Min-based fusion of possibilistic networks

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Abstract

The problem of merging miltiple-source uncertain information is a crucial issue in many applications. This paper proposes an analysis of possibilistic merging operators where uncertain information are encoded by means of possibilistic networks. We first show that the min-based merging of possibilistic networks having the same DAG structures can be easily achieved in a polynomial time. We then propose solutions to merge possibilistic networks having different structures.

1 Introduction

Possibilistic networks [14, 10, 16] are important tools proposed for an efficient representation and analysis of uncertain information. Their success is due to their simplicity and their capacity of representing and handling independence relationships which are important for an efficient management of uncertain pieces of information. Possibilistic networks are directed acyclic graphs (DAG), where each node encodes a variable and every edge represents a relationship between two variables. Uncertainties are expressed by means of conditional possibility distributions for each node in the context of its parents.

This paper deals with the problem of combining pieces of information issued from different sources. This problem can be encountered in various fields of applications such as databases, multi-agent systems, expert opinion pooling, etc.

Several works have been recently achieved to fuse propositional or weighted logical knowledge bases issued from different sources [1, 4, 6, 7, 8, 9, 2].

The aim of this paper is to propose approaches for fusing uncertain pieces of information represented by possibilistic networks.

The rest of this paper is organised as follows. Next section gives a brief background on possibility theory and possibilistic networks. Section 3 presents the conjunctive combination mode. Section 4 discusses the fusion of possibistic networks having same graphical structures. Section 5 deals with fusion of possibilistic networks having different structures but the union of their DAGs is free of cycles. Section 6 concludes the paper.

2 Basics of possibility theory

Let $V = \{A_1, A_2, ..., A_N\}$ be a set of variables. We denote by $D_A = \{a_1, .., a_n\}$ the domain associated with the variable A. By a we denote any instance of A. $\Omega = \times_{A_i \in V} D_{A_i}$ denotes the universe of discourse, which is the Cartesian product of all variable domains in V. Each element $\omega \in \Omega$ is called a state of Ω. In the following, we only give a brief recalling on possibility theory, for more details see [12].

2.1 Possibility distributions and possibility measures

A possibility distribution π is a mapping from Ω to the interval [0, 1]. It represents a state of knowledge about a set of possible situations distinguishing what is plausible from what is less plausible. Given a possibility distribution π

defined on the universe of discourse Ω, we can define a mapping grading the *possibility measure* of an event $\phi \subseteq \Omega$ by $\Pi(\phi) = \max_{\omega \in \phi} \pi(\omega)$. A possibility distribution π is said to be *normalized*, if $\max_{\omega \in \Omega} \pi(\omega) = 1$.

In a possibilistic setting, conditioning consists in modifying our initial knowledge, encoded by a possibility distribution π, by the arrival of a new *sure* piece of information $\phi \subseteq \Omega$. There are different definitions of conditioning. In this paper, we use the so-called min-based conditioning proposed in an ordinal setting [12] [18], and defined by:

$$\Pi(\psi \mid \phi) = \begin{cases} \Pi(\psi \wedge \phi) & if\ \Pi(\psi \wedge \phi) < \Pi(\phi) \\ 1 & \text{otherwise} \end{cases} \tag{1}$$

There are also several definitions of independence relations in the possibilistic framework [11, 15]. In this paper, we use the so-called *Non-Interactivity* independence [19] and defined by:

$$\Pi(x \wedge y \mid z) = min(\Pi(x \mid z), \Pi(y \mid z)), \forall x, y, z. \tag{2}$$

2.2 Min-based possibilistic graphs

This section defines min-based possibilistic graphs. A *min-based possibilistic graph* over a set of variables V, denoted by $\mathbb{N} = (\pi_{\mathbb{N}}, G_{\mathbb{N}})$, consists of:

