The fuzzy sets in a maintenance problem

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Abstract

Abstract. Many ranking methods have been proposed so far. However, there is not a method that can always give a satisfactory solution to every situation; some method are counterintuitive, others methods give different rankings solutions for the same conditions, this is not the case of Liou and Wang method, however, this method can give the same ranking for different fuzzy number, so that, we propose a ranking fuzzy numbers with the introduction of an inside point in the fuzzy number that resolve this situation. We apply this method in a maintenance problem.

Keywords: Ranking fuzzy numbers, index of optimism, index of modality

1 Introduction

Most of the times, the decision-maker is not able to define the importance of the criteria or the goodness of the alternatives with respect to each criterion in a strict way. In general for the decision maker it is easier when he/she evaluates their judgements by means of linguistic terms.

Decisions in maintenance management, focused to any aspect of maintenance function, its objectives, organizational structure and actions give up the limits of the department and need information from different sources. These decisions could change in magnitude; form the high cost equipment replacement through minim reparation. It is important to understand the structure of the process of decisions maintenance problem. We treat it with the Analytic Hierarchy Process (AHP).

As it is well known, the (AHP) is a simple MCDA to deal with unstructured and multi attribute problems which was developed by Saaty [16, 17] and consists of decomposing a complex problem into its components, organizing the components into levels to generate a hierarchical structure. The aim of constructing this hierarchy is to determine the impact of the lower level on an upper level, which is achieved by paired comparisons provided by the decision maker. In this case the AHP was used for the propose to structuring and clarifying the preferences in a decision problem in maintenance.

In the pairwise comparisons method criteria and alternative are presented in pairs to one or more referees (e.g., experts or decision makers). It is necessary to evaluate individual alternatives, derive weights for the criteria, construct the overall rating of the alternatives and identify the best alternative. Let us denote the alternatives by $A_1, A_2,.., A_n$ (n is the number of compared alternatives), their actual weights by $w_1, w_2,.., w_n$ and the matrix of the ratios of all weights by $W=[w_i/w_j]$. The matrix of pairwise comparisons A=[a_{ij}] represents the intensities of the expert's preference between individual pairs of alternatives (A_i versus A_j, for all i,j=1,2,..,n) chosen usually from a given scale. The element a_{ij} are considered to be estimates of the ratios w_i/w_j where w is the vector of actual weights of the alternatives, which is what we want to find. All the ratios are positive and satisfy the reciprocity property: $a_{ij}=1/a_{ji}$ i,j=1,2,..,n. Saaty´s eigenvector solution of $Aw=\lambda_{max}w$ always exists if the consistency (or transitivity) condition $a_{ij} * a_{jk} = a_{ik}$ (i,j,k=1,2,..,n) is satisfied.

Decision makers usually find that it is more confident to give fuzzy triangular judgements than fixed values judgements as Saaty. This is because usually he/she is unable to explicit about his/her preferences due to the vagueness, ambiguity nature of the comparison process.

In practical use, ranking fuzzy numbers is a very important procedure for decision making in a fuzzy environment. To resolve the task of comparing fuzzy numbers, many authors have proposed fuzzy ranking methods which give a totally ordered set or ranking.

Bortolan and Degani [3] provided not only a systematical review of fuzzy ranking research but also gave results of comparisons among various ranking methods.

Chen and Hwang [6] were the first to present a twenty ranking method classified into three major classes according the media each method use. There are preference relation methods, a fuzzy mean and spread method, fuzzy scoring methods, and linguistic methods. Each main class is further divided according to the techniques used.

Wang and Kerre [17, 18] to organize more than thirty ordering indices classified them into three categories. In the first class, each index is associated with a mapping F from the set of fuzzy quantities to the real line $\Re$ in order to transform the involved fuzzy quantities into real numbers. Fuzzy quantities are then compared according to the corresponding real numbers. In the second class, reference sets are set up and all the fuzzy quantities to be ranked are compared with the reference sets. In the last class, a fuzzy relation is constructed to make pairwise comparisons between the fuzzy quantities involved. These pairwise comparisons serve as a basis to obtain the final ranking orders.

