MYRIAD: a tool suite for MCDA

Christophe Labreuche & Fabien Le Huédé
Thales Research & Technology
Domaine de Corbeville, 91404 Orsay Cedex, France
christophe.labreuche@thalesgroup.com, fabien.lehuede@thalesgroup.com

Abstract

We present a software called Myriad for MCDA, based on a two-additive Choquet integral. The parameters of the model are determined from the preferential information provided by the decision maker. When options are assessed by the model, the software helps the decision maker in understanding the main reasons of the recommendations made.

Keywords: Choquet integral, Interaction, Capacity.

1 Introduction

Multi-Criteria Decision Aiding (MCDA) aims at helping a decision maker (DM) in making up his mind about the assessment of an option or the selection of the best option among several alternative options, on the basis of several decision criteria. This difficult task requires the use of a preference model and a process. The model represents the way the options are assessed and compared one another. It formalizes the expertise constructed from the interview of a DM. The process formalizes the interaction between the DM and the preference model. On the one hand, the DM provides some preferential information from which the optimal values of the parameters of the preference model are deduced. This is the *disaggregation* phase. On the other hand, the preference model is run on prototypical or real options, and the results are presented to the decision makers. This is the *aggregation* phase. These two phases need to be instrumented by a software.

Within the Thales group, we deal with many MCDA applications:

- Evaluation problems such as trainee's evaluation in which one shall give an assessment of each option together with a synthesis of its main assets and flaws,
- Acquisition problems (products' design) in which one shall assess the quality of a product on the basis of some criteria and provide some recommendations about the *most efficient* way to improve the product,
- Classification problems such as threat or risk assessment, or classification from information coming from several sources, in which each option shall be assigned to a category,
- Optimization problems in which the cost function depends on multiple criteria and the set of options is so wide that it is described by combinatoric techniques.

Since these applications concern mainly experts know-how, the underlying model must be versatile and elaborate enough to encompass most commonly encountered decisional behaviors. Conversely, the model shall not be too complicated so that the DM is able to understand the model and the recommendations made from it. This leaded us to construct an approach based on Multi-Attribute Utility Attribute where an overall utility is computed for each option, and the use of the two-additive Choquet integral as an aggregation function. The 2-additive Choquet integral is a good compromise between versatility and ease to understand. We present here a tool named MYRIAD developed at Thales for MCDA applications based on a two-additive Choquet integral.

Section 2 describes the model used. The disaggregation phase is dealt with in Section 3 whereas the aggregation one is considered in Section 4. Section 5 presents an application in cosmetics.

2 General framework

For the sake of simplicity, we assume in this part that the set $N = \{1, \dots, n\}$ of criteria are organized in a single level of aggregation. The set of attributes is denoted by $X_1, \dots, X_n$. All the attributes are made commensurate thanks to the introduction of partial utility functions $u_i : X_i \to [0,1]$. The $[0,1]$ scale depicts the satisfaction of the DM regarding the values of the attributes. An option x is identified to an element of $X = X_1 \times \cdots \times X_n$, with $x = (x_1, \dots, x_n)$. Then the overall assessment of x is given by

$$U(x) = H(u_1(x_1), \dots, u_n(x_n)) \tag{1}$$

where $H : [0,1]^n \to [0,1]$ is the aggregation function. The overall preference relation $\succeq$ over X is then

$$x \succeq y \iff U(x) \geq U(y) \ .$$

The two-additive Choquet integral is defined for $(z_1, \dots, z_n) \in [0,1]^n$ by [4]

$$\begin{aligned} H(z_1, \dots, z_n) &= \sum_i \left(v_i - \frac{1}{2} \sum_{j \neq i} |I_{i,j}| \right) z_i \\ &+ \sum_{I_{i,j} > 0} I_{i,j} \, (z_i \wedge z_j) + \sum_{I_{i,j} < 0} |I_{i,j}| \, (z_i \vee z_j) \end{aligned} \tag{2}$$

where v_i is the relative importance of criterion i and $I_{i,j}$ is the interaction between criteria i and j. Assume that $z_i < z_j$. A positive interaction between criteria i and j depicts complementarity between these criteria (positive synergy) [4]. Hence, the lower score of z on criterion i conceals the positive effect of the better score on criterion j to a larger extent on the overall evaluation than the impact of the relative importance of the criteria taken independently of the other ones. In other words, the score of z on criterion j is *penalized* by the lower score on criterion i. Conversely, a negative interaction between criteria i and j depicts substitutability between these criteria (negative synergy) [4]. The score of z on criterion i is then *saved* by a better score on criterion j.

3 The disaggregation phase

As said earlier, the aim of the disaggregation phase is to construct a preference model such as (1) combined with (2) from interviews of the DM. Three stages are needed.

The first stage is the structuring phase. It consists in determining the stakes that are involved and identifying the potential viewpoints. Cognitive maps can be used to help in making the right criteria emerge in a bottom-up approach. We obtain a hierarchy structure of the criteria where the root corresponds to the overall aggregation (highest level of aggregation) and the leaves are the attributes. This hierarchy is entered in Myriad (see Figure 1).