- a *graphical component*, denoted by $G_{\mathbb{N}}$, which is a DAG (Directed Acyclic Graph). Nodes represent variables and edges encode the link between the variables. The parent set of a node A is denoted by U_A.
- a *numerical component*, denoted by $\pi_{\mathbb{N}}$, which quantifies different links. For every root node A ($U_A = \emptyset$), uncertainty is represented by the a priori possibility degree $\pi_{\mathbb{N}}(a)$ of each instance $a \in D_A$, such that $\max_a \pi_{\mathbb{N}}(a) = 1$.
  For the rest of the nodes ($U_A \neq \emptyset$) uncertainty is represented by the conditional possibility degree $\pi_{\mathbb{N}}(a \mid u_A)$ of each instances $a \in D_A$ and $u_A \in D_{U_A}$. These conditional distributions satisfy the following normalization condition:
  $\max_a \pi_{\mathbb{N}}(a \mid u_A) = 1$, for any u_A.

The set of a priori and conditional possibility degrees induces a unique joint possibility distribution defined by:

Definition 1 *Given a min-based possibilistic graph $\mathbb{N} = (\pi_{\mathbb{N}}, G_{\mathbb{N}})$, we define its associated joint possibility distribution, denoted by π_m, using the following* min-based chain rule *:*

$$\pi_m(A_1, .., A_N) = \min_{i=1..N} \Pi_{\mathbb{N}}(A_i \mid U_{A_i}) \tag{3}$$

3 Conjunctive merging

One of the important aims in merging uncertain information is to exploit complementarities between sources in order to get a more complete and preciseglobal point of view.

In possibility theory, given a set of possibility distributions $\pi_i's$, one of the basic combination mode is the conjunction (i.e., the minimum) of possibility distributions (for more details on the semantic fusion of possibility distributions see [5]). Namely:

$$\forall \omega, \pi_{\oplus(\omega)} = min_{i=1,n} \pi_i(\omega).$$

The conjunctive aggregation makes sense if all the sources are regarded as equally and fully reliable since all values that are considered as impossible by one source but possible by the others are rejected. Besides, if two sources provide the same information $\pi_1 = \pi_2$, the result of the conjunctive combination is still the same distribution; indeed minimum is the only idempotent conjunction connective.

Let $\mathbb{N}1$ and $\mathbb{N}2$ be two possibilistic networks. Our aim is to directly construct from $\mathbb{N}1$ and $\mathbb{N}2$ a new possibilistic network, denoted by $\mathbb{N}\oplus$. The new possibilistic network should be such that:

$$\forall \omega, \pi_{\mathbb{N}\oplus}(\omega) = min(\pi_{\mathbb{N}1}(\omega), \pi_{\mathbb{N}2}(\omega)).$$

In [3] equivalent transformations between possibilistic graphs and possibilistic knowledge bases have been provided. Besides, it has been shown in [2] that at the syntactic level the conjunctive combination mode is equivalent to consider the union of possibilistic knowledge bases.

Hence, from these existing transformations, it is possible to compute the fused possibilistic network associated with the conjunctive comnination mode. This can be done by first transforming each possibilistic network into its equivalent possibilistic knowledge base. Then we compute a possibilistic network associated with the union of the two possibilistic knowledge bases.

This procedure is not interesting and has two limitations. The first one is that the transformation step from a possibilistic base into a possibilistic graph is generally a hard problem. For instance, if the two graphs to fuse have exactly the same singly connected DAG, then there is no garantee that the result will also be a same singly connected DAG. The second limitation is that existing transformations only deal with binary variables.

This paper proposes alternative approaches which overcome these two limitations. We assume that the two networks are defined on the same set of variables. This is not a limitation, since any possibilistic networks can be extended with additional variables, as it is shown by the following proposition:

Proposition 1 *Let $\mathbb{N} = (\pi_{\mathbb{N}}, G_{\mathbb{N}})$ be a possibilitic network defined on a set variables V. Let A be a new variable. Let $\mathbb{N}1 = (\pi_{\mathbb{N}1}, G_{\mathbb{N}1})$ be a new possibilistic networks such that :*

