Enhancing OLAP Querying with the aid of H-IFS

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Abstract— *Over the past years we have witnessed an increasing interest in expressing user or domain preferences or knowledge inside database queries. First, it appeared to be desirable property of a query system to offer more expressive query languages that can be more faithful to what a user intends to say. Second, a classical query in the sense of relational paradigm may also have a restricted answer or sometimes an empty set of answers, while a relaxed version of the query enhanced with background or domain knowledge might be matched by some items in the database.*

Keywords— Query answering, OLAP, IFS, multidimensional modelling.

1 Introduction

The need for flexible systems to manage value uncertainty has been the focus for database researchers [1], mainly at theoretical level and in the context of the relational model.

At the same time OLAP technology required [2] the extension of the relational systems with the inclusion of the data-cube and operators to operate over it. Alternatively, new models [3] were proposed to support OLAP based querying on top of multidimensional views. Both approaches support the organisation of data around several axes of analysis. In OLAP based systems, when it comes to the model level, support for value uncertainty will be required at the fact level as well at the level of dimensions with the support of non-rigid hierarchies [4]. Still [4] considers that facts and dimensions as in [5], [6] represent structural information [7].

Current research issues for OLAP systems can be summarised as follows

- Flexible models are required to support value uncertainty at fact level as well as at the dimension level with the provision of non rigid dimensions
- Flexibility should not be limited at the structural level. It should be allowed also at the query level. Users should be allowed to synthesise their own model of dimensions for analysis purposes based on existing structure. Dimensions may be based in either rigid or non-rigid hierarchies.

Recently in OLAP systems a need has been identified for enhancing the query scope with the aid of "kind-of" relation that describe knowledge as well as ordering of the elements of a domain or a hierarchical universe.

However, in our context, the terms of the hierarchy [8], and the relations between terms are not fuzzy and do not represent [9] only kind-of relations. These observations led us to introduce the concept of closure of the H-IFS which is a developed form defined on the whole hierarchy.

The rest of the paper is organised as follows; In Section two we define the basic properties of Intuitionistic Fuzzy sets and H-IFS. In Section 3 the semantics of the H-IFS cubic representation [14], [15] in contrast to the basic multidimensional-cubic structures are presented we also define the extended SQL-OLAP aggregators. In Section 4 we present the main concepts involved in the designing and implementation the 'IF-Oracle' OLAP utility and also demonstrate the potential of 'IF-Oracle' utility. Finally we conclude and compare our proposal with other school of thoughts.

2 Intuitionistic Fuzzy Sets A-IFS

2.1 Foundations

Each element of an Intuitionistic fuzzy [10, 11] set has degrees of membership or truth (μ) and non-membership or falsity (ν), which don't sum up to 1.0 thus leaving a degree of hesitation margin (π).

As opposed to the classical definition of a fuzzy set given by

$A' = \widetilde{A} = \{< x, \mu_A(x) > | x \in X\}$ where $\mu_A(x) \in [0,1]$ is the membership function of the fuzzy set A′, an Intuitionistic fuzzy set A is given by

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$$
$$\mu_A : X \to [0,1] \text{ and } \nu_A : X \to [0,1]$$

Such that

$$0 \le \mu_A(x) + \nu_A(x) \le 1$$

and $\mu_A : X \to [0,1], \nu_A : X \to [0,1]$ denote a degree of membership and a degree of non-membership of X ∈ A, respectively. Obviously, each fuzzy set may be represented by the following Intuitionistic fuzzy set;

$$A = \{< x, \mu_A(x), 1 - \mu_A(x) > | x \in X\}$$

For each Intuitionistic fuzzy set in X, we will call $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, Intuitionistic fuzzy index of $x \in A$ which expresses a lack of knowledge of whether x belongs to A or not. For each $x \in A$ $0 \le \pi_A(x) \le 1$.

