A linguistic fuzzy model with a monotone rule base is not always monotone

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Abstract

In this paper experiments are described with three linguistic fuzzy models sharing the same monotone rule base and the same membership functions for the two input variables, but applying different membership functions in the output domain. We investigated which inference methods result in a monotonic input-output behaviour. Apart from the conventional Mamdani–Assilian inference method with Center of Gravity (COG) defuzzification, three implicator-based inference methods combined with COG-like defuzzification are discussed.

Keywords: linguistic fuzzy model, monotone rule base, monotone model, implicator-based inference method.

1 Introduction

When identifying fuzzy models of real-world systems, one is often confronted with a small number of data points. In these cases it is very important to fully exploit the additional non-quantitative knowledge about the system, in order to obtain meaningful, interpretable models. Qualitative knowledge about a system can guide the selection of the model type and inference procedure as well as lead to a set of constraints the parameters of the membership functions should satisfy, and therefore largely reduce the search space of the data-driven model identification. An example of this additional qualitative information is the monotonicity of the model output with respect to an input variable, i.e. the model output is either increasing or decreasing in the variable for all combinations of values of other input variables, for instance when modelling the assignment of a global quality score based on several criteria.

The final goal of our study is to formulate a list of properties linguistic fuzzy models should satisfy in order to be monotone with respect to an input variable. We investigate linguistic fuzzy models as their framework allows for the design of interpretable models for non-experts. Moreover, the monotonicity of Takagi–Sugeno models has already been discussed in detail in [4, 6, 7]. In this text we do not deal with the monotonicity problem in a theoretical way. We just describe results of preliminary experiments with the three linguistic fuzzy models described in Section 2, that are meant to guide the theoretical part of our study.

With the term *linguistic fuzzy models* we indicate all fuzzy models with rules containing linguistic values in both antecedent and consequent. Apart from the t-norm-based Mamdani–Assilian inference method, described in Section 3, three implicator-based inference methods were applied. The first implicator-based inference method, introduced in Section 4, is directly applied on the models described in Section 2. The two other implicator-based inference methods, described in Section 5, are applied on two sets of rules derived from the rule base of the *conventional* linguistic fuzzy model. These rules form the rule base of the so-called ATL (resp. ATM) model and are obtained by applying the modifier ATL (resp. ATM) to all linguistic values in both antecedent as consequent of each rule of the linguistic fuzzy model.

Conclusions and further work are given in Section 6.

2 Model description

The three models have two input variables X_1 and X_2 and one output variable Y. Their rule base is identical and consists of the following 6 rules:

IF x_1 IS A_1^1 AND x_2 IS A_1^2 THEN y IS B_1,
IF x_1 IS A_1^1 AND x_2 IS A_2^2 THEN y IS B_1,
IF x_1 IS A_2^1 AND x_2 IS A_1^2 THEN y IS B_1,
IF x_1 IS A_2^1 AND x_2 IS A_2^2 THEN y IS B_2,
IF x_1 IS A_3^1 AND x_2 IS A_1^2 THEN y IS B_2,
IF x_1 IS A_3^1 AND x_2 IS A_2^2 THEN y IS B_3.

The rule base is monotone and increasing in both input variables, i.e. for any two rules whose antecedents only differ in one input variable, the linguistic value in the consequent of the rule containing the largest linguistic input value is never smaller than the linguistic value in the consequent of the second rule.

For all variables, the piece-wise linear membership functions describing the linguistic values form a fuzzy partition. As preliminary experiments pointed out that the monotonicity of linguistic fuzzy models with a rule base monotone in all variables and fuzzy partitions for all input variables, strongly depends on the shape, and in particular the symmetry, of the output membership functions, three different sets of output membership functions will be compared. The membership functions $A_{i_l}^l$ and B_j of the three models are shown in Fig. 1.

