality

Remembering the nature of $h(\vec{x})$ we have:

$$\begin{array}{l} R_1 : \text{If } \sum_{k=1}^{m} \alpha_i y_i K(\vec{x},\vec{x_i}) \text{ is } S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x) \\ \quad \text{Then } Y_1 = 1 \\ R_2 : \text{If } \sum_{k=1}^{m} \alpha_i y_i K(\vec{x},\vec{x_i}) \text{ is } S^*_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x) \\ \quad \text{Then } Y_2 = -1 \end{array}$$

Applying the operator $*$, taking in count that sigmoid function is bijective,

$$\begin{array}{l} R_1 : \text{If } \alpha_1 y_1 K(\vec{x},\vec{x_1}) \text{ is } S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x) \; * \\ \qquad \alpha_2 y_2 K(\vec{x},\vec{x_2}) \text{ is } S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x) \; *...* \\ \qquad \alpha_m y_m K(\vec{x},\vec{x_m}) \text{ is } S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x) \\ \quad \text{Then } Y_1 = 1 \\ R_2 : \text{If } \alpha_1 y_1 K(\vec{x},\vec{x_1}) \text{ is } S^*_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x) \; * \\ \qquad \alpha_2 y_2 K(\vec{x},\vec{x_2}) \text{ is } S^*_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x) \; *...* \\ \qquad \alpha_m y_m K(\vec{x},\vec{x_m}) \text{ is } S^*_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x) \\ \quad \text{Then } Y_2 = -1 \end{array}$$

$$\begin{array}{l} R_1 : \text{If } K(\vec{x},\vec{x_1}) \text{ is } S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x\alpha_1 y_1) \; * \\ \qquad K(\vec{x},\vec{x_2}) \text{ is } S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x\alpha_2 y_2) \; *...* \\ \qquad K(\vec{x},\vec{x_m}) \text{ is } S_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x\alpha_m y_m) \\ \quad \text{Then } Y_1 = 1 \\ R_2 : \text{If } K(\vec{x},\vec{x_1}) \text{ is } S^*_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x\alpha_1 y_1) \; * \\ \qquad K(\vec{x},\vec{x_2}) \text{ is } S^*_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x\alpha_2 y_2) \; *...* \\ \qquad K(\vec{x},\vec{x_m}) \text{ is } S^*_{(\eta-\frac{\delta}{2},\eta+\frac{\delta}{2})}(x\alpha_m y_m) \\ \quad \text{Then } Y_2 = -1 \end{array}$$

$$\begin{array}{l} R_1 : \text{If } K(\vec{x},\vec{x_1}) \text{ is } S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_1 y_1},\frac{\eta+\frac{\delta}{2}}{\alpha_1 y_1})}(x) \; * \\ \qquad K(\vec{x},\vec{x_2}) \text{ is } S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_2 y_2},\frac{\eta+\frac{\delta}{2}}{\alpha_2 y_2})}(x) \; *...* \\ \qquad K(\vec{x},\vec{x_m}) \text{ is } S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_m y_m},\frac{\eta+\frac{\delta}{2}}{\alpha_m y_m})}(x) \\ \quad \text{Then } Y_1 = 1 \\ R_2 : \text{If } K(\vec{x},\vec{x_1}) \text{ is } S^*_{(\frac{\eta-\frac{\delta}{2}}{\alpha_1 y_1},\frac{\eta+\frac{\delta}{2}}{\alpha_1 y_1})}(x) \; * \\ \qquad K(\vec{x},\vec{x_2}) \text{ is } S^*_{(\frac{\eta-\frac{\delta}{2}}{\alpha_2 y_2},\frac{\eta+\frac{\delta}{2}}{\alpha_2 y_2})}(x) \; *...* \\ \qquad K(\vec{x},\vec{x_m}) \text{ is } S^*_{(\frac{\eta-\frac{\delta}{2}}{\alpha_m y_m},\frac{\eta+\frac{\delta}{2}}{\alpha_m y_m})}(x) \\ \quad \text{Then } Y_2 = -1 \end{array}$$

and then $K(\vec{x},\vec{x_i})$ can be seen as $K(O(\vec{x},\vec{x_i}))$ where $O(\vec{x},\vec{x_i})$ is a function like a $< \vec{x},\vec{x_i} >$ or $\|\vec{x}-\vec{x_i}\|$. For example,

$$tanh(\xi < \vec{x},\vec{x_i} > +\gamma) = K(O(\vec{x},\vec{x_i})) \text{ where}$$

$O(\vec{x},\vec{x_i}) =< \vec{x},\vec{x_i} >$ and $K(x) = tanh(\xi(x)+\gamma)$.

