

A New Conditioning Rule, Its Generalization and Evidential Reasoning

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Abstract - In Evidence theory, several conditioning rules for updating belief have been proposed, including Dempster's rule of conditioning. The paper views the conditioning rules proposed so far and proposes a new rule of conditioning based on three requirements. Then, it generalizes the rule to be applied to the case where condition is given by an uncertain belief. The paper also discusses a few interpretations of an equation used for evidential reasoning, one of which is interpreted as conditioning with an uncertain condition.

Keywords— Dempster-Shafer theory of evidence, evidence theory, conditioning rule, evidential reasoning, uncertainty.

1 Introduction

The paper discusses a new conditioning rule of Evidence theory that we proposed recently [1,2] and generalizes the rule to be applied to the case where condition is given by an uncertain belief. Then, it discusses a few interpretations of an equation used for evidential reasoning, one of which is interpreted as conditioning with an uncertain condition.

The first and well-known conditioning rule in the theory is Dempster's rule of conditioning [3]. However, it has been pointed out that the rule has a few problems, and several alternatives have been proposed so far [4-13]. A well-known problem is that a small change of prior belief, which is represented as basic belief assignment (*bba*) or mass function in the paper, may produce a large change of posterior belief [4]. Another is a practical problem that conditional belief is undefined, when a given condition *totally conflicts* with the prior belief in the sense of $Pl(B) = 0$, where B is the condition represented by a subset of a frame of discernment Θ and Pl is the plausibility measure equivalent to the prior belief.

Dempster's rule of *conditioning* is derived as a special case of Dempster's rule of *combination*. Suppose there are two distinct beliefs represented by *bba*'s m , m' , and the total mass of m' is assigned to a set B , i.e. $m'(B) = 1$. In the case, the belief combined from m and m' is the same as the conditional belief $m(\bullet \mid B)$. This means that both information of the prior belief m and the condition B are treated equivalently or symmetrically.

The paper deals with the two pieces of information asymmetrically. In the basic conditioning rule, which is one with a sure condition without any uncertainty in the paper, it is assumed that the condition is a piece of information given *a posteriori* and always certain, while the prior belief is given *a priori* and may not be correct. When they *partially*

conflict with each other (i.e. $X \cap B = \emptyset$ for a X , where X is a focal element of the prior belief m , and \emptyset is the empty set), the proposed conditioning trusts in B rather than X , and reassigns the mass $m(X)$ to B . The simple idea produces a conditioning rule different from conventional ones proposed so far. Then, the paper expands the basic conditioning to the one with an uncertain condition, and discusses the relation between the conditioning rule and evidential reasoning.

The paper is composed as follows; section 2 reviews Dempster-Shafer theory of evidence and major conditioning rules proposed so far including those with an uncertain condition represented by a belief. Section 3 discusses requirements that the basic conditioning rule should satisfy, and proposes a new basic conditioning. Section 4 expands the conditioning rule and proposes another generalized rule by removing a requirement. Section 5 discusses interpretations of an equation for evidential reasoning, one of which is a conditioning rule with an uncertain condition.

2 Conditioning Rules in Evidence Theory

2.1 Dempster's Theory of Evidence [3]

Let Θ be a frame of discernment. A function $m: 2^\Theta \rightarrow [0,1]$ satisfying the following equations represents a *bba*;

$$m(\emptyset) = 0, \quad (1)$$

$$\sum_{A \in 2^\Theta} m(A) = 1, \quad (2)$$

where 2^Θ is the power set of Θ . $m(A)$ is a degree of belief that a variable whose domain is Θ has the value in A , but has no information about elements and subsets of A . Subsets A satisfying $m(A) > 0$ are called focal elements.

The belief could be represented in other forms, such as Belief and Plausibility measures defined below.

$$Bel(A) = \sum_{X \subseteq A} m(X). \quad (3)$$

$$Pl(A) = \sum_{X \cap A \neq \emptyset} m(X). \quad (4)$$

Both measures are functions from 2^Θ to $[0,1]$, and satisfy $Bel(\emptyset) = Pl(\emptyset) = 0$ and $Bel(\Theta) = Pl(\Theta) = 1$. Functions $m(\bullet)$, $Bel(\bullet)$ and $Pl(\bullet)$ are all transformable one another, and represent the same belief state.