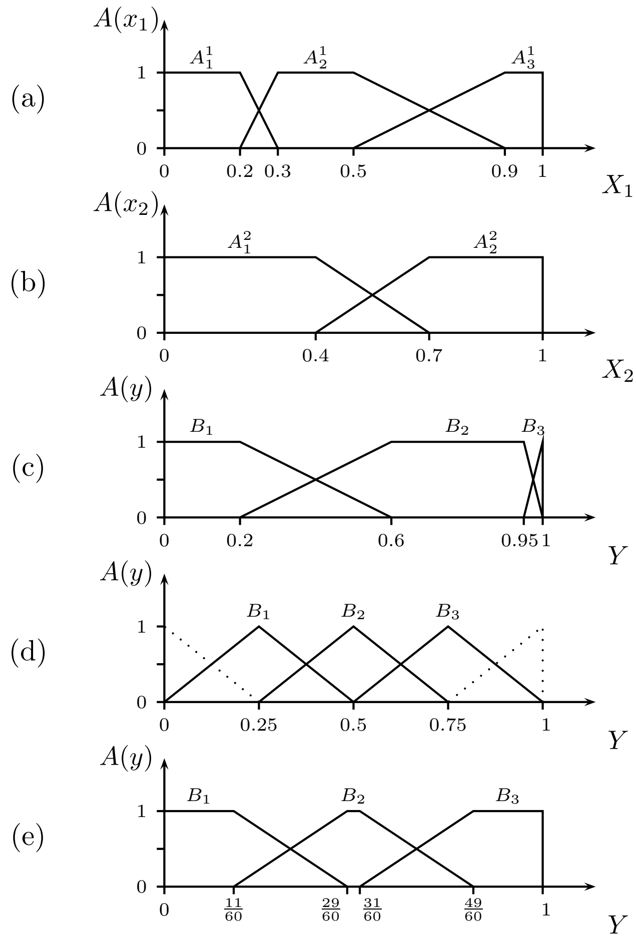

Figure 1: Membership functions assigned to the input variables X_1 (a) and X_2 (b), and the output variable Y in the first (c), second (d) and third (e) model.

3 Mamdani-Assilian inference method

The kernel of a linguistic fuzzy model is the rule base consisting of r rules of the following form:

$$R_s : \text{IF } x_1 \text{ IS } A_s^1, \ldots, x_m \text{ IS } A_s^m \text{ THEN } y \text{ IS } B_s$$

where A_s^l (resp. B_s) are linguistic values of variable X_l (resp. Y) described by membership functions A_s^l (resp. B_s) and $\mathbf{x} = [x_1, \ldots, x_m]$ are the input values.

When determining the model output via the Mamdani–Assilian inference method [1, 5], first the membership degrees $A_s^l(x_l)$ of the (fuzzified) model input $\mathbf{x}$ to the linguistic values in the antecedents of the rules are calculated. In the following step, the fulfilment degrees α_s of the r rules ($s = 1, \ldots, r$) are computed from the membership degrees $A_s^l(x_l)$ via a t-norm T. In this work, the basic t-norms $T_\mathbf{M}$, $T_\mathbf{P}$ and $T_\mathbf{L}$ are considered:

$$\alpha_s = \mathop{T}_{l=1}^{m} A_s^l(x_l)\,. \tag{1}$$

Next, the adapted membership functions $B_s'(y)$ are computed using the same t-norm as for the fulfilment degrees α_s:

$$B_s'(y) = \begin{cases} \min(\alpha_s, B_s(y)) & \text{, if } T = T_\mathbf{M}, \\ \alpha_s \cdot B_s(y) & \text{, if } T = T_\mathbf{P} \text{ and} \\ \max(\alpha_s + B_s(y) - 1, 0) & \text{, if } T = T_\mathbf{L}, \end{cases} \tag{2}$$

and the global fuzzy output $B(y)$ is determined as follows:

$$B(y) = \max_{s=1}^{r} B_s'(y)\,. \tag{3}$$

Finally, the crisp model output y^* is obtained by defuzzifying the fuzzy output, for instance

Table 1: Overview of the monotonicity property of the three models when applying the Mamdani–Assilian inference method.

