Analysis of a mean operator for data fusion

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Abstract

We present in this article an improvement of the operator triples π of Yager and Rybalov: it is a new operator called mean 3π. Whereas triple π of Yager is an operator completely reinforced this new operator is a mean operator, which makes it more robust to the noise

Keywords: fusion, mean operators, noise.

1 Introduction

Modeling a non-linear dynamic system can be done starting from the differential equation describing it or time series which represent the behavior of the system, as a link between inputs and response of the system. In the second case, the analyze of this kind of systems is carried out either by statistical studies or by methods of classifications. Among the methods of classification we are interesting by the neural methods and the fuzzy methods. The difference between the neural networks and fuzzy logic is related to the modeling of the knowledge of the expert in the last case. One of the advantages of fuzzy logic is that beyond fuzzification of the predicates, we can apply the fuzzification of the operators to connect the predicates. In this paper we introduce a new operator of aggregation called the mean triples π which seems well adapted to analyze the time series describing the strongly non-linearity of biological systems.

2 Data Fusion and Aggregation Operators

The goal of the fusion operators (or aggregation) is to carry out the fusion of information resulting from various and varied sources. There is a great number of fusion operators and the exhaustive list of those would be certainly very important. However let us quote the work completed by Bloch [1] which listed the majority of these operators and classified them according mathematical properties. The choice and utilization of an aggregation operator depend on many parameters more or less objective. But this choice is above all dependent on the *fusion* itself. Before going further it seems important to figure out the definition of fusion. The definition suggested by Bloch, Hunter and al. [2] which was given at the time of work of the European Group of Work on Fusion (FUSION)is:
The fusion consists to join together or aggregate the information coming from various sources, and to exploit this new resulting information in various applications like the decision, numerical estimation, etc.
This definition points out two principal elements. First at all, the definition emphasis the combination of the information. Then the accent is put on the fusion. To define this combination and the goal subjacently, it is advisable to know what kind of data we seek to aggregate. We point out the types of data proposed by Bloch, Hunter and al [2] and later presented by Dubois and Prade [4]:

- *the observations.* They describe the world from more or less particular point of view.

We speak in generally about numerical data provided by sensors.

- *the knowledge*. It describes how the world is *in general*. In this case we speak often about data from human observations that from sensors.
- *the preferences.* It is information which describes how we like the world to be. Of course, this information is coming from persons.
- *the regulations*. We speak about generic information presented on the form of laws.

Concerning the aggregation operators they need to verify the monotony conditions. We could interpret this condition by the fact if the marginal information increase the numerical value also increases or in a less strong condition does not decrease. Among these operators we quote:

- the triangular norms (t-norm). This kind of operators are used when all the sources are reliable. The most used are the minimum and the product. Copulas are generalization of the conjunctives operators (see [4]).
- the t-conorms. They are disjunctive operators. They are used when at least one source is reliable. The other sources could be uncertain. The most known is the maximum.
- the hybrid connections. They are the combinations of t-norms and t-conorms. The goal of these operators is to get the advantages of t-norms and t-conorms by the variation of a parameter. These operators have been studied by Zimmermann et Zynso [15] and later by Piera-Carreté et al. [8].

  Let us note T the t-norm and C the t-conorm dual. Connective mixed the most known is:

  - connective the mixed linear one:

  $$\alpha T(x_1 ..., x_n) + (1 - \alpha)C(x_1 ..., x_n) \quad (1)$$

  - the geometrical connective:

  $$T(x_1 ..., x_n)^{\alpha} + C(x_1 ..., x_n)^{(1-\alpha)} \quad (2)$$

  where $0 \leq \alpha \leq 1$.
  Piera-Carreté and Aguilar-Martin [7] showed that the number of classes provided by the operators connective mixed varies according to the value of the parameter α: if α is closer to 1, the number of classes increases.

- uninorms proposed by Yager and Rybalov [13]. They are commutative, associative and they have a neutral element $e \in [0, 1]$ which the user may fixed. In practice, an uninorm is often defined by a T-norm on the interval $[0, e]$ and by a T-conorm on the interval $[e, 1]$. We point out that some uninorms are symmetrical sums [4]. It is pointed out that the symmetrical sums, introduced by Sylvert [9], are operators whose characteristic is to be symmetrical concerning a sub-set and his complement.
- the zero norms (*nullnorms*) defined by Calvo and al. [3]. They are commutative, associative operators and having an absorbent element $a \in [0, 1]$ which the user could fixed a priori.
- means. Like their name, they are operators which provide a value between the minimum and the possible maximum. Let us quote for example the arithmetic and geometrical mean or the median. Weighted averages ordered suggested by Yager [11] (*Ordered Weighted Averaging: OWA*) are also mean operators into whom it is possible to introduce a weighting depending of the importance and of the reliability of the sources.
- completely reinforced operators. It is a particular class of operators whose have the characteristic to be both positively reinforced and negatively reinforced. We will reconsider more amply these concepts of reinforcement positive and negative in the next paragraphe. The concept of reinforcement was presented by Yager and Rybalov [14]. The only completely reinforced operator that we know is triple Π developed by these two authors [14]. We figure out this triple Π is also a symmetrical sum.