When two distinct beliefs m_1 and m_2 are derived from two independent information sources respectively, they could be combined by the famous rule, Dempster's rule of combination;

$$m_D(A) = \frac{\sum_{A=X \cap Y} m_1(X) \cdot m_2(Y)}{1 - \sum_{X \cap Y = \emptyset} m_1(X) \cdot m_2(Y)}. \quad (5)$$

where $A \neq \emptyset$ and $m_D(\emptyset) = 0$. If the denominator is zero (i.e. m_1 and m_2 totally conflict with each other) the combination is not defined.

When $m_1(\bullet) = m(\bullet)$ and $m_2(B) = 1$, the combination gives the next equation called Dempster's rule of conditioning.

$$m_D(A|B) = \frac{\sum_{A=X \cap B} m(X)}{1 - \sum_{X \cap B = \emptyset} m(X)}, \quad (6)$$

where $A' \in$ and $m_D(A|B) = 0$. When $Pl(B) = 0$ or m and B totally conflict with each other, the denominator becomes zero and $m_D(A|B)$ is undefined.

2.2 Alternative Conditioning Rules

As discussed in Introduction, many alternative conditioning rules have been proposed so far. Major rules are shown below ($B \neq \emptyset$ is assumed).

(i) Focusing rule

The rule is proposed in [7, 8] separately and is called focusing rule in [10].

$$Bel_F(A|B) = \frac{Bel(A \cap B)}{Bel(A \cap B) + Pl(\overline{A \cap B})}. \quad (7a)$$

$$Pl_F(A|B) = \frac{Pl(A \cap B)}{Pl(A \cap B) + Bel(\overline{A \cap B})}. \quad (7b)$$

Representation with *bba* is studied in [13].

(ii) Zhang's rule

Dempster's rule reassigns all mass of $m(X)$ to $X \cap B$. Zhang asserts that it is "radical" and proposes the next rule [11].

$$m_Z(A|B) = k \sum_{X \cap B = A} \frac{|X \cap B|}{X} m(X). \quad (8)$$

where k is a constant for normalization.

(iii) Geometric rule [5]

This is a strong rule in the sense that all mass of focal elements that are not included in B is abandoned.

$$m_G(A|B) = \begin{cases} \sum_{X \in B} m(X), & \text{if } A \in B, \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

(iv) Planchet's Conditional rule [6]

This is a weak rule in the sense that only mass of focal elements that have no intersection with B is abandoned.

$$m_P(A|B) = \begin{cases} \sum_{X \in B} m(X), & \text{if } A \in B, \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

(v) Ito&Inagaki's rule

Ito and Inagaki proposed a rule assuming that the condition is just an assertion of an information source [12].

a) In the case of $Bel(\overline{B}) < 1$,

$$Bel_I(A|B) = \begin{cases} \{1 - k_B Bel(\overline{B})\} \{Bel(A \cup \overline{B}) - Bel(\overline{B})\}, & \text{if } A \neq \emptyset, \\ 1, & \text{if } A = \emptyset, \end{cases} \quad (11a)$$

where k_B is a constant that satisfies $0 \leq k_B \leq 1/(1 - Bel(\overline{B}))$.

b) In the case of $Bel(\overline{B}) = 1$,

$$Bel_I(A|B) = \begin{cases} 1, & \text{if } A = \emptyset, \\ 0, & \text{otherwise.} \end{cases} \quad (11b)$$

(vi) Rules with an uncertain condition

Ichihashi and Tanaka proposed three Jeffrey-like conditioning rules with an uncertain condition [14].

a) Plausible conditioning

$$m_{pl}(A|m_b) = \sum_{A=X \in Y} \frac{m(X) \cdot m_b(Y)}{1 - \sum_{X \in Y} m(X)}, \quad (12)$$

where $m_b(\bullet)$ is a *bba* on Θ and $m_{pl}(\emptyset|m_b) = 0$. It is assumed $\sum_{X \cap Y = \emptyset} m(X) < 1$ for any focal element Y of m_b .