Definition 1. Let A and B be two Intuitionistic fuzzy sets defined on a domain X. A is included in B (denoted A ⊆ B) if and only if their membership functions and non-membership functions satisfy the condition:

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$(\forall X \in X)\ (\mu_A(x) \leq \mu_B(x)\ \&\ \nu_A(x) \geq \nu_B(x))$

Two scalar measures are classically used in classical fuzzy pattern matching to evaluate the compatibility between an ill-known datum and a flexible query, known as

- a possibility degree of matching, **Π(Q/ D)**
- a necessity degree of matching, **N(Q/ D)**

Definition 2. Let Q and D be two Intuitionistic fuzzy sets defined on a domain X and representing, respectively, a flexible query and an ill-known datum.

The possibility degree of matching between Q and D, denoted Π(Q/D), is an "optimistic" degree of overlapping that measures the maximum compatibility between Q and D, and is defined by:

$$\Pi(Q/D) = \left\langle \sup_{x \in X} \min(1-\nu_Q(x), \nu_Q(x)), \inf_{x \in X} \max(1-\nu_D(x), \nu_D(x)) \right\rangle$$

The necessity degree of matching between Q and D, denoted N(Q/D), is a "pessimistic" degree of inclusion that estimates the extent to which it is certain that D is compatible with Q, and is defined by:

$$N(Q/D) = \left\langle \inf_{x \in X} \max(\mu_Q(x), 1-\mu_Q(x)), \sup_{x \in X} \min(\mu_D(x), 1-\mu_D(x)) \right\rangle$$

The problem occurring from defining Intuitionistic fuzzy sets based on the "kind-of" relation is that two different Intuitionistic fuzzy sets on the same hierarchy do not necessarily have the same definition domain, which means they cannot be compared using the classic comparison operations **Π**(Q/ D), N(Q/ D).

2.2 From IFS to H-IFS

The definition domains of the hierarchical fuzzy sets [12, 13] that we propose below are subsets of hierarchies composed of elements partially ordered by the "kind of" relation. An element l_i is more general than an element l_j (denoted $l_i \sim l_j$), if l_i is a predecessor of l_j in the partial order induced by the "kind-of" relation of the hierarchy. An example of such a hierarchy is given in 'Figure 1'.

Definition 3. Let F be a H-IFS defined on a subset D of the elements of a hierarchy L. It degree is denoted as <μ, ν>. The closure of F, denoted clos(F), is a H-IFS defined on the whole set of elements of L and its degree $<\mu, \nu>_{clos(F)}$ is defined as follows.

For each element **l** of L, let $S_L = \{l_1, \ldots, l_n\}$ be the set of the smallest super-elements in D.

If S_L is not empty,

$$< \mu, \nu >_{clos(F)} (S_L) = < \max_{1 \leq i \leq n} \mu(L_i), \min_{1 \leq i \leq n} \nu(L_i) >$$

else

$$< \mu, \nu >_{clos(F)} (S_L) = < 0,0 >$$

In other words, the closure of a H-IFS F is built according to the following rules. For each element l_1 of L:

- If l_I belongs to F, then l_I keeps the same degree in the closure of F (case where $S_L = \{ l_I \}$).
- If l_I has a unique smallest super-element l_I in F, then the degree associated with l_I is propagated to L in the closure of F, $S_L = \{ l_I \}$ with $l_{I} > l_I$)

If L has several smallest super-elements $\{l_1, \ldots, l_n\}$ in F, with different degrees, a choice has to be made concerning the degree that will be associated with l_I in the closure. The proposition put forward in definition 3, consists of choosing the maximum degree of validity μ and minimum degree of non validity v associated with $\{l_1, \ldots, l_n\}$. We refer to as the *Optimistic strategy*.

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We can also utilise a *Pessimistic strategy* which consists of choosing the minimum degree of validity μ and maximum degree of non validity ν associated with $\{l_1, \ldots, l_n\}$.Alternatively, an *Average strategy* could be utilised, which consists of calculating the IF-Average and applying it to the degrees of validity μ and non-validity ν.

It has been observed that two different H-IFSs, defined on the same hierarchy, can have the same closure, as in the following example.