Liou and Wang [15] proposed a method of ranking fuzzy numbers with integral values. The method, which is independent of the type of membership functions used and the normality of the functions, can rank more than two fuzzy numbers simultaneously. It is relatively simple in computation, especially in ranking triangular and trapezoidal fuzzy numbers. Further, an index of optimism is used to reflect the decision maker's optimistic attitude.

The paper structure is as follows: Section 2 define some fuzzy numbers. Section 3 establishes a ranking fuzzy number with an inside point that resolves the sample situation. Due to lack of space we do not introduce a section for the AHP approach, but we do expose the more important characteristics in this introduction. For the same reason we only present in section 4 the results of the maintenance problem where Liou and Wang's index was modified. Finally, in Section 5 the most important conclusions and issues for further research are presented.

2 Fuzzy Numbers

The concept of fuzzy number can be defined as follows:

Definition 1. A real fuzzy number A is described as any fuzzy subset of the real line $\Re$ with membership function f_A which processes the following properties:
(a). f_A is a continuous mapping from $\Re$ to the closed interval [0, 1];

(b). $f_A(x) = 0$, for all $x \in (-\infty, a]$;

(c). f_A is strictly increasing on $[a, b]$;

(d). $f_A(x) = w$, for all $x \in [b, c]$;

(e). f_A is strictly decreasing on $[c, d]$;

(f). $f_A(x) = 0$, for all $x \in (d, \infty]$,

where $a \le b \le c \le d$ are real numbers. We may let $a = -\infty$, or $a = b$, or $b = c$, or $c = d$, or $d = +\infty$.

Unless elsewhere specified, it is assumed that A is convex and bounded; i.e., $-\infty < a, d < \infty$. If w = 1 in (d), A is a normal fuzzy number. We adopted this definition in this study and for convenience, the fuzzy number in Definition 1 can be denoted by $A = (a, b, c, d)$.

*Definition 2***.** The membership function f_A of A can be expressed as:

$$f_A(x) = \begin{cases} f_A^L(x), & a \le x \le b, \\ w, & b \le x \le c, \\ f_A^R(x), & c \le x \le d, \\ 0, & otherwise, \end{cases} \qquad (1)$$

where $f_A^L: [a, b] \to [0, w]$ and $f_A^R: [c, d] \to [0, w]$.

Since $f_A^L: [a, b] \to [0, w]$ is continuous and strictly increasing, the inverse function of f_A^L exists.

Similarly, since f_A^R : [c, d] $\rightarrow$ [0, w] is continuous and strictly decreasing, the inverse function of f_A^R also exists. The inverse functions of f_A^L and f_A^R can be denoted by g_A^L and g_A^R, respectively. Since f_A^L : [a, b] $\rightarrow$ [0, w] is continuous and strictly increasing, g_A^L : [0, w] $\rightarrow$ [a, b] is also continuous and strictly increasing. Similarly, since f_A^R: [c, d] $\rightarrow$ [0, w] is continuous and strictly decreasing, g_A^R : [0, w] $\rightarrow$ [c, d] is also continuous and strictly increasing, g_A^L and g_A^R are continuous on [0, w]; they are integrable on [0,w]. That is, both $\int_0^1 g_A^L\, dy$ and $\int_0^1 g_A^R\, dy$ exist.

For a trapezoidal fuzzy number A=[a,b,c,d;1], its membership function f_A is the same given in (1) with the value w=1.

From Definition 2, it is obvious that f_A^L it is the left membership function of the fuzzy number A, it is continuous and strictly increasing in [a,b], and f_A^R it is the right membership function of the fuzzy number A, it is continuous and strictly decreasing in [c,d].