- *$G_{\mathbb{N}1}$ is equal to $G_{\mathbb{N}}$ plus a root node A, and*
- *$\pi_{\mathbb{N}1}$ is identical to $\pi_{\mathbb{N}}$ for variables in V, and is equal to a uniform possibility distribution on the root node A (namely, $\forall a \in D_A$, $\pi_{\mathbb{N}1}(a)=1$).*

Then, we have :
$\forall \omega \in \times_{A_i \in V} D_{A_i}$, $\pi_{\mathbb{N}}(\omega) = max_{a \in D_A} \pi_{\mathbb{N}1}(a\omega)$, where $\pi_{\mathbb{N}}$ and $\pi_{\mathbb{N}1}$ are respectively the possibility distributions associated with $\mathbb{N}$ and $\mathbb{N}1$ using Definition 1

4 Fusion of the same-structure networks

This section presents the procedure of merging causal networks having a same DAG structures. For sake of simplicity and without loss of generality, we restrict ourselves to the case of the fusion of two causal networks.

The two possibilistic networks to merge, denoted by $\mathbb{N}1$ and $\mathbb{N}2$, only differ on conditionnal possibility distributions assigned to variables.

The following definition and proposition show that the result merging of networks having same structure is immediate.

Definition 2 *Let $\mathbb{N}1 = (\pi_{\mathbb{N}1}, G_{\mathbb{N}1})$ and $\mathbb{N}2 = (\pi_{\mathbb{N}2}, G_{\mathbb{N}2})$ be two possibilistic networks such that $G_{\mathbb{N}1} = G_{\mathbb{N}2}$. We define the result of merging $\mathbb{N}1$ and $\mathbb{N}2$ a possibilistic network, denoted by $\mathbb{N}\oplus = (\pi_{\mathbb{N}\oplus}, G_{\mathbb{N}\oplus})$, where :*

- *$G_{\mathbb{N}\oplus} = G_{\mathbb{N}1} = G_{\mathbb{N}2}$ and*
- *$\pi_{\mathbb{N}\oplus}$ are defined by $\forall A$:*
  $\pi_{\mathbb{N}\oplus}(A \mid U_A) = min(\pi_{\mathbb{N}1}(A \mid U_A), \pi_{\mathbb{N}2}(A \mid U_A))$,

where A is a variable and U is the set of parents of A.

Proposition 2 *Let $\mathbb{N}1 = (\pi_{\mathbb{N}1}, G_{\mathbb{N}1})$ and $\mathbb{N}2 = (\pi_{\mathbb{N}2}, G_{\mathbb{N}2})$ be two possibilistic networks having exactly the same associated DAG. Let $\mathbb{N}\oplus = (\pi_{\mathbb{N}\oplus}, G_{\mathbb{N}\oplus})$ be the result of merging $\mathbb{N}1$ and $\mathbb{N}2$ using the above definition. Then, we have :*

$\forall \omega \in \Omega, \pi_{\mathbb{N}\oplus}(\omega) = \min(\pi_{\mathbb{N}1}(\omega), \pi_{\mathbb{N}2}(\omega))$,
where $\pi_{\mathbb{N}\oplus}, \pi_{\mathbb{N}1}, \pi_{\mathbb{N}2}$ are respectively the possibility distributions associated with $\mathbb{N}\oplus, \mathbb{N}1, \mathbb{N}2$ using Definition 1.

Example 1 *Let $\mathbb{N}1$ and $\mathbb{N}2$ be two possibilistic networks. We assume that they have the same associated graphs represented by figure1. The possiblity distributions associated with $\mathbb{N}1$ and $\mathbb{N}2$ are given respectively by Table1 and Table2.*

Then fused possibilistic network $\mathbb{N}\oplus$ is such that its associated graph is also the DAG of Figure1 and its conditionnal possibility distributions are given by Table3.