	$T_{\mathbf{M}}$	$T_{\mathbf{P}}$	$T_{\mathbf{L}}$
model 1	no	**yes**	no
model 2	no	**yes**	no
model 3	no	**yes**	no

with the Center of Gravity (COG) defuzzification method resulting in the crisp model output y^*_{COG}:

$$y^*_{\mathrm{COG}} = \frac{\int_Y y \cdot B(y)\,dy}{\int_Y B(y)\,dy} \,. \tag{4}$$

The model output y^*_{COG} of the three models is determined for 7007 inputs $(X_1, X_2) \in [0 : 0.001 : 1] \times [0.4 : 0.05 : 0.7]$ for $T_{\mathbf{M}}$, $T_{\mathbf{P}}$ and $T_{\mathbf{L}}$. A model *is* not monotone if there exists a pair of two points not satisfying the desired monotonic input-output behaviour. In this experimental setup, a model *is said to be* monotone, if no such pair can be formed among the data points considered. All three models were monotone when applying $T_{\mathbf{P}}$ and none of the models were monotone when applying $T_{\mathbf{M}}$ or $T_{\mathbf{L}}$ (Table 1).

4 Implicator-based inference method

When applying implicator-based inference methods, the fulfilment degrees α_s are calculated as described in (1), but the adapted membership functions B'_s are computed using an implicator instead of a t-norm. In this study five implicators are considered: three R-implicators $I_{\mathbf{M}}$, $I_{\mathbf{P}}$ and $I_{\mathbf{L}}$ and two S-implicators $I_{\mathbf{M},\mathcal{N}}$ and $I_{\mathbf{P},\mathcal{N}}$ ($I_{\mathbf{L},\mathcal{N}}$ is equal to $I_{\mathbf{L}}$).

For $I_{\mathbf{M}}$ the adapted membership functions are obtained by

$$B'_s(y) = \begin{cases} 1 & \text{, if } \alpha_s \le B_s(y) \text{ and} \\ B_s(y) & \text{, otherwise}\,, \end{cases} \tag{5}$$

for $I_{\mathbf{P}}$ by

$$B'_s(y) = \begin{cases} 1 & \text{, if } \alpha_s \le B_s(y) \text{ and} \\ \frac{B_s(y)}{\alpha_s} & \text{, otherwise}\,, \end{cases} \tag{6}$$

for $I_{\mathbf{L}}$ (and $I_{\mathbf{L},\mathcal{N}}$) by

$$B'_s(y) = \min(1 - \alpha_s + B_s(y), 1)\,, \tag{7}$$

for $I_{\mathbf{M},\mathcal{N}}$ by

$$B'_s(y) = \max(1 - \alpha_s, B_s(y))\,, \tag{8}$$

and finally, for $I_{\mathbf{P},\mathcal{N}}$, by

$$B'_s(y) = 1 - \alpha_s + \alpha_s B_s(y)\,. \tag{9}$$

The global fuzzy output B is the intersection of the r adapted membership functions B'_s:

$$B(y) = \min_{j=1}^{r} B'_s(y)\,, \tag{10}$$

which is defuzzified by the modified center of gravity defuzzification method introduced for $I_{\mathbf{L}}$ by Dvořák and Jedelský [3], returning $\frac{1}{2}(y_0 + y_{end})$ in case the fuzzy output is the empty or the universal set, and otherwise, y^*_{COGDJ}:

$$y^*_{\mathrm{COGDJ}} = \frac{\int_y y \cdot (B(y) - \min_y B(y))\,dy}{\int_Y (B(y) - \min_y B(y))\,dy} \tag{11}$$