In this paper, we treat with fuzzy triangular numbers. The fuzzy number A is triangular if its membership function f_A is given by:

$$f_A(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{x-c}{b-c}, & b \le x \le c \\ 0, & otherwise \end{cases} \qquad (2)$$

where a,b and c are real numbers. In this case the triplet [a,b,c;1] represents the number and with the same meaning that the trapezoidal ones.

3 Ranking fuzzy numbers by three areas

We consider two triangular fuzzy numbers A_1=(1,4,5), with membership function given by

$$\mu_{A_1}(x) = \begin{cases} \dfrac{x-1}{3}, & 1 \le x \le 4 \\ 5-x, & 4 \le x \le 5 \\ 0, & elsewhere \end{cases} \qquad (3)$$

whose,

$S_L(A_1) = \int_0^1 (y+2)dy = \dfrac{5}{2}$ and,

$S_R(A_1) = \int_0^1 (6-3y)dy = \dfrac{9}{2}$

and the A_2 = (2,3,6) with membership function

$$\mu_{A_2}(x) = \begin{cases} x-2, & 2 \le x \le 3 \\ \dfrac{6-x}{3} & 3 \le x \le 6 \\ 0, & elsewhere \end{cases} \qquad (4)$$

whose,

$S_L(A_2) = \int_0^1 (3y+1)dy = \dfrac{5}{2}$ and ,

$S_R(A_2) = \int_0^1 (5-y)dy = \dfrac{9}{2}$

It is clear that by the Liou and Wang' method, the two fuzzy numbers are equal.

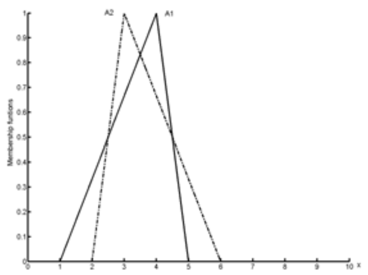

Figure 1: Representation of fuzzy numbers A_1 y A_2

Remark: For us the interpretation of the fuzzy number A_2=(2,3,6), could be the linguistic label "rather more than around 3".

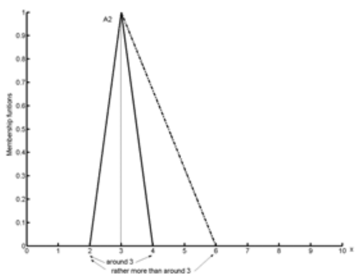

Figure 2:Representation "rather more than around 3"

It is logical to think that the label "rather less than around 4" is better than "rather more than around 3" so that A_1=(1,4,5) $\succ$ A_2=(2,3,6).

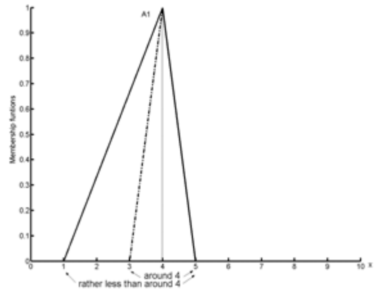

Figure 3: Representation “rather less than around 4”

It is for this reason that we try to find a solution to these cases that it does not solve in a satisfactory way the method of Liou and Wang. So that we propose a modification in the Liou and Wang approach.

Definition 3: Let A a fuzzy number defined by f_A^L and f_A^R with the associated inverse functions g_A^L and g_A^R respectively, then the area generated by f_A^L in [-∞,b] is defined by:

$$S_L(A) = a + \int_a^b f_A^L(x)dx = \int_0^1 g_A^L(y)dy \quad (5)$$

This area is related to the left side of the fuzzy number.

And the area generated by f_A^R en [-∞,d] is defined by:

$$S_R(A) = b + \int_c^d f_A^R(x)dx = \int_0^1 g_A^R(y)dy \quad (6)$$

This area is related to the right side of the fuzzy number.

Definition 4: Let A be a fuzzy number, we define the area to the point that represents the height b as,

$$S_H(A) = b \quad (7)$$

this area is related to the mode of the fuzzy number.