The H-IFSs Q={Wine<1,0>, Red Wine<0.7,0.1>, Brown Wine<1,0>, White Wine <0.4,0.3>} and R={Wine<1,0>, Red Wine<0.7,0.1>, Brown Wine<1,0>, Pinot Noir <1.0,0.0>} have the same closure, represented 'Figure 1' below.

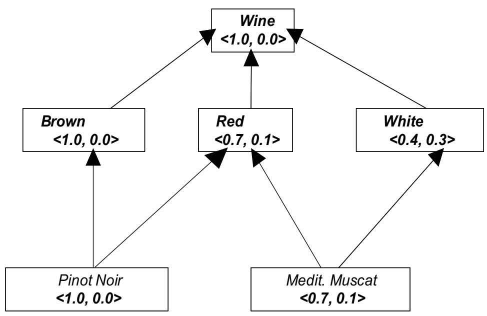

Figure 1: Common closure of the H-IFS's Q and R

Such H-IFS's form equivalence classes with respect to their closures.

Definition 4. Let F be a hierarchical fuzzy set, with dom(F) = $\{l_1, \ldots, l_n\}$, and F_{-k} the H-IFS resulting from the restriction of F to the domain dom(F) \ $\{l_k\}$. l_k is deducible in F if

$$<\mu, \nu>clos_{(F-k)} (l_k) = <\mu, \nu>clos_{(F)} (l_k)$$

As a first intuition, it can be said that removing a derivable element from a hierarchical fuzzy set allows one to eliminate redundant information. But, an element being derivable in F does not necessarily mean that removing it from F will have no consequence on the closure: removing k from F will not impact the degree associated with k itself in the closure, but it may impact the degrees of the sub-elements of k in the closure.

For instance, if the element Brown Wine is derivable in Q, according to *definition 4*, removing Brown Wine <1,0> from Q would not modify the degree of Brown Wine itself in the resulting closure, but it could modify the degree of its

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sub-element Pinot Noir. Thus, Brown Wine <1,0> cannot be derived or removed. This remark leads us to the following definition of a minimal hierarchical fuzzy set.

Definition 5. In a given equivalence class (that is, for a given closure C), a hierarchical fuzzy set is said to be **minimal** if its closure is C and if none of the elements of its domain is derivable**.**

Obtaining the Minimal H-IFS

Step 1: Assign Min-H-IFS $\leftarrow \emptyset$. Establish an order so that the sub-elements $\{l_1,\ldots,l_n\}$ of the hierarchy L are examined after its super-elements.

Step 2: Let l_1 be the first element & $(\mathbf{l_1})/<\mu, \nu> \neq (\mathbf{l_1})/<0, 0>$ then add $\mathbf{l_1}$ to Min-H-IFS and

$$<\mu, \nu>_{clos(Min\text{-}HIFS)} (\mathbf{l_1}) = (\mathbf{l_1})/<\mu, \nu>.$$

Step 3: Let us assume that K elements of the hierarchy L satisfy the condition $<\mu, \nu>_{clos(Min\text{-}HIFS)} (\mathbf{l_i}) = (\mathbf{l_i})/<\mu, \nu>$.

In this case the Min-H-IFS do not change. Otherwise go to next element $\mathbf{l_{k+1}}$ and execute Step 4.

Step 4: The $\mathbf{l_{k+1}}/<\mu_{k+1}, \nu_{k+1}>$ associated with $\mathbf{l_{k+1}}$.

In this case $\mathbf{l_{k+1}}$ is added to Min-H-IFS with the corresponding $<\mu_{k+1}, \nu_{k+1}>$.

Step 5: Repeat steps three and four until $\mathbf{clos_{(Min\text{-}HIFS)}=C}$.

For instance S_1 and S_2 are **minimal** (none of their elements is derivable). They cannot be reduced further.

S_1= Wine<1,0>

S_2={Wine<1,0>, Red Wine<0.7,0.1>, Pinot Noir<1,0>, White Wine <0.4, 0.3>}

3 Representing H-IFS Inside Cubes

In this section we present the semantics of the H-IFS cubic representation [14], [15] in contrast to the basic multidimensional-cubic structures. The basic cubic operators are extended and enhanced with the aid of Intuitionistic Fuzzy Logic.