The second stage aims at constructing the partial utility functions u_i. When aggregation function H is a weighted sum, the independence of the criteria makes it possible to *separate* the criteria and focus on a criterion i for the construction of its associated utility function u_i, *forgetting* the other criteria and the multi-criteria nature of the problem during this phase. The MACBETH approach [1] is a method that is consistent in a measurement standpoint for the construction of the interval scale u_i. Due to the use of an aggregation function allowing interaction between criteria, isolating the criteria cannot be carried out so that one cannot ask to the DM, information regarding directly u_i. It has been showed in [8] that the utility functions u_i can be constructed from information relating on the overall preference relation $\succeq$, generalizing the MACBETH approach (see Figure 2).

Let us give a little bit more details on that. An interval scale is given up to a dilation and a shift. In order to fix the two degrees of freedom in each utility function u_i, the idea is to identify two elements of X_i that are *perfectly satisfactory* for the DM (denoted by $\mathbf{1}_i$) and *unacceptable* for the DM (denoted by $\mathbf{0}_i$), and to fix $u_i(\mathbf{1}_i) = 1$, $u_i(\mathbf{0}_i) = 0$. We have shown in [8] that asking questions about the difference of satisfaction between the two acts $(x_i, \mathbf{0}_{N \setminus i})$ and $(y_i, \mathbf{0}_{N \setminus i})$, for all $x_i, y_i \in X_i$ enables us to construct u_i whatever the interaction between criteria may be. In the Macbeth methodology, the decision maker is asked to give an assessment of the difference of satisfaction between any two acts $(x_i, \mathbf{0}_{N \setminus i})$ and $(y_i, \mathbf{0}_{N \setminus i})$ (for all $x_i, y_i \in X_i$) in the ordinal scale composed of 6 elements: {*very small, small, mean, large, very large, extreme*} [1].

The last stage concerns the determination of the parameters of the aggregation model, that is the importance and interaction indices of the two-additive Choquet integral. The DM enters in MYRIAD preferential information about each aggregation level composed of a mix of the following three types of data.

- the DM prefers an option to another one;
- the DM gives an overall assessment to an option;
- the DM gives some information directly on the importance or the interaction indices, for instance a criterion is more important than another one, a criterion is important (i.e. $v_i > 1/n$), or the interaction between two criteria is positive.

An algorithm then finds the optimal parameters associated with previous information. The algorithm implemented in MYRIAD is close to the method developed by J.L. Marichal [4]. The information provided by the DM may be inconsistent in the sense that there might be no value of the parameters satisfying the information provided by the DM. In this case, the preferential information that is at the origin of the inconsistency are extracted and showed to the DM.

Once the model is thoroughly specified, an interpretation of this model can be displayed to the DM, in terms of the most/less important criteria, and the pairs of criteria for which the interaction is positive/negative. The DM has then a better insight on the preference model obtained. He can turn back to stage 2 or 3 if he desires to change the model.

4 The aggregation phase

The aggregation step consists in applying the preference model obtained previously on one or several options. This step is not restricted to the computation of the utilities of each option on all elementary criteria and aggregation functions. In order that the assessments and comparisons carried out during the aggregation phase help the DM in validating or rejecting some preference information, the results must be explained. The DM wants to understand precisely the results of the computations by the model. The major point concerns the aggregation part.

A graphical representation of the aggregation by the two-additive Choquet integral is presented in MYRIAD. Let us look at expression (2). From the monotonicity properties on the importance and interaction indices, one has

$$\forall i \in N \ , \quad v_i - \frac{1}{2}\sum_{j\neq i} |I_{i,j}| \geq 0 \ ,$$

$$\sum_{I_{i,j}>0} I_{i,j} + \sum_{I_{i,j}<0} |I_{i,j}| + \sum_{i} \left(v_i - \frac{1}{2}\sum_{j\neq i} |I_{i,j}| \right) = 1$$

Hence all coefficients appearing in (2) are non-negative and they sum-up to one. Expression (2) is thus a convex sum $H(z) = \sum_k \alpha_k \, h_k(z)$, where only three types of decisional behaviors are present: $z_i \wedge z_j$ (intolerant behavior characterizing a positive synergy between i and j), $z_i \vee z_j$ (tolerant behavior characterizing a negative synergy between i and j), and z_i (linear term corresponding to criterion i taken alone).

One can "plot" the result of H in a pie-chart in which each segment represents an elementary behavior h_k (see Figure 5). The aperture of the segment related to h_k is $2\pi\alpha_k$, and this segment is covered at rate $h_k(z)$. Hence, the surface covered by this segment is $\alpha_k h_k(z)$ so that the overall covering of the disk is precisely $H(z)$. This graphical representation makes it easy to understand why result $H(z)$ is rather high (the disk is pretty filled up) or low (the disk is almost empty). This graphical representation is displayed in MYRIAD(see Figure 5).

A semantic explanation is also determined. This argumentation aims at presenting to the elementary decision behaviors that are really at the origin of the evaluation made [7]. These arguments are returned in one or more sentences in MYRIAD (see Figure 4).

In some circumstances, the options are not fixed and can be modified and improved in some ways. We can think of trainees that can improve themselves, or of an industrial product that we want to be as close as possible to the customers' needs. In this case, the DM is not only interested in an assessment of the options. This appears essential in the acquisition cycle. Some recommendations shall provide the most promising improvement ways. We develop an approach based on a sensitivity analysis performed on each coalition of criteria [3, 6]. The determination of the criteria on which it is the most rewarding to improve an