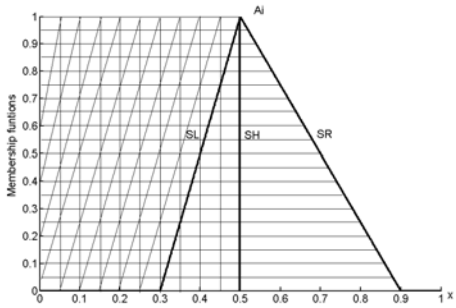

Figure4: Representation of a triangular fuzzy number and S_L (A$_i$), S_H (A$_i$), and S_R (A$_i$)

Suppose that S ={A_1,A_2,…,A_n} is a set of convex fuzzy numbers, and that I is a mapping from S to the real numbers $\Re$, where a natural order exists. Hence

$$I : S \rightarrow \Re$$

For A_i, $A_j \in$ S , the ranking function has the following properties.

i. if $I(A_i) < I(A_j)$, then $A_i < A_j$

ii. if $I(A_i) = I(A_j)$, then $A_i = A_j$

iii. if $I(A_i) > I(A_j)$, then $A_i > A_j$

With this definition we propose the following ranking function $I_{\beta,\lambda}$

Definition 5: If A is a fuzzy number with membership function f_A , defined as in (2), then the ordering index by three areas is defined as:

$$I_{\beta,\lambda}(A) = \beta S_H(A) + (1-\beta)\left[LW_i^\lambda(A)\right] = \\ = \beta S_H(A) + (1-\beta)\left[\lambda S_R(A) + (1-\lambda)S_L(A)\right] \quad (8)$$

when $S_R(A)$ represents the upper mean value associated with the inverse function of f_A^R , $S_L(A)$ is the lower mean value of the f_A^L in the fuzzy number and $S_H(A)$ is the area to the middle point of core of the fuzzy number, β is the index of modality β ∈ [0,1] that represents the importance of the central value front extreme values and λ is the decision-maker’s index of optimism λ ∈ [0,1].

Therefore, $S_L(A)$, $S_H(A)$, and $S_R(A)$ must be considered when ranking fuzzy numbers.

Remark: Given two fuzzy numbers A_i and A_j verify that

$$I_{\beta,\lambda}(A_i + A_j) = I_{\beta,\lambda}(A_i) + I_{\beta,\lambda}(A_j) \quad (9)$$

Proof. It is straightforward, taking into account the properties of integral functions.

4. Case of study

We are going to study a decision problem in maintenance in an engine factory that is specialized in production, sale and maintenance of four stroke engines, medium and slow speed.

One of the most important steps that should be done in a maintenance process and in the engine reparation is the cleaning of every its component. The testing process and reconditioning of every component request that every piece has a high quality cleaning; if not the reparation process will not be appropriate.

The problem concerned with is pieces with diverse degrees of dirt, with very different geometry, and a work process that demands speed and flexibility. Must be taken into account criteria like the total cost of annual operation, the productivity of the used system, the load capacity of the system, the cleaning efficiency and the healthiness of the used products. The global objective of the problem is to decide which is the best system for cleaning pieces.

Depending on the type of dirt a procedure is applied. The processes for the cleaning of pieces are very diverse and each one has their advantages and their drawbacks.

We are dealing with a problem characterized by the following components:

1. Objective: Choose the best cleaning system
2. Alternatives:
   - A1: Mechanical Cleaning
   - A2: Conventional Cleaning
   - A3: Ultrasonic Cleaning
   - A4: Chemical Cleaning
3. Criteria:
   - C1: Total cost of annual operation
   - C2: Productivity volumetric of the system
   - C3: Capacity of load of the system
   - C4: Efficiency in the cleaning
   - C5: Healthiness

We consider the AHP approach in which the decision-maker compare these five criteria and the four alternatives with respect to the criteria by using an analytical hierarchy process.