Since the emergence of the OLAP technology [16] different proposals have been made to give support to different types of data and application purposes. One of these is to extend the relational model (ROLAP) to support the structures and operations typical of OLAP. Further approaches [17], are based on extended relational systems to represent data-cubes and operate over them.

Nowadays, information and knowledge-based systems need to manage imprecision in the data and more flexible structures are needed to represent the analysis domain. New models have appeared to manage incomplete datacube [18], imprecision in the facts and the definition of fact using different levels in the dimensions.

Nevertheless, these models continue to use inflexible hierarchies thus making it difficult to merge reconcilable data from different sources with some incompatibilities in their schemata. These incompatibilities arise due to different perceptions-views about a particular modelling reality.

In addressing the problem of representing flexible hierarchies we propose a new multidimensional model that is able to treat with imprecision over conceptual hierarchies based on Intuitionistic Fuzzy logic. The use of conceptual hierarchies enables us to:

- define the structures of a dimension in a more perceptive way to the final user, thus allowing a more perceptive use of the system.
- query information from different sources or even use information or preferences given by experts to improve the description of hierarchies, thereby getting more knowledgeable query results. We outline a unique way for incorporating "H-IFS" relations, or conceptual imprecise hierarchies as dimensions with respect to the model proposed in [19].

3.1 *Semantics of the IF-Cube in contrast to Crisp Cube*

In this section we review the semantics of Multidimensional modelling and Intuitionistic Fuzzy Logic and based on these we propose a unique concept named as Intuitionistic Fuzzy Cube (IF-Cube). The IF-Cube is the basis for the representation of flexible hierarchies and thus flexible facts.

3.1.1 *Overview of the Cube Model*

A logical model that influences the database design and the query engines is the *multidimensional-cubic* view of data in the warehouse. In a multidimensional data model, there is a set of *numeric measures* that are the objects of analysis. Examples of such measures are sales, budget, etc. Each of the numeric measures depends on a set of *dimensions,* which provide the context for the measure. The attributes of a dimension may be related via a hierarchy of relationships. In the above example, the product name is related to its category and the industry attribute through a hierarchical relationship, see Fig. 2

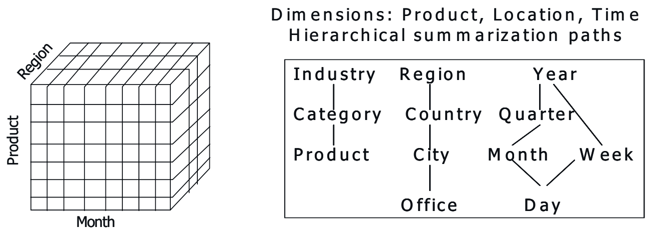

Figure 2: Cube_Sales, Rigid Hierarchies for Product, Location, Time Dimensions

According to [19] a cube structure is defined as a 4-tuple, <D, M, A, f> where the four components indicate the characteristics of the cube. These characteristics are:

a set of n dimensions $D = \{d_1, d_2, \ldots, d_n\}$ where each d_i is a dimension name, extracted from a domain $dom_{dim(i)}$. A set of k measures $M = \{m_1, m_2, \ldots, m_k\}$ where each m_i is a measure name, extracted from a domain $dom_{measure(i)}$. The set of dimension names and measures names are disjoint; i.e., $D \cap M = 0$. A set of t attributes $A = \{a_1, a_2, \ldots, a_t\}$ where each a_i is an attribute name, extracted from a domain $dom_{attr(i)}$. A one-to-many mapping $f : D \rightarrow A$, i.e. there exists, corresponding to each dimension, a set of attributes.