b) Credible conditioning

$$m_{cr}(A|m_b) = \sum_{A \subset Y} \frac{m(A) \cdot m_b(Y)}{1 - \sum_{X \not\subset Y, X \neq Y} m(X)}, \quad (13)$$

where $\sum_{X \in Y, X' \subset Y} m(X) < 1$ for any focal element Y of m_b . $X \subset Y$ means that X is a proper subset of Y in the paper.

c) Possible conditioning

$$m_{po}(A|m_b) = \sum_{A \cap Y \neq \emptyset} \frac{m(A) \cdot m_b(Y)}{1 - \sum_{X \cap Y = \emptyset} m(X)}, \quad (14)$$

where $\sum_{X \cap Y = \emptyset} m(X) < 1$ for any focal element Y of m_b .

When m is Bayesian, and focal elements of m_b partition Θ , all of m_{pl} , m_{cr} and m_{po} equal to Jeffrey's rule. When $m_b(B) = 1$ for a specific B , m_{pl} , m_{cr} and m_{po} equal to

Dempster's rule, Geometric rule and Planchet's rule, respectively.

3 Basic Conditioning Rule

In the paper, the basic conditioning rule means the one with a sure condition without any uncertainty. It assumes the following:

The given condition is exactly true information given temporally after the prior belief. It puts a strict restriction on focal elements of the posterior belief.

In short, the condition B is an established fact, and the true value actually exists in B . Thus, we trust in B rather than the prior belief m , when they partially conflict with each other, i.e. $B \cap X = \emptyset$ for an X , where X is a focal element of m . Based on the above idea, the requirements that the posterior belief must satisfy are discussed below.

(1) Requirement-1: $m(A|B) = 0$ when $A \cap \bar{B} \neq \emptyset$, where \bar{B} is the complement of B . Assumed the condition B is an exact fact, the requirement might be self-evident. When the requirement holds, the following equations also hold.

$$m(A|B) > 0 \in A' B, \quad (15)$$

$$B' A \in Bel(A|B) = 1, \quad (16)$$

$$Bel(A|B) = Bel(A \cap B|B), \quad (17)$$

where $'$ is implication.

(2) Requirement-2: $m(A|B)$ must be defined except the case where $B = \emptyset$. This requirement comes from the assertion by Shafer [15] and Smets [9] that Evidence theory must be interpreted as a theory of belief, not as a theory of frequency. If bba , Bel and Pl represent degrees of a human belief, they should not be recognized as completely and precisely correct scales. Even if B is not plausible, i.e. $Pl(B) = 0$, a fact B may happen in the actual world. If the posterior belief conditioned by B is undefined in those cases, it might be a useless theory in real applications.

(3) Requirement 3: $m(\bullet|\Theta) = m(\bullet)$. The condition Θ represents the true value is in $'$. The condition is clearly unnecessary, unless we deal with an open world problem like Transferable Belief Model [16], which is an expanded evidence theory in the sense that $m(') > 0$ is allowed.

No conditioning rule in the previous section satisfies all the above requirements. All rules satisfy the 3rd one. Dempster's, Focusing, Zhang's and geometric rules satisfy the 1st one. However, nothing satisfies the 2nd requirement [1,2].

Conditioning rules could be derived from various combination rules in the same way as how Dempster's conditioning rule is derived as shown in section 2.1. However, such rules derived from combination rules proposed in [20-23] do not satisfy all the requirements, either. They do not satisfy the 1st requirement [1,2].

In this section, a new conditioning rule, which satisfies all the requirements, is proposed. In the proposed rule, the mass of focal elements of the prior belief are reassigned to another subset in the posterior belief as follows:

(a) When a focal element X of the prior belief and the condition B are consistent (i.e. $X \cap B \neq \emptyset$), the condition B imposes restriction on the focal element, the possible area where the true value may exist. That is, the mass $m(X)$ is re-assigned to $X' B$.