In this way we get,

R_1 : If $O(\vec{x},\vec{x_1})$ is $S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_1 y_1},\frac{\eta+\frac{\delta}{2}}{\alpha_1 y_1})}(K(x))\ *$
$O(\vec{x},\vec{x_2}))$ is $S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_2 y_2},\frac{\eta+\frac{\delta}{2}}{\alpha_2 y_2})}(K(x))\ *...*$
$O(\vec{x},\vec{x_m})$ is $S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_m y_m},\frac{\eta+\frac{\delta}{2}}{\alpha_m y_m})}(K(x))$
Then $Y_1 = 1$
R_2 : If $O(\vec{x},\vec{x_1})$ is $S^*_{(\frac{\eta-\frac{\delta}{2}}{\alpha_1 y_1},\frac{\eta+\frac{\delta}{2}}{\alpha_1 y_1})}(K(x))\ *$
$O(\vec{x},\vec{x_2})$ is $S^*_{(\frac{\eta-\frac{\delta}{2}}{\alpha_2 y_2},\frac{\eta+\frac{\delta}{2}}{\alpha_2 y_2})}(K(x))\ *...*$
$O(\vec{x},\vec{x_m})$ is $S^*_{(\frac{\eta-\frac{\delta}{2}}{\alpha_m y_m},\frac{\eta+\frac{\delta}{2}}{\alpha_m y_m})}(K(x))$
Then $Y_2 = -1$

with output $Y = \frac{\sum_{i=1}^{2} Y_i g_i}{(\sum_{i=2}^{m} g_i)}$, where

$$g_1 = *(S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_1 y_1},\frac{\eta+\frac{\delta}{2}}{\alpha_1 y_1})}(K(x)), S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_2 y_2},\frac{\eta+\frac{\delta}{2}}{\alpha_2 y_2})}(K(x)), ..., S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_m y_m},\frac{\eta+\frac{\delta}{2}}{\alpha_m y_m})}(K(x)))$$

$$g_2 = *(S^*_{(\frac{\eta-\frac{\delta}{2}}{\alpha_1 y_1},\frac{\eta+\frac{\delta}{2}}{\alpha_1 y_1})}(K(x)), S^*_{(\frac{\eta-\frac{\delta}{2}}{\alpha_2 y_2},\frac{\eta+\frac{\delta}{2}}{\alpha_2 y_2})}(K(x)), ..., S^*_{(\frac{\eta-\frac{\delta}{2}}{\alpha_m y_m},\frac{\eta+\frac{\delta}{2}}{\alpha_m y_m})}(K(x))$$

This kind of rules, which represent Support Vector Machines, offer us the knowledge that we were looking for.

In the following example we exhibit an particular case of theory developed above, solving the X-Or classification problem.

6 Examples

6.1 the X-Or problem

After training, A Support Vector Machine which solves the problem in an optimal way has the following form:

$$f(\vec{x}) = sign\left(\sum_{k=1}^{m} \alpha_i y_i (< \vec{x},\vec{x_i} > +1)^3 + b\right)$$

being $(< \vec{x},\vec{x_i} > +1)^3 = K(\vec{x},\vec{x_i})$, $\eta = 5.55e-017$ and

$\vec{x_i}$	y_i	α_i
$\vec{x_1} = (-1,-1)$	-1	$\alpha_1 = 0.038461538461538464$
$\vec{x_2} = (-1,+1)$	$+1$	$\alpha_2 = 0.038461538461538464$
$\vec{x_3} = (+1,-1)$	$+1$	$\alpha_3 = 0.038461538461538464$
$\vec{x_4} = (+1,+1)$	-1	$\alpha_4 = 0.038461538461538464$

With these values, we get our δ-FRBS with the following membership functions:

$$S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_i y_i},\frac{\eta+\frac{\delta}{2}}{\alpha_i y_i})}((x+1)^3) = \frac{1}{1+e^{(-\lambda_\delta(\alpha_i y_i (x+1)^3 - \eta))}}$$

which are sigmoid functions centered in $\sqrt[3]{\frac{\eta}{\alpha_i y_i}}-1$.