3.1.2 *Semantics of the IF-Cube*

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In contrast, an **IF-Cube** is an abstract structure that serves as the foundation for the multidimensional data cube model. Cube C is defined as a five-tuple *(D, l, F, O, H)* where:

- *D* is a set of dimensions
- *l* is a set of levels $l_1, \ldots, l_n$,
- A dimension $d_i = (l \leq O, l\perp, l_\top)$ $dom(d_i)$ where
- $l = l_i$ $i=1 \ldots n$. l_i is a set of values and $l_i \cap l_j = \{\}$
- $\leq O$ is a partial order between the elements of *l*.
- To identify the level *l* of a dimension, as part of a hierarchy we note it as *dl.* $l\perp$: base level $l_\top$: top levelfor each pair of levels l_i and l_j we have the relation :

$\mu_{ij} : l_i \times l_j \rightarrow [0,1]$ $\quad v_{ij} : l_i \times l_j \rightarrow [0,1]$ $0 < \mu_{ij} + v_{ij} < 1$

- *F* is a set of fact instances with schema $F = \{<x, \mu_F(x), v_F(x)> | x \in X\}$, where $x = <att_1, \ldots, att_n>$ is an ordered tuple belonging to a given universe *X*, $\mu_F(x)$ *and* $v_F(x)$ are the degree of membership and non-membership of *x* in the fact table *F* respectively.
- *H* is an object type history that corresponds to a cubic structure*(l, F, O, H′)* which allows us to trace back the evolution of a cubic structure after performing a set of operators i.e. aggregation.

The example below provides a sample imprecise cube (*D, l, F, O, H) i.e. sales* and a conceptual non-rigid hierarchy product with reference to wine consisting of $l_i, \ldots, l_n$ levels with respective levels of membership and non membership < μ_{ij} v_{ij}, >.

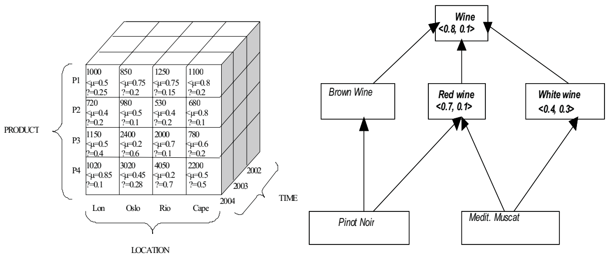

Figure 3: *Imprecise Cube Sales, IF Hierarchy 'Wine*

3.1.3 Cube Aggregation

Aggregation (A): An aggregation operator *A* is a function *A(G)* where $G = \{<x, \mu_F(x), v_F(x)> | x \in X\}$ where $x = <att_1, \ldots, att_n>$ is an ordered tuple belonging to a given universe.

X, $\{att_1, \ldots, att_n\}$ is the set of attributes of the elements of *X*, $\mu_F(x)$ *and* $v_F(x)$ are the degree of membership and non-membership of *x*.

The result is a bag of the type $\{<x', \mu_F(x'), v_F(x')> | x' \in X\}$. To this extent, the bag is a group of elements that can be duplicated and each one has a degree of μ and ν.

Input: $C_i =$ *(D, l, F, O, H)* and the function *A(G)*

Output: $C_o = (D, l_o, F_o, O_o, H_o)$

The definition of the extended group operators allows us to define the extended group operators ***Drill up (Δ), and Drill Down (Ω).***

Drill up (Δ): The result of applying Drill up over dimension d_i at level dl_r using the aggregation operator A over a datacube $C_i = (D_i, l_i, F_i, O, H_i)$ is another datacube $C_o = (D_o, l_o, F_o, O, H_o)$.

Input: $C_i = (D_i, l_i, F_i, O, H_i)$

Output: $C_o = (D_o, l_o, F_o, O, H_o)$

An object of type history is a recursive structure. The structured history of the datacube allows us to keep all the information when applying *Drill up* and get it all back when *Drill Down* is performed. To be able to apply the operation of *Drill up* we need to make use of the IF_{SUM} aggregation operator.