3 The triple Π operator

Suppose that for a given class, a vector form has a membership degrees important for all features considered. In the human reasoning, an aggregation of all this marginal information will be higher than each degree taken separately [14]. In this type of reasoning, the membership degrees which are strong will be reinforced mutually. This behavior is called *positive reinforcement*. Of a similar reasoning, if for a given class, an object has small membership degree the aggregation will be weaker than weakest of the membership degrees values. We speak in this case about *negative reinforcement*.

The total reinforcement is a property which translates certain aspects of the human reasoning. The using of the operator having this property can thus be interesting in measurement where we seek a system close to this type of reasoning.

Definition

An aggregation operator L whose arguments are within the interval $[0,1]$, has the property of positive reinforcement if when all its attributes are affirmative (i.e. higher or equal to 0,5) it checks the conditions:

$$L(x_1..., x_n) \geq \max_i [L(x_i)] \tag{3}$$

Of similar way, an aggregation operator L whose arguments are within the interval $[0,1]$, has the property of negative reinforcement if when all its attributes are not-affirmative (i.e. lower or equal as 0,5), it checks:

$$L(x_1..., x_n) \leq \min_i [L(x_i)] \tag{4}$$

an operator who has the above two properties is defined as being *total reinforced* (*full reinforced*).

The T-norms are negative reinforcement operators ($T(x_1, ..., x_n) \leq \min_i[T(x_i)]$) but they are not positive reinforcement. In addition the t-conorms are positive reinforcement ($C(x_1, ..., x_n) \geq \max_i[C(x_i)]$) but they are not negative reinforcement. We could hope that combinations of T-norms and T-conorms (as the connective mixed one) is completely reinforced but Yager and Rybalov found counterexamples which proves that is not true [14].

The means operators are not positively reinforced or reinforced negatively by definition. Indeed, there is for an average $\min_i(x_i) \leq M(x_1, ..., x_n \leq \max_i(x_i))$.

The only operator who is (to our knowledge) completely reinforced is triple Π defined by Yager and Rybalov [14].

triple Π is defined as follows:

$$PI(x_1..., x_n) = \frac{\Pi_{j=1}^n x_j}{\Pi_{j=1}^n x_j + \Pi_{j=1}^n (1 - x_j)} \tag{5}$$

Remember that this operator is also a symmetrical sum [9]. We must also note that if there are several studies and works on the general properties of the symmetrical sums [9][5], but are few works done on the differences between symmetrical sums.

The property of the total reinforcement is thus particularly interesting because it makes it possible to obtain a good modeling of the human behavior, which is often the goal of many systems of fusion d'informations. It should be noted that triple Π incorporates information of the type of the *observations in order to refine the information related to the real world* [2] and can be used in this type of information fusion.

4 The *Mean Triple* Π

4.1 The mean triple Π

The triple Π is an interesting operator by the fact it is a completely reinforced, it is sometimes more judicious to use operators of the type *mean*. As underline it Yager [12], then Bloch, Hunter and al [2], and Dubois and Prade [4], when the signals used represent the same phenomenon (these signals may be independent ones to others or not), it is more relevant, in a conceptual point of view, to use a mean operator in order to synthesize the information.

The basic idea which leds to the definition of this new mean operator has been to seek a mean operator which has properties close to that of triple

Π. Which means:

$$PI(x_1,...,x_n) = \frac{\Pi_{j=1}^n G(x_j)}{\Pi_{j=1}^n G(x_j) + \Pi_{j=1}^n G(1-x_j))} \quad (6)$$

where $G(x)$ is a function named *generatrix function* which is positive et increasing [10][9]. To obtain the idempotent property, we have considered the function $G(x) = x^{1/n}$ where n is the dimension of the vector x. We could defined a new aggregation operator:

$$MPI(x_1,...,x_n) = \quad (7)$$

$$= \frac{\Pi_{j=1}^n (x_j)^{(1/n)}}{\Pi_{j=1}^n (x_j)^{(1/n)} + \Pi_{j=1}^n (1-x_j)^{(1/n)}} \quad (8)$$

$$= \frac{1}{1 + \Pi_{j=1}^n \left[\frac{1-x_j}{x_j}\right]^{1/n}}$$

We named this new operator *mean triple* Π, by reference to the triple Π from which has been obtained.

Proposition
The mean triple Π defined above is a mean operator.