The rules are:

R_1 : If $< \vec{x},\vec{x_1} >$ is $S_{(\frac{\eta-\frac{\delta}{2}}{-\alpha_1},\frac{\eta+\frac{\delta}{2}}{-\alpha_1})}((x+1)^3)\ *$
$< \vec{x},\vec{x_2} >$ is $S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_2},\frac{\eta+\frac{\delta}{2}}{\alpha_2})}((x+1)^3)\ *$
$< \vec{x},\vec{x_3} >$ is $S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_3},\frac{\eta+\frac{\delta}{2}}{\alpha_3})}((x+1)^3)\ *$
$< \vec{x},\vec{x_4} >$ is $S_{(\frac{\eta-\frac{\delta}{2}}{-\alpha_4},\frac{\eta+\frac{\delta}{2}}{-\alpha_4})}((x+1)^3)$
Then $Y_1 = 1$
R_2 : If $< \vec{x},\vec{x_1} >$ is $S^*_{(\frac{\eta-\frac{\delta}{2}}{-\alpha_1},\frac{\eta+\frac{\delta}{2}}{-\alpha_1})}((x+1)^3)\ *$
$< \vec{x},\vec{x_2} >$ is $S^*_{(\frac{\eta-\frac{\delta}{2}}{\alpha_2},\frac{\eta+\frac{\delta}{2}}{\alpha_2})}((x+1)^3)\ *$
$< \vec{x},\vec{x_3} >$ is $S^*_{(\frac{\eta-\frac{\delta}{2}}{\alpha_3},\frac{\eta+\frac{\delta}{2}}{\alpha_3})}((x+1)^3)\ *$
$< \vec{x},\vec{x_4} >$ is $S^*_{(\frac{\eta-\frac{\delta}{2}}{-\alpha_4},\frac{\eta+\frac{\delta}{2}}{-\alpha_4})}((x+1)^3)$
Then $Y_2 = -1$

The membership functions of these rules are illustrated in Fig.3 and Fig.4.

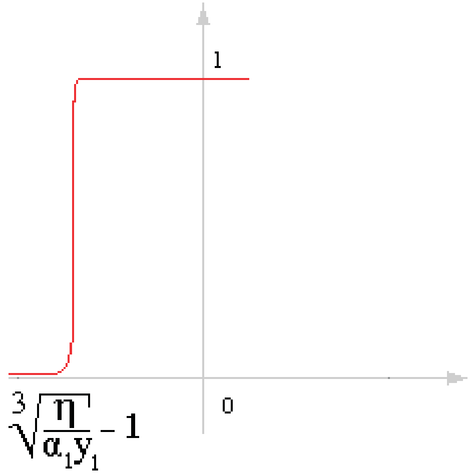


Figure 3: The membership function $S_{(\frac{\eta-\frac{\delta}{2}}{-\alpha_1},\frac{\eta+\frac{\delta}{2}}{-\alpha_1})}((x+1)^3-\eta) = S_{(\frac{\eta-\frac{\delta}{2}}{-\alpha_4},\frac{\eta+\frac{\delta}{2}}{-\alpha_4})}((x+1)^3-\eta)$

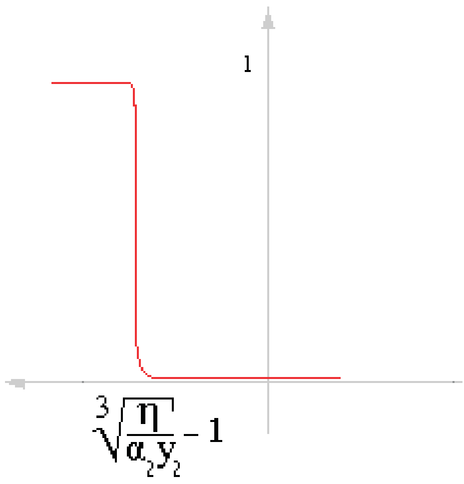


Figure 4: The membership function $S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_2},\frac{\eta+\frac{\delta}{2}}{\alpha_2})}((x+1)^3) = S_{(\frac{\eta-\frac{\delta}{2}}{\alpha_3},\frac{\eta+\frac{\delta}{2}}{\alpha_3})}((x+1)^3)$

7 Conclusions

As shown in X-Or example we contribute by means of this research to interpretability of SVMs and its conversion in simple rules. It represents an important step in the knowledge extraction/insertion of SVM. In future works we will show how to replace the operator $*$ in the IF-part by t-norms and t-conorms in a commonly fashion.

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