In Fig. 2 some fuzzy outputs of the first model are shown. As the implicators $I_{\mathbf{M}}$ and $I_{\mathbf{P}}$ do not extend the support of the adapted membership functions B'_s, the empty set is obtained as fuzzy output when all three linguistic output values have a non-zero fulfilment degree (e.g. for $(X_1, X_2) = (0.6, 0.6)$), whereas very strangely shaped fuzzy outputs are obtained for these implicators when only two linguistic output values have a non-zero fulfilment degree (e.g. for $(X_1, X_2) = (0.4, 0.6)$). The fuzzy outputs obtained for the implicators $I_{\mathbf{L}}$, $I_{\mathbf{M},\mathcal{N}}$ and $I_{\mathbf{P},\mathcal{N}}$ are quite similar and have acceptable shapes.

The model output y^*_{COGDJ} of the three models is determined for $(X_1, X_2) \in [0 : 0.001 : 1] \times [0.4 : 0.05 : 0.7]$ for all 15 combinations of t-norm and implicator. In Table 2 an overview is given for which combinations of t-norm and implicator a monotone model is obtained. For the first model no monotonic input-output behaviour is obtained. The second and third model are monotone for $T_{\mathbf{P}}$ combined with $I_{\mathbf{P},\mathcal{N}}$. The second model is also monotone for $I_{\mathbf{M},\mathcal{N}}$ and all t-norms. The model outputs are piecewise continuous for $I_{\mathbf{M}}$, $I_{\mathbf{P}}$ and $I_{\mathbf{M},\mathcal{N}}$, whereas the outputs obtained with $I_{\mathbf{L}}$ and $I_{\mathbf{P},\mathcal{N}}$ tend to be smoother.

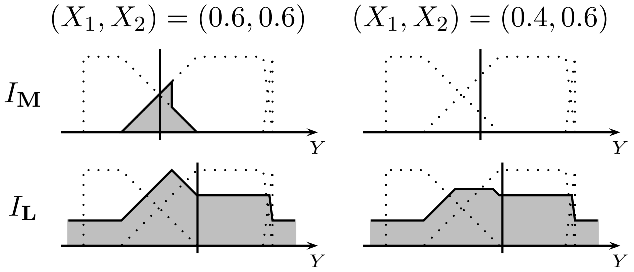

Figure 2: Illustration of fuzzy outputs obtained for the first model with the implicator-based inference method with $T = T_{\mathbf{M}}$. The vertical line indicates y^*_{COGDJ}.

Table 2: Overview of the monotonicity property of the three models when applying an implicator-based inference method and the defuzzification method of Dvořák and Jedelský

		$I_{\mathbf{M}}$	$I_{\mathbf{P}}$	$I_{\mathbf{L}}$	$I_{\mathbf{M},\mathcal{N}}$	$I_{\mathbf{P},\mathcal{N}}$
model 1	$T_{\mathbf{M}}$	no	no	no	no	no
	$T_{\mathbf{P}}$	no	no	no	no	no
	$T_{\mathbf{L}}$	no	no	no	no	no
model 2	$T_{\mathbf{M}}$	no	no	no	**yes**	no
	$T_{\mathbf{P}}$	no	no	no	**yes**	**yes**
	$T_{\mathbf{L}}$	no	no	no	**yes**	no
model 3	$T_{\mathbf{M}}$	no	no	no	no	no
	$T_{\mathbf{P}}$	no	no	no	no	**yes**
	$T_{\mathbf{L}}$	no	no	no	no	no

5 ATL and ATM models

As mentioned in Section 1, the last two inference procedures discussed in this study are applied to ATL and ATM models. The modifiers ATL and ATM used in these models are defined as in [2]:

$$\mathrm{ATL}(A)(x) = \sup\{A(t) \mid t \leq x\}\,, \qquad (12)$$

$$\mathrm{ATM}(A)(x) = \sup\{A(t) \mid t \geq x\}\,. \qquad (13)$$

The rules of the ATL (resp. ATM) model are derived from the rules in the rule base of the linguistic fuzzy model in Section 2 by applying the modifier ATL (resp. ATM) to all linguistic values in the antecedent and consequent of each rule, as illustrated for the fourth rule of the ATL model:

IF x_1 IS ATL(A_2^1) AND x_2 IS ATL(A_2^2)
THEN y IS ATL(B_2).