Drill Down (Ω): This operator performs the opposite function of the *Drill up* operator. It is used to Drill Down from the higher levels of the hierarchy with a greater degree of generalization, to the leaves with the greater degree of precision. The result of applying *Drill Down* over a datacube C_i = *(D, l, F, O, H)* having H=*(l', D', A', H')* is another datacube C_o= *(D', l', F', O, H').*

Input: C_i=*(D, l, F, O, H)*

Output C_o=*(D',l',F',O,H')* where *F'* → *set of fact instances defined by operator A.*

The defined IF OLAP Cube model allows us to:

- accommodate imprecise facts
- utilise *conceptual hierarchies* used for aggregation purposes in the cases of roll-up and Drill Down operations.
- offer a unique feature such as keeping track of the history when we move between different levels of a hierarchical order.

In the next section we demonstrate the usefulness of the H-IFS notion and the extended aggregation operators for extending the query capabilities of Oracle10g. We developed an ad-hoc utility 'IF-Oracle' implemented on top of Oracle10g that allow us to:

- Define an H-IFS hierarchy
- Incorporate hierarchical knowledge in the form of H-IFS as part of the standard OLAP queries.
- Enhance the scope of query answers against the Oracle10g standard query answers.

4 Embedding IF Cubes in Oracle10g

IF-Oracle has been developed using Visual Studio.Net as an ad-hoc utility that is attached to and enhances Oracle10g DBMS query capabilities.

For demonstrating the functionality of IF-Oracle let us consider a sample multidimensional model, Fig.4 in the form of a star schema that describes sales of Vitis Vinifera type wines.

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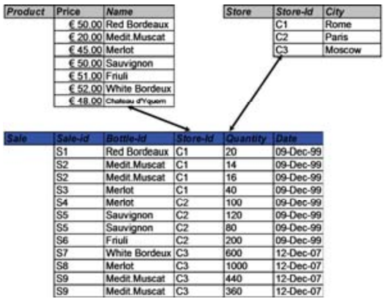

Figure 4: Sample of a Star Schema

After forming the structure and storing it as a concept relation in Oracle10g, we perform the calculation of the hierarchical closure of the H-IFS and its weights.
The user now has the choice of selecting three different strategies: ***Optimistic, Pessimistic*** or ***Average*** as defined on section 2.2.
Let's assume that the user's interest lays on finding information about Red wines. Fig.5 below shows the hierarchy after weights have been estimated and assigned reflecting the user's intent.

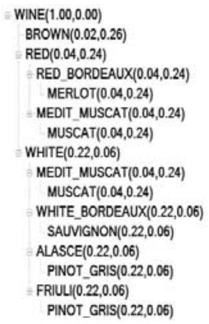

Figure 5: Vitis Vinifera sub-hierarchy view with weights

We can observe that the principle of the H-IFS closure *(see definition 3)* has been preserved when propagating the degree of validity μ and non-validity ν from super-elements to sub-elements by using the optimistic strategy.
The degree of validity and non-validity <μ, ν > are calculated as follows:

$$\mu = \frac{|c_l|}{|c_{l-1}|} \qquad \nu = \frac{|\neg c_l|}{|c_{l-1}|}$$

Where c_l corresponds to those elements from the fact table that absolutely satisfy the selection criteria with reference to a node in the hierarchy. C_{l-1} represents the elements children elements of that selection on a lower level that satisfy the selection condition to some extent. It is obvious that $\pi = 1-(\mu+\nu)$.
After adding the hierarchy into the repository and automatically calculating the weights for the requested nodes, the user can utilize the ad-hoc interface for execution of queries either in standard SQL or make use of the enhanced Select clause and features that IF-Oracle provides.