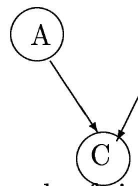

Figure 1: Example of similar networks

Table 1: Initial possibility distributions associated with $\mathbb{N}1$

a	$\pi_1(a)$	b	$\pi_1(b)$	a	b	c	$\pi_1(c \mid a \wedge b)$
a_1	1	b_1	1	a_1	b_1	c_1	1
a_2	0.2	b_2	0.5	a_1	b_1	c_2	0.3
				a_1	b_2	c_1	0.1
				a_1	b_2	c_2	0.2
				a_2	b_1	c_1	0.1
				a_2	b_1	c_2	1
				a_2	b_2	c_1	0.6
				a_2	b_2	c_2	0

Table 2: Initial possibility distributions associated with $\mathbb{N}2$

a	$\pi_2(a)$	b	$\pi_2(b)$	a	b	c	$\pi_2(c \mid a \wedge b)$
a_1	1	b_1	0.1	a_1	b_1	c_1	1
a_2	0.3	b_2	0.2	a_1	b_1	c_2	0
				a_1	b_2	c_1	0.7
				a_1	b_2	c_2	0.1
				a_2	b_1	c_1	0
				a_2	b_1	c_2	0.5
				a_2	b_2	c_1	1
				a_2	b_2	c_2	0.4

Table 3: Conditionnal possibility distributions associated with $\mathbb{N}\oplus$

a	$\pi_\oplus(a)$	b	$\pi_\oplus(b)$	a	b	c	$\pi_\oplus(c \mid a \wedge b)$
a_1	1	b_1	0.1	a_1	b_1	c_1	1
a_2	0.2	b_2	0.2	a_1	b_1	c_2	0
				a_1	b_2	c_1	0.1
				a_1	b_2	c_2	0.1
				a_2	b_1	c_1	0
				a_2	b_1	c_2	0.5
				a_2	b_2	c_1	0.6
				a_2	b_2	c_2	0

5 Fusion of U-acyclic networks

The above section has shown that the fusion of possibilistic networks can be easily achieved if they share the same DAG.

This section considers the case when the networks have not the same structure. However we assume that their union does not contain a cycle .

A union of two DAGs $(\mathrm{G}_1, \mathrm{G}_2)$ is a graph where :

- its variables is the union of variables in G_1 and G_2 and
- for each variable A, its parents are those in G_1 and G_2.

If the union of G_1 and G_2 does not contain cycles, we say that G_1 and G_2 are U-acyclic networks. In this case the fusion can be easily obtained. We first provides a proposition which shows how to add links to a possibilistic network without changing its possibility distribution.

Proposition 3 *Let $\mathbb{N} = (\pi_\mathbb{N}, G_\mathbb{N})$ be a possibilistic network. Let A be a variable, and let Par(A) be parents of A in $G_\mathbb{N}$. Let $B \notin Par(A)$. Let $\mathbb{N}1 = (\pi_{\mathbb{N}1}, G_{\mathbb{N}1})$ be a new possibilistic network obtained from $\mathbb{N} = (\pi_\mathbb{N}, G_\mathbb{N})$ by adding a link from B to A, with the new conditionnal possibility associated with A is:*
$\forall a \in D_a, b \in D_B, u \in D_{Par(A)}, \pi_{\mathbb{N}1}(a \mid ub) = \pi_\mathbb{N}(a \mid u)$.
Then, we have :
$\forall \omega, \pi_\mathbb{N}(\omega) = \pi_{\mathbb{N}1}(\omega)$.

Given this proposition the fusion of two U-acyclic networks $\mathbb{N}1$ and $\mathbb{N}2$ is immediate. Let $G_{\mathbb{N}\oplus}$ be the union of $G_{\mathbb{N}1}$ and $G_{\mathbb{N}2}$. Then the fusion of $\mathbb{N}1$ and $\mathbb{N}2$ can be obtained using the following two steps:

Step 1 Using Proposition 3, expand $\mathbb{N}1$ and $\mathbb{N}2$ such that $G_{\mathbb{N}1} = G_{\mathbb{N}2} = G_{\mathbb{N}\oplus}$.

Step 2 Apply Proposition 2 to the possibilistic networks obtained from Step 1.