The procedure to obtain the fuzzy outputs of ATL and ATM models is the same as for the implicator-based inference method. After the calculation of the fulfilment degrees α_s by (1), the adapted membership functions $B'_s(y)$ are determined by (5)–(9), which in the final step are joined together in order to obtain a global fuzzy output (10). The fuzzy output of an ATL (resp. ATM) model is either the universal set or a set described by a non-decreasing (resp. non-increasing) membership function to which the upper (resp. lower) bound of the output domain has membership degree 1, as illustrated in Fig. 3 for the first model.

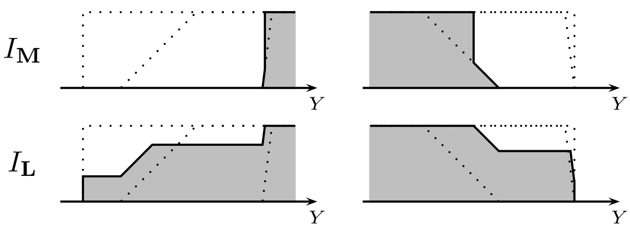

Figure 3: Model outputs of the ATL and ATM models (model 1) for $X = [0.6, 0.6]$ and $T = T_{\mathbf{M}}$

Two approaches were considered to derive a crisp output from the fuzzy outputs of the ATL and ATM models: one could first defuzzify each fuzzy output individually and then combine the two crisp outputs, or one could first combine the fuzzy outputs and defuzzify the resulting fuzzy set. Regardless of the defuzzification strategy applied, the crisp output should, in our opinion, be at least as large as the largest output value $y^*_{\mathrm{ATL,LB}}$ with minimum membership degree to the fuzzy output of the ATL model and at most as large as the smallest output value $y^*_{\mathrm{ATM,UB}}$ with minimum membership degree to the fuzzy output of the ATM model. This guarantees that the intersection of both models has a compact kernel, and implies $y^*_{\mathrm{ATL,LB}} \leq y^*_{\mathrm{ATM,UB}}$. Note that, in case the fuzzy output is the universal set, $y^*_{\mathrm{ATL,LB}}$ and $y^*_{\mathrm{ATM,UB}}$ are equal to respectively the lower bound y_0 and upper bound y_{end} of the output domain.

In that region of the input space where the three linguistic output values of the *conventional* linguistic fuzzy model all have a non-zero fulfilment degree, i.e. when $(X_1, X_2) \in [0.5, 0.9] \times [0.4, 0.7]$, it holds that $y^*_{\mathrm{ATL,LB}} > y^*_{\mathrm{ATM,UB}}$ for model 1 and model 3 and $y^*_{\mathrm{ATL,LB}} = y^*_{\mathrm{ATM,UB}}$ for model 2 for

$T_{\mathbf{M}}$ and $T_{\mathbf{P}}$ combined with $I_{\mathbf{M}}$ or $I_{\mathbf{P}}$. So when using ATL and ATM models the implicators $I_{\mathbf{M}}$ and $I_{\mathbf{P}}$ should not be used in combination with the t-norms $T_{\mathbf{M}}$ and $T_{\mathbf{P}}$. All other combinations of T and I considered satisfy the constraint. Furthermore $y^*_{\mathrm{ATL,LB}}$ and $y^*_{\mathrm{ATM,UB}}$ are both monotone on $[0 : 0.001 : 1] \times [0.4 : 0.05 : 0.7]$ for all 15 combinations for all three models.