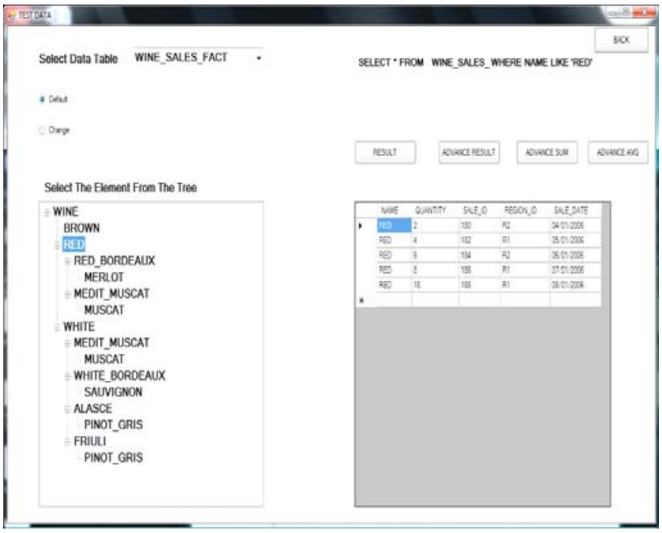

Figure 6: Standard SQL output for "Red" wine

Fig. 6 shows the results of a user request for "Red" wine executed in standard SQL provided by Oracle10g.
In contrast, Fig. 7 shows the output after executing the same query, but this time using the IF-Oracle utility.

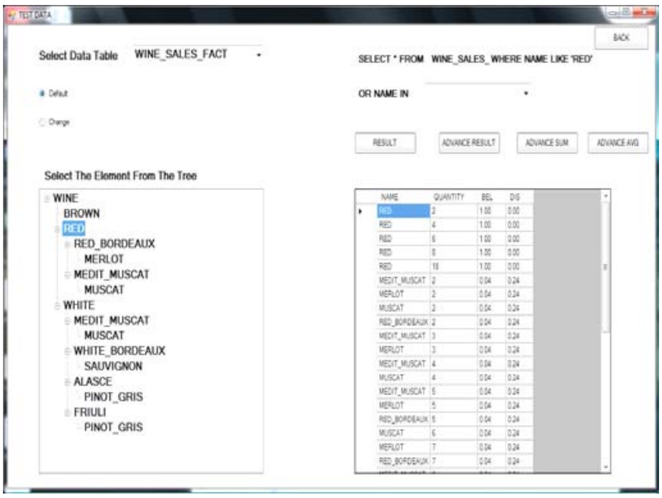

Figure 7: Enhanced SQL output for "Red" wine

By comparing the two figures, one can observe that IF-Oracle produces a knowledge-based answer instead of mindlessly matching the records against the word "Red".
The results show that IF-Oracle not only retrieves sales of "Red" bottles, but also sales of bottles that are classified as red wines by the knowledge represented in the H-IFS hierarchy as "Merlot", "Red Bordeaux", "Medit. Muscat", etc. with indicative degrees of <μ, ν> relevant to the user's preference.
At this point one may decide to further enhance the query capabilities of the IF-Oracle utility by allowing versions of hierarchies. In such case similarities [20] and dissimilarities [21] between different versions should be reflected in the query results, or in the common acceptable DWH schema.

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5 Conclusions & Comparisons

In this paper, we focus on integrating hierarchical users preferences or intent in OLAP queries with the aim on enhancing the OLAP scope and in return to get richer answer, closer to user requests. We provide a means of using background knowledge to re-engineer query processing and answering with the aid of H-IFS and Intuitionistic Fuzzy relational representation.

The hierarchical links defined on the basis of the H-IFS closure are representing hierarchical knowledge in different forms. A cubic OLAP model presented that allows dimensions to be defined as H-IFS.

In comparing our framework with the bipolar querying school of thought as presented in [22], [23], and [24] the reader may take into account the following:

- The bipolar querying school of thought is the main vehicle for allowing users to express their preferences as part of a query formulation. Allowing thus queries to come up with more knowledgeable answers.

However our framework is resolving a different problem, related specifically to OLAP in the following ways:

- It allows users to import knowledge from external sources in order to redefine the axis of analysis and eventually the cubic structure for different business scenarios.
- The IF-Oracle utility can operate using either cubic structures-MOLAP or relations-ROLAP.
- The IF-Oracle utility assumes that stored data are precise. However it can also cope with imprecise data. The importance here is not on the goodness of data, is mainly on how well the stored data do fit in different scenarios or changing dimensions.

Future research efforts will concentrate on incorporating knowledge arriving from external sources either semi structured or unstructured, considering the web as such source.

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