Demonstration
We show that this operator is a mean by checking the properties of the mean operators (voir [12]), therefore:

1. the commutativity: $MPI(x,y) = MPI(y,x)$
2. the monotonic: $MPI(x,y) \geq MPI(z,t)$ si $x \geq z$ et $y \geq t$
3. the idempotence: $M(x,...,x) = x$
4. the self identity : $MPI[B, \langle MPI(B)\rangle] = MPI(B)$

The first three conditions could be deduce easily from the properties of the product function and n-square function. The most difficult property is the self identity shows in the next lines. We want demonstrate that:
$MPI(x_1,...,x_n, MPI(x_1,...,x_n)) =$
$= MPI(x_1,...,x_n, MPI) = MPI(x_1,...,x_n)$.

Therefore:
$MPI(x_1,...,x_n, MPI) =$

$$\frac{\Pi_{j=1}^n (x_j)^{1/(n+1)} \times (MPI)^{1/(n+1)}}{D}$$

with $D = [\Pi_{j=1}^n (x_j)^{1/(n+1)} \times (MPI)^{1/(n+1)} + \Pi_{j=1}^n (1-x_j)^{1/(n+1)} \times (1-MPI)^{1/(n+1)}]$
and
$(MPI)^{1/(n+1)} =$
$= \left(\frac{\Pi_{j=1}^n (x_j)^{(1/n)}}{\Pi_{j=1}^n (x_j)^{(1/n)} + \Pi_{j=1}^n (1-x_j)^{(1/n)}}\right)^{1/(n+1)}$
$= \frac{\Pi_{j=1}^n (x_j)^{1/n(n+1)}}{\left(\Pi_{j=1}^n (x_j)^{(1/n)} + \Pi_{j=1}^n (1-x_j)^{(1/n)}\right)^{1/(n+1)}}$
By simplification with:
$\frac{1}{\left(\Pi_{j=1}^n (x_j)^{(1/n)} + \Pi_{j=1}^n (1-x_j)^{(1/n)}\right)^{1/(n+1)}}$
we have:

$MPI(x_1,...,x_n, MPI) =$
$= \frac{\Pi_{j=1}^n (x_j)^{[1/(n+1)+1/n(n+1)]}}{\Pi_{j=1}^n (x_j)^{[1/(n+1)+1/n(n+1)]} + \Pi_{j=1}^n (1-x_j)^{[1/(n+1)+1/n(n+1)]}}$
$= \frac{\Pi_{j=1}^n (x_j)^{[(n+1)/n(n+1)]}}{\Pi_{j=1}^n (x_j)^{[(n+1)/n(n+1)]} + \Pi_{j=1}^n (1-x_j)^{[(n+1)/n(n+1)]}}$
$= \frac{\Pi_{j=1}^n (x_j)^{(1/n)}}{\Pi_{j=1}^n (x_j)^{(1/n)} + \Pi_{j=1}^n (1-x_j)^{(1/n)}}$
$= MPI = MPI(x_1,...,x_n)$.

The mean triple Π is a new mean operator of aggregation obtaining from triple Π. It is obvious that the mean triple Π cannot be completely reinforced since by definition it is a mean: the numerical value is between the maximum and the minimum. However, it has a property similar to the total reinforcement of triple Π, which is based on a comparison with the classic arithmetic mean. The definition of the property is given in below.

4.2 The *mean reinforcement*

Propriety
Let MPI be the mean triple Π. We consider the classic arithmetic mean $\frac{1}{n}\sum_{j=1}^n x_j$. Then:
If $\forall j \in 1,...,n$, with $x_j \geq 0,5$,then we have:

$$MPI(x_1,...,x_n) \geq \frac{1}{n}\sum_{j=1}^n x_j \quad (9)$$

If $\forall j \in 1, ..., n$, with $x_j \leq 0,5$, then we have:

$$MPI(x_1, ..., x_n) \leq \frac{1}{n}\sum_{j=1}^{n} x_j \qquad (10)$$

This property has named mean reinforcement by reference to the total reinforced of the triple Π.