To defuzzify the individual fuzzy outputs of the ATL and ATM models, we propose COG-like defuzzification methods, returning y_0 in case the fuzzy output of the ATL model is the empty or universal set and y_{end} in case the fuzzy output of the ATM model is the empty or universal set, otherwise, they return:

$$y^*_{\mathrm{ATL,COG}} = \frac{\int_{y^*_{\mathrm{ATL,LB}}}^{y^*_{\mathrm{ATL,UB}}} y \cdot B_{ATL}(y)\, dy}{\int_{y^*_{\mathrm{ATL,LB}}}^{y^*_{\mathrm{ATL,UB}}} B_{ATL}(y)\, dy}, \qquad (14)$$

$$y^*_{\mathrm{ATM,COG}} = \frac{\int_{y^*_{\mathrm{ATM,LB}}}^{y^*_{\mathrm{ATM,UB}}} y \cdot B_{ATM}(y)\, dy}{\int_{y^*_{\mathrm{ATM,LB}}}^{y^*_{\mathrm{ATM,UB}}} B_{ATM}(y)\, dy}. \qquad (15)$$

Table 3 gives an overview which combinations of t-norm and implicator resulted in monotone $y^*_{\mathrm{ATL,COG}}$- and $y^*_{\mathrm{ATM,COG}}$-values on $[0 : 0.001 : 1] \times [0.4 : 0.05 : 0.7]$ for which the constraint $y^*_{\mathrm{ATL,COG}} \leq y^*_{\mathrm{ATM,COG}}$ holds. This is only the case for all models, for $T = T_{\mathbf{L}}$ combined with $I_{\mathbf{M}}$ or $I_{\mathbf{P}}$. The model outputs obtained with $I_{\mathbf{M}}$ and $I_{\mathbf{P}}$ are piecewise continuous. The use of the implicators $I_{\mathbf{L}}$, $I_{\mathbf{M},\mathcal{N}}$ and $I_{\mathbf{P},\mathcal{N}}$ results in smoother model outputs, which are however only monotone and satisfying the above constraint for $I = I_{\mathbf{L}}$ for the second and third model. For those combinations of t-norm and implicator satisfying the above monotonicity properties and inequality, the global model output is computed according to the following monotonicity-preserving operation:

$$y^*_{\mathrm{ATL-ATM,COG}} = \frac{y^*_{\mathrm{ATL,COG}} + y^*_{\mathrm{ATM,COG}}}{2}. \qquad (16)$$

When following the second approach, we also apply a defuzzification method inspired on the center of gravity defuzzification on the global fuzzy output B:

Table 3: Overview for which combinations of T and I the inequality $y^*_{\mathrm{ATL,COG}} \leq y^*_{\mathrm{ATM,COG}}$ holds

		$I_{\mathbf{M}}$	$I_{\mathbf{P}}$	$I_{\mathbf{L}}$	$I_{\mathbf{M},\mathcal{N}}$	$I_{\mathbf{P},\mathcal{N}}$
model 1	$T_{\mathbf{M}}$			no	no	no
	$T_{\mathbf{P}}$			no	no	no
	$T_{\mathbf{L}}$	**yes**	**yes**	no	no	no
model 2	$T_{\mathbf{M}}$	no	no	**yes**	no	no
	$T_{\mathbf{P}}$	no	no	**yes**	no	no
	$T_{\mathbf{L}}$	**yes**	**yes**	**yes**	no	no
model 3	$T_{\mathbf{M}}$			**yes**	no	no
	$T_{\mathbf{P}}$			**yes**	no	no
	$T_{\mathbf{L}}$	**yes**	**yes**	**yes**	no	no

$$B(y) = \min(B_{\mathrm{ATL}}(y), B_{\mathrm{ATL}}(y)), \qquad (17)$$

returning $0.5 * (y_0 + y_{end})$ in case the intersection of fuzzy outputs of the ATL and ATM model is the empty or the universal set and otherwise returning:

$$y^*_{\mathrm{ATLATM,COG}} = \frac{\int_{y_0}^{y_{end}} y \cdot B(y)\, dy}{\int_{y_0}^{y_{end}} B(y)\, dy}. \qquad (18)$$