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{pmatrix} EI & EI & SmI & SmI & SmI \\ EI & EI & SmI & EI & SmI \\ SlI & SlI & EI & VSLI & EI \\ SlI & EI & VSMI & EI & SmI \\ SlI & SlI & EI & SlI & EI \end{pmatrix} \end{array}$$

Table 1: Decision-maker's preferences in the pairwise comparison process

Verbal judgements	Decision Maker Intervals
(EI)	[0.58,1,1.71]
(SmI)/(SlI)	[1.71,2.61,3.69] / [0.27,0.38,0.58]
(SMI)/(SLI)	[3.69,4.77,5.67] / [0.176,0.21,0.27]
(VSMI)/(VSLI)	[5.49,6.66,9] / [0.11,0.15,0.18]
(EMI)/(ELI)	[6.84,7.29,9] / [0.11,0.137,0.146]

Table 2: Global priorities and areas values

	Global priorities	$[S_L(A_i), S_H(A_i), S_R(A_i)]$
A1	[0.454,1.043,2.487]	[0.748, 1.043,1.765]
A2	[1.382,2.729,5.7706]	[2.0553,2.729,4.2498]
A3	[3.145,5.6408,10.585]	[4.3929,5.6408,8.1131]
A4	[4.393,4.518,11.833]	[4.4555,4.518,8.1755]

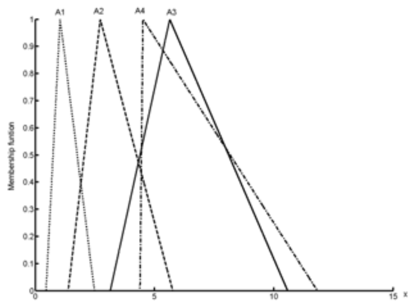

Figure 5: Results $[S_L(A_i), S_H(A_i), S_R(A_i)]$

The decision-maker have considered as modality value β=0.5 and also as degree of optimism λ=0.5, the result is:

$$I_{0.5,0.5}(A_1) = \beta S_H(A_1) + (1-\beta)[\lambda S_R(A_1) + (1-\lambda)S_L(A_1)]$$
= 0.5x1.043+0.5x[0.5x1.765+0.5x0.748]= 1.1498

$$I_{0.5,0.5}(A_2) = \beta S_H(A_2) + (1-\beta)[\lambda S_R(A_2) + (1-\lambda)S_L(A_2)]$$
=0.5x2.729+0.5x[0.5x4.2498+0.5x2.0553]= 2.9408

$$I_{0.5,0.5}(A_3) = \beta S_H(A_3) + (1-\beta)[\lambda S_R(A_3) + (1-\lambda)S_L(A_3)]$$
=0.5x5.6408+0.5x[0.5x8.1131+0.5x4.3929]= 5.9469

$$I_{0.5,0.5}(A_4) = \beta S_H(A_4) + (1-\beta)[\lambda S_R(A_4) + (1-\lambda)S_L(A_4)]$$
=0.5x4.518+0.5x[0.5x8.1755+0.5x4.4555]= 5.4167

Then $A_3 \succ A_4 \succ A_2 \succ A_1$, being the final weights the following.

Table 3: Final weights associated with alternatives.

	Weights normalized
A1	0.0744
A2	0.1903
A3	0.3845
A4	0.3505

5. Conclusions

The proposed method is simple to apply when is being defined as the sum of three areas, whose are associated with three continuous functions and delimited in [0,1].

The outlined approach presents all the advantages that it already presented the method of Liou and Wang, improving this in some cases when the index of Liou and Wang are not sufficiently discriminatory.

This proposal produces satisfactory results, not only in very defined problems, even it gives solution to some of the cases that this method could not discriminate, at least the discrimination outlined by the index of Liou and Wang is guarantied.

Also, this method includes the possibility to order *n* fuzzy numbers at the same time, some ranking methods has not got this performance.

Finally, this methodology does not require a big computational effort for great scale problems. Also is very simple and fast to apply when triangular and trapezoidal fuzzy number are used.

We apply this method to a problem in maintenance where we can prove the advantages of this approach.

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