Demonstration
Show that $\forall j \in 1, ..., n$, with $x_j \geq 0,5$, then:

$$\frac{\Pi_{j=1}^{n}(x_j)^{(1/n)}}{\Pi_{j=1}^{n}(x_j)^{(1/n)} + \Pi_{j=1}^{n}(1-x_j)^{(1/n)}} \geq \frac{1}{n}\sum_{j=1}^{n} x_j$$

We have:

$$\frac{\Pi_{j=1}^{n}(x_j)^{(1/n)} + \Pi_{j=1}^{n}(1-x_j)^{(1/n)}}{\Pi_{j=1}^{n}(x_j)^{(1/n)}} \leq \frac{n}{\sum_{j=1}^{n} x_j} \quad \text{if all } x_i \in [\frac{1}{2}, 1]$$

let:

$$1 + \frac{\Pi_{j=1}^{n}(1-x_j)^{(1/n)}}{\Pi_{j=1}^{n}(x_j)^{(1/n)}} \leq \frac{n}{\sum_{j=1}^{n} x_j} \quad \text{if all } x_i \in [\frac{1}{2}, 1]$$

of course:

$$\frac{\Pi_{j=1}^{n}(1-x_j)^{(1/n)}}{\Pi_{j=1}^{n}(x_j)^{(1/n)}} \leq \frac{n}{\sum_{j=1}^{n} x_j} - 1 \quad \text{if all } x_i \in [\frac{1}{2}, 1]$$

and:

$$(\prod_{i=1}^{n}(\frac{1}{x_i} - 1))^{\frac{1}{n}} \leq \frac{1}{\frac{\sum_{i=1}^{n} x_i}{n}} - 1 \quad \text{if all } x_i \in [\frac{1}{2}, 1]$$

Taking the logarithme of the both terms we prove that :

$$\frac{1}{n}\sum_{i=1}^{n} ln(\frac{1}{x_i} - 1) \leq ln(\frac{1}{\frac{\sum_{i=1} nx_i}{n}} - 1) \quad \text{if all } x_i \in [\frac{1}{2}, 1]$$

Or the function $f(x) = ln(\frac{1}{x} - 1)$ has the first derivative:

$$\frac{df(x)}{x} = \frac{1}{x^2 - x}$$

and the second one is:

$$\frac{d^2 f(x)}{dx^2} = \frac{1-2x}{(x^2 - x)^2}$$

The inequality above is du to the concavity of f within interval $[\frac{1}{2}, 1]$ (Jensen inequality) on the interval $[\frac{1}{2}, 1]$, the sign of the second derivative of f is negative: $\frac{d^2 f(x)}{dx^2} \leq 0$.
the second inequality comes from the fact that on $[0, \frac{1}{2}]$ f is convex.
This property translates the fact that when the signals are in accord, the average triple Π discriminates the classes better that the arithmetic mean. This property could be interesting when we want evaluate the correlations between different the sources.

4.3 Experimental results

The application that we treat relates to fusion of information during a processus of classification in a biotechnology. We need to fusion information resulting from various measured biochemical parameters (pH, dioxygene, carbon dioxide, etc.) allowing to carry out a classification whose classes correspond to the physiological states of these micro-organisms (note we have *information from real world*). Classification used is method LAMDA (Learning Algorithm for Multivariate Went back Analysis) [6] [10].
Comparatively to triple Π, the average triple Π provides a ranging value between the minimum and the maximum of the marginal membership degrees, making the synthesis between the various manifestations of the same event. In practice, we noted that the results of classification of the two operators were generally similar.

A notable exception between triple Π and the average triple Π is the classification of the noisy signals. The mean triple Π du to its property of smoothing is more robust with the noisy signal than triple Π. On the figure 1, 4 parameters (pH, rO_2, rO_2 and Luminance) have been used . The two operators were tested on these noisy signals and two classifications are disturbed by the presence of noise. Nevertheless even in the presence of noise, the classification using the average triple Π provides at least a class which characterizes the fermentation state (state 1) see the panel 1.

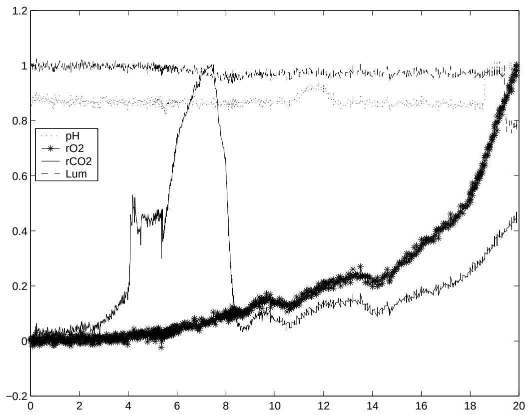

Figure 1: The 4 biochemical parameters with noise(SNR=40dB)

	% classif. MPI	% classif. PI
SNR=46,02 dB	13,47%	CTN
SNR=40 dB	11,56%	CTN
SNR=30,40 dB	CTN	CTN

Table 1: Comparaison of the classification using the mean triple Π (MPI) and the triple Π (PI). CTN signifies that the Classification is Too Noisy to obtain significant result

5 Conclusion

We introduced a new operator who combines the properties of the completely reinforced operators and the mean operators. The results obtained on the time series show that this operator is less sensitive to the noise as the operator of Yager and Rybalov [14]. Therefore in the case of biological signals which have important response time we can use it as aggregation operator to find the classes associated in physiological states.

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