Table 4 summarizes for which combinations of T and I monotone model outputs are obtained on $[0 : 0.001 : 1] \times [0.4 : 0.05 : 0.7]$ for the three models. The first model is only monotone when combining $T_{\mathbf{L}}$ with $I_{\mathbf{P}}$, whereas the second and third model are always monotone except for $T_{\mathbf{P}}$ and $T_{\mathbf{L}}$ combined with $I_{\mathbf{M}}$.

6 Conclusions and further work

From our experiments with COG defuzzification and mean of maximum (MOM) defuzzification (not discussed in this work due to lack of space) we can conclude that if one wants to obtain a monotone model with the Mamdani–Assilian inference method, one should not use the t-norms $T_{\mathbf{M}}$ or $T_{\mathbf{L}}$. Further investigations have to point out if linguistic fuzzy models with a monotone rule base are always monotone if $T_{\mathbf{P}}$ is applied for

Table 4: Overview of the combinations of T and I for which $y^*_{\text{ATLATM,COG}}$ is monotone on $[0:0.001:1]\times[0.4:0.05:0.7]$

		$I_{\mathbf{M}}$	$I_{\mathbf{P}}$	$I_{\mathbf{L}}$	$I_{\mathbf{M},\mathcal{N}}$	$I_{\mathbf{P},\mathcal{N}}$
model 1	$T_{\mathbf{M}}$			no	no	no
	$T_{\mathbf{P}}$			no	no	no
	$T_{\mathbf{L}}$	no	**yes**	no	no	no
model 2	$T_{\mathbf{M}}$	**yes**	**yes**	**yes**	**yes**	**yes**
	$T_{\mathbf{P}}$	no	**yes**	**yes**	**yes**	**yes**
	$T_{\mathbf{L}}$	no	**yes**	**yes**	**yes**	**yes**
model 3	$T_{\mathbf{M}}$			**yes**	**yes**	**yes**
	$T_{\mathbf{P}}$			**yes**	**yes**	**yes**
	$T_{\mathbf{L}}$	no	**yes**	**yes**	**yes**	**yes**

both conjunction and inference, regardless of the fuzzy partition applied in the output domain.

When opting for an implicator-based inference procedure, we recommend not to apply the implicators $I_{\mathbf{M}}$ and $I_{\mathbf{P}}$, as the empty set is obtained as fuzzy output for the procedure described in Section 4 and the inequality $y^*_{\text{ATL,LB}} \leq y^*_{\text{ATM,UB}}$ does not hold for the procedures discussed in Section 5, if two output values whose supports have an empty intersection, have non-zero fulfilment degrees. Only for one T-I combination, $T_{\mathbf{P}}$ and $I_{\mathbf{P},\mathcal{N}}$, monotone models with continuous model outputs are obtained with the inference method described in Section 4 for the second and third model using symmetric output membership functions.

The results obtained with the ATL and ATM models, especially when applying the 'first combine, then defuzzify' inference procedure, are more promising. With the fourth discussed inference method, the second and third model are always monotone when applying $I_{\mathbf{L}}$, $I_{\mathbf{M},\mathcal{N}}$ and $I_{\mathbf{P},\mathcal{N}}$ as implicator with any of the three t-norms considered. Similar results were obtained with MOM defuzzification for the first and last implicator-based inference methods, except that the obtained model outputs are piece-wise continuous instead of continuous.

Future work will include the development of alternative defuzzification procedures for the ATL and ATM models, as well as a more theoretical-based formulation of the properties linguistic fuzzy models should satisfy in order to be monotone. Furthermore, linguistic fuzzy models being monotone with respect to some, but not all, input variables, and models with more than two input variables will be the subject of future investigations.

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