(b) When X and B partially conflict with each other (i.e. $X \cap B = \emptyset$), the grounds for the mass given to X are lost. Thus, the mass is temporarily given to $'$ representing total ignorance. Then, the restriction by B is imposed. As a result, the mass $m(X)$ is re-assigned to $\Theta \cap B = B$.

From the above (a) and (b), we get the next equations.

$$m_K(A|B) = \begin{cases} \in \in m(X), \text{ if } A' | , A \} B, \\ \in m(X) + \in m(X), \text{ if } A = B, \\ 0, \text{ otherwise,} \end{cases} \quad (18a)$$

where $B \neq \emptyset$. This equation could be written as

$$m_K(A|B) = \sum_{A=X \cap B} m(X) + \sum_{\substack{X \cap B = \emptyset \\ A=B}} m(X), \quad (18b)$$

where $A \neq \emptyset, B \neq \emptyset$ and $m_K(\emptyset|B) = 0$.

The idea in (b) that the mass is given to $'$ in case of partial conflict is the same as one used in Yager's combination rule [21]. However, the conditioning rule derived from Yager's rule is different from the above, and does not satisfy the 1st requirement [1,2].

It is proved easily that $m_K(A|B)$ is a bba and that it satisfies the three requirements. It is also clear that $m_K(A|B) = m_D(A|B)$ when m and B are consistent (= all focal elements of m and B are consistent).

Then, the next inequality could be proved [2].

$$Pl_K(A|B)' Pl_D(A|B)' Bel_D(A|B)' Bel_K(A|B), \quad (19)$$

where $Pl_K(A|B)$ and $Pl_D(A|B)$ ($Bel_K(A|B)$ and $Bel_D(A|B)$) are plausibility (belief) functions calculated using eq.s (18) and (6), respectively.

$m_K(A|B)$ is not a generalization of Bayesian conditioning. When m is Bayesian, $Pl(B) = P(B)$ holds. In the case of $Pl(B) = P(B) = 0$ where $B \neq \emptyset$, $P(A|B)$ is undefined, while $m_K(A|B)$ must be defined due to Requirement-2. In addition, $m_K(A|B)$ is not Bayesian in general, when $|B| > 1$.

Associativity does not hold in this conditioning, namely, $m_K(A|B|C) \neq m_K(A|C|B) \neq m_K(A|B \cap C)$ for $B \cap C \neq \emptyset$, where $m_K(A|B|C) = m_K^B(A|C)$ and $m_K^B(A) =$

$m_K(A|B)$. Non-associativity is caused by asymmetric nature of the conditioning between the two pieces of information, the prior belief and the condition.

Suppose the case where $X \cap B \cap C = \emptyset$, $X \cap B \neq \emptyset$, $X \cap C \neq \emptyset$ and $B \in C' \in$, where X is a focal element of the prior belief m . When calculating $m_K(\bullet|B)$, $m(X)$ is reassigned to $X \cap B$. Then, when calculating $m_K(\bullet|B|C)$, the mass $m(X)$ given to $X' B$ is reassigned again to C , because $X \cap B \cap C = \emptyset$. On the other hand, the same mass $m(X)$ is reassigned to B when calculating $m_K(\bullet|C|B)$, and to $B \cap C$ when calculating $m_K(\bullet|B \cap C)$.

4 Generalized Rule of Conditioning

4.1 Conditioning Rule with Unreliable Condition

The basic conditioning rule is generalized by moderating the 1st requirement. First, we introduce an index λ ($0 \leq \lambda \leq 1$) to represent reliability of the condition. When $\lambda < 1$, the condition is not completely correct, therefore requirement-1 is abandoned.

The mass reassignment in this case is an expansion of the one in the basic conditional rule, and is conducted as described below.

(a') When $X \in B' \in$, reassign mass of ' $m(X)$ to $X' B$, and $(1 - \lambda)m(X)$ to $X \cap \emptyset = X$.

(b') When $X \cap B = \emptyset$, reassign mass of $\lambda m(X)$ to $\in' B = B$, and $(1 - \lambda)m(X)$ to $X' \in = X$.