Example 2 *Let us consider two causal networks, where their DAG are given by Figure2. These two DAG have a different structure structure.*

The conditionnal possibility distributions associated with above networks are given by Tables 4 and 5:

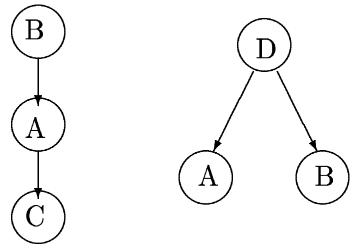

Figure 2: Example of U-acyclic networks

Table 4: Initial conditional possibility distributions $\pi_{\mathbb{N}1}$

b	$\pi_1(b)$	a	b	$\pi_1(a \mid b)$	a	c	$\pi_1(c \mid a)$
b_1	0.1	a_1	b_1	0.3	a_1	c_1	1
b_2	1	a_1	b_2	0.6	a_1	c_2	0.5
		a_2	b_1	1	a_2	c_1	0
		a_2	b_2	0	a_2	c_2	0.2

Table 5: initial conditional possibility distributions $\pi_{\mathbb{N}2}$

d	$\pi_2(d)$	a	d	$\pi_2(a \mid d)$	b	d	$\pi_2(b \mid d)$
d_1	1	a_1	d_1	1	b_1	d_1	0.1
d_2	0	a_1	d_2	0.1	b_1	d_2	0.8
		a_2	d_1	0	b_2	d_1	1
		a_2	d_2	0	b_2	d_2	0.7

We see clearly, from figure2, that the union of the two DAG's is free of cycles. Figure3 provides the DAG of $G_{\mathbb{N}\oplus}$ which is obtained by using proposition3 and proposition1 .

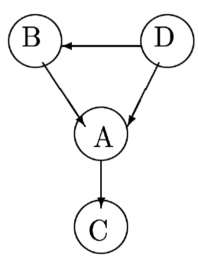

Figure 3: the DAG $G_{\mathbb{N}\oplus}$

Table6 gives values of conditionnal possibility distribution for each variable .

From these different tables of conditionnal distributions, we can easily check that the joint

Table 6: Merged conditionnal distributions $\pi_{\mathbb{N}\oplus}$

d	$\pi_\oplus(d)$	a	c	$\pi_\oplus(c \mid a)$	b	d	$\pi_\oplus(b \mid d)$
d_1	1	a_1	c_1	1	b_1	d_1	0.1
d_2	0	a_1	c_2	0.5	b_1	d_2	0.1
		a_2	c_1	0	b_2	d_1	1
		a_2	c_2	0.2	b_2	d_2	0.7

a	b	d	$\pi(a \mid b \wedge d)$
a_1	b_1	d_1	0.3
a_1	b_1	d_2	0.1
a_1	b_2	d_1	0.6
a_1	b_2	d_2	0.1
a_2	b_1	d_1	0
a_2	b_1	d_2	0
a_2	b_2	d_1	0
a_2	b_2	d_2	0

possibility $\pi_{\mathbb{N}\oplus}$ computed by chain rule, is equal to the minimum of the joint possibility distributions of $\pi_{\mathbb{N}1}$ and $\pi_{\mathbb{N}2}$. For instance, let $\omega = a_1b_1c_1d_1$ be a possible interpretation. Using chain rules, we have:

$\pi_{\mathbb{N}1}(a_1b_1c_1d_1) = min(0.3, 0.1, 1) = 0.1.$
$\pi_{\mathbb{N}2}(a_1b_1c_1d_1) = min(1, 0.1, 1) = 0.1.$
$\pi_{\mathbb{N}\oplus}(a_1b_1c_1d_1) = min(0.3, 1, 0.1, 1) = 0.1.$

6 conclusion

This paper has proposed a syntactic fusion of possibilistic networks. We showed that when the possibilistic networks have the same structure or when the union of their DAGs is free of cycles, then the fusion can be achieved efficently. For lack of space, we did not present the fusion of U-cyclic possibilistic networks, which can be achieved with the help of additional variables. This is left for further research. This paper only considered min-based conjunctive combination modes. A future work is to extend results of this paper to other possibilistic combination modes.

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