In (a'), the degree that B is trusted is λ . Thus, only $\lambda m(X)$ is reassigned to $X \cap B$. The rest $(1 - \lambda)m(X)$ is reassigned to $X \cap \emptyset = E$, with replacing B by \emptyset , because B cannot be trusted at the degree of $1 - \lambda$.

In the case of (b'), $\lambda m(X)$ is reassigned to $\emptyset \cap B = B$, because the mass $\lambda m(X)$ is temporarily given to total ignorance \emptyset due to $X \cap B = \emptyset$, then restriction B is imposed. The rest $(1 - \lambda)m(X)$ is reassigned to $X \cap \emptyset = X$, because B cannot be trusted at the degree of $1 - \lambda$.

From the above re-assignment, we get the next equation.

$$m_\lambda(A|B) = \begin{cases} \sum_{A=X \cap B} \lambda m(X) + (1 - \lambda)m(A), & \text{if } B \supset A \neq \emptyset, \\ \sum_{A=B \subseteq X} \lambda m(X) + (1 - \lambda)m(B) & \\ \quad + \sum_{X \cap B = \emptyset} \lambda m(X), & \text{if } A = B, \\ (1 - \lambda)m(A), & \text{if } A \not\subseteq B, A \neq B, A \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases} \quad (19)$$

where $B' \in$. It is proved easily that $m_\lambda(A|B)$ is a *bba*. In addition, it is also proved that $m_\lambda(A|B)$ satisfies both the requirements 2 and 3, and that the next equation holds [2].

$$m_\lambda(A|B) = \lambda m_K(A|B) + (1 - \lambda)m(A). \quad (20)$$

The equation could be understood as discounting of condition in conditioning, while discounting of information source in combination rule was proposed in [3].

4.2 Conditioning Rule with Uncertain Condition

The basic conditional rule is also generalized so that an uncertain condition given by *bba* m_b would be accepted in the same way as plausible, credible and possible conditioning rules in Section 2.

The mass reassignment in the case is similar to (a') and (b'). The difference is i) reliability λ of condition B is replaced by *bba* $m_b(Y)$ of a focal element Y of m_b , and that ii) re-assignment of unreliable mass corresponding to $(1 - \lambda)m(X)$ is unnecessary. The mass reassignment is done as follows:

(a'') When a focal element X of the prior belief m and a focal element Y of the uncertain condition m_b are consistent (i.e. $X \cap Y \neq \emptyset$ for any X and Y), the condition Y imposes restriction on the focal element X with uncertainty $m_b(Y)$. As a result, mass $m(X) \bullet m_b(Y)$ is re-assigned to $X \cap Y$.

(b'') When X and Y partially conflict with each other (i.e. $X \cap Y = \emptyset$), the grounds for the mass given to X are lost. Thus, the mass is temporarily given to ' \in ' representing total ignorance. Then, the restriction by Y is imposed with uncertainty $m_b(Y)$. As a result, the mass $m(X) \bullet m_b(Y)$ is re-assigned to $\in' Y = Y$.

From (a'') and (b''), we get the following conditioning rule, if $A \neq \emptyset$.

$$\begin{aligned} m_{KYU}(A|m_b) &= \sum_{A=X \cap Y} m(X) \bullet m_b(Y) + \sum_{\substack{X \cap Y = \emptyset \\ A=Y}} m(X) \bullet m_b(Y) \\ &= \sum_Y m_b(Y) \left(\sum_{A=X \cap Y} m(X) + \sum_{\substack{X \cap Y = \emptyset \\ A=Y}} m(X) \right) \\ &= \sum_{Y \subseteq \emptyset} m_K(A|Y) \bullet m_b(Y). \end{aligned} \quad (21)$$

It is proved easily that $m_{KYU}(A|m_b)$ is a *bba* and that it satisfies both the requirements 2 and 3.

The above equation shows that posterior belief with uncertain condition can be obtained by weighted sum of the basic conditioning rule $m_K(A|B)$ with different conditions B , where weight is given by the mass of the conditional set B of the belief representing the uncertain condition.

The following is obtained easily from eq. (20) and (21).

$$\begin{aligned} m_\in(A|B) &= \in n_K(A|B) + (1' \in)m(A) \\ &= \in n_K(A|B) + (1' \in)m_K(A|\in) \\ &= m_{KYU}(A|m_{(B, \in)}), \end{aligned} \quad (22)$$

where $m_{(B, \cdot)}(B) = \lambda$ and $m_{(B, \lambda)}(\Theta) = 1 - \lambda$.

In addition, it is proved easily that $m_{KYU}(A | m_b) = m_{pl}(A | m_b)$, when m and m_b are consistent each other, (i.e. $X \cap Y \neq \emptyset$ for all combination of X and Y , where X is a focal element of m and Y is of m_b). However, $m_{KYU}(A | m_b)$ is not an extension of Jeffrey's rule unlike $m_{pl}(A | m_b)$, because $m_K(A | B)$ is not Bayesian in general, even if the prior belief m is Bayesian.

5 Interpretations of Evidential Reasoning

The section shows that the conditioning represented by eq. (21) could be used as an interpretation of evidential reasoning. First, we review a few conventional discussions about evidential reasoning.

5.1 Representation of Rules

There are two ways to represent rules in evidential reasoning: joint form and conditional form [17]. The former represents a rule by a joint belief function on a product $\Theta_y \times \Theta_x$, where Θ_y, Θ_x are frames of discernment for variables y, x in antecedent and consequent of rules, respectively. The latter represents a rule by a conditional belief such as $m(A | B)$, where $B \subseteq \Theta_y, A \subseteq \Theta_x$. Though papers adopting joint form seem published more than those with conditional form, conditional form is more natural and easier for users to understand rules than joint form [17,18]. In addition, conditional form needs fewer values as knowledge than joint form; number of values conditional form needs is $2^{|x|} \cdot 2^{|y|} = 2^{|x|+|y|}$, while joint form needs $2^{|\Theta_x| \cdot |\Theta_y|}$, where $|\cdot|$ represents cardinality [17, 18].

5.2 Conventional Interpretations with Conditional Form

We have two kinds of knowledge. One is a set of rules described as

if $y \in B$ then $x \in A$ with belief $m^{(1)}(A | B)$,

and the other is a prior belief $m^{(2)}(B)$ about y . We know all values of $m^{(1)}(A | B)$ and $m^{(2)}(B)$ for all combinations of $B \in \Theta_y$ and $A \in \Theta_x$ except for $m^{(1)}(A | \emptyset)$.

There are a few theories for evidential reasoning with conditional form of rules. Liu, *et al.* proposes the next equation for evidential reasoning [19].

$$m(A) = \sum_{B \subseteq \Theta_y} m^{(1)}(A | B) \cdot m^{(2)}(B). \quad (23)$$

The theoretical support of the equation is Bayesian formula of probabilistic reasoning with cylindrical extension.

$$P(A_E) = \sum_{B_E \in 2^{\Theta_y} \times 2^{\Theta_x}} P^{(1)}(A_E | B_E) \cdot P^{(2)}(B_E), \quad (24)$$

where $P, P^{(1)}, P^{(2)}$ are probability measures on $2^{\Theta_y} \times 2^{\Theta_x}$, and $B_E = \{B\}' \cdot 2^{\Theta_x}, A_E = 2^{\Theta_y} \times \{A\}$. It is assumed that $P(A_E) = m(A), P^{(1)}(A_E | B_E) = m^{(1)}(A | B)$ and $P^{(2)}(B_E) = m^{(2)}(B)$.

Smets proposed evidential reasoning for Transferable Belief Model [16] based on conjunctive rule of combination in [18]. Conjunctive combination is equivalent to Dempster's rule combination without normalization and is defined by

$$m_{\cap}(A) = \sum_{A=X \cap Y} m'(X) \cdot m''(Y), \quad (25)$$

where m' and m'' are distinct beliefs defined on the same frame of discernment Θ .

The equation is proved to be equivalent to the next equation in [5].

$$m_{\cap}(A) = \sum_{Y \subseteq \Theta} m'_{\cap}(A | Y) \cdot m''(Y), \quad (26)$$

where $m_{\cap}(A | B)$ is conjunctive rule of conditioning, which is equal to $m_D(A | B)$ without normalization.

What should be noted is that reasoning with eq. (26) deviates from the standard evidential reasoning, if m' and m'' partially conflict with each other (i.e. $X \cap Y = \emptyset$ for a pair of X and Y , which are focal elements of $m'(\cdot)$ and $m''(\cdot)$, respectively), because $m(\epsilon) > 0$ is derived.

Fortunately, the deviation from the standard evidential reasoning never happens, if m', m'' are *bba*'s on $\Theta = \Theta_y \times \Theta_x$, and A, X and Y are cylindrical extensions of $A' \subseteq \Theta_x, X \in \Theta_x$ and $Y' \subseteq \Theta_y$, respectively (i.e. $A = \epsilon_y \cdot A', X = \Theta_y \times X', Y = Y' \times \Theta_x$), because $X \cap Y = Y' \times X' \neq \emptyset$ for any non-empty X' and Y' . Therefore, m_{\cap} obtained from eq. (26) is surely a *bba* on $\Theta_y \times \Theta_x$. Then, we can derive the following reasoning equation for the standard evidence theory.

$$m(A') = \sum_{Y' \subseteq \Theta_y} m^{(1)}(A' | Y') \cdot m^{(2)}(Y'), \quad (27)$$

where it is assumed that $m^{(1)}(A' | Y') = m'_{\cap}(A | Y), m^{(2)}(Y') = m''(Y), m(A') = m_{\cap}(A)$.

The two reasoning equations (23) and (27) are actually the same. That is, the evidential reasoning can be interpreted from two viewpoints: one is from probabilistic reasoning, and the other is from conjunctive combination. Note that conjunctive combination and conjunctive conditioning could be replaced by Dempster's combination and Dempster's conditioning, respectively in this case, because m' and m'' defined on $\Theta_y \times \Theta_x$ are consistent and normalization of Dempster's rule is unnecessary.

5.3 New Interpretation as Uncertain Conditioning

Let us recall the conditioning rule with an uncertain condition given by eq. (21). We re-write the equation here.

$$m_{KYU}(A|m_b) = \sum_{Y \subseteq \Theta} m_K(A|Y) \cdot m_b(Y).$$

The equation is apparently similar to eq. (26). However, it is different, because $m_{\cap}(\emptyset) > 0$ may occur in eq. (26), while $m_{KYU}(\emptyset) = 0$ always holds.

However, if $m_K(A|Y)$ and $m_b(Y)$ are *bba's* on $\epsilon = \epsilon_y, \epsilon_x$, and $A = \epsilon_y, A \in \epsilon_x$, $Y = Y' \times \Theta_x$, $A \in \epsilon_x$, $Y' \subseteq \Theta_y$ similar to the case of eq. (26), $m_K(A|Y) = m_D(A|Y) = m_{\cap}(A|Y)$ holds as discussed in Section 3. Thus, equations (21) and (26) are actually the same in this case.

As a result, the equation (23) or (27) of evidential reasoning could be interpreted from the viewpoint of conditioning given a belief including uncertainty (eq. (21)), as well as from the viewpoint of probabilistic reasoning (eq. (24)) and conjunctive combination (eq. (25)).

6 Conclusions

The paper discussed and proposed a new basic conditioning rule of Evidence theory and generalizes the rule to be applied to the case where condition is given by an uncertain belief. Then, it discusses a few interpretations of an equation used for evidential reasoning, one of which is interpretation by conditioning with an uncertain condition.

The paper first discussed three requirements that should be satisfied by a basic conditioning rule. Then, it proposed a new basic rule that satisfies all of them, however, none of major conventional rules does.

The basic conditioning rule was generalized by abandoning one of the three requirements in order to cope with the case where condition is given as an uncertain belief. The generalized rule is obtained by weighted sum of the basic posterior beliefs with different conditions, where weight is the mass of conditional set in the belief given as an uncertain condition.

Then the paper discussed theoretical backgrounds of an equation used for evidential reasoning, which could be supported by probabilistic reasoning or by conjunctive combination of evidence. The paper showed that it could also be supported by a viewpoint of a conditioning rule with an uncertain condition.

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