Scientific Theories and the Computational Theory of Perceptions – A Structuralis View Including Fuzzy Sets

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Abstract

Fuzzy Logic, Computing with words, and the Computational Theory of Preceptions build a stack of methodologies to help bridge the gap between systems and phenomena in reality and scientific theories. Bridging this gap has been a problem in all philosophical approaches, and the so-called structuralist view of theories in one that was established in the 20th century. In this paper we present a "fuzzy extension" of this view on scientific research. For this scientific work in progress we show that the "fuzzy structuralist approach" can be fruitful to interpret quantum mechanics and evolutionary biology.

Keywords: Fuzzy Sets, Computational Theory of Perceptions, Philosophy of Science.

1 Introduction

In this Introduction, to deal with basic definitions of basic terms, why not use WIKIPEDIA? – What is Science? – WIKIPEDIA's article "history of science" explains: "Science is a body of empirical and theoretical knowledge, produced by a global community of researchers, making use of specific techniques for the observation and explanation of real phenomena, this *techne* summed up under the banner of *scientific method*." [1] Here we notice that science and technology have a close connection and we can find many examples for interactions of science and technology in its history.

The mathematical theory of fuzzy sets is very young compared to others e.g. calculus, group theory, topology, and probability theory, and we know that fuzzy sets became well-known because of the enormous large number of application systems the field of applications in industry, economy and other areas since the first fuzzy steam engine that was conceptualized by Sedrak Assilian and Ebrahim Mamdani in the 1970s [2]. Nevertheless, the theory of fuzzy sets has also a great theoretical potential: this consists – in my point of view – a new interpretation of the scientific process using fuzzy sets. In the paper at hand we will have a look on both sciences – the body of empirical and theoretical knowledge – and scientists – members of the community of researchers who observe real phenomena and produce scientific theories – the protagonists in the process of knowledge production.

Since Galileo and Descartes science is extensively mathematical. Results of measurements, natural laws and scientific theories have got mathematical expressions, "the book of nature is written in the language of mathematics" and the scientific method incorporates its rigor.

One of the earliest areas of mathematics is computing. – What is computing? – WIKIPEDIA explains: "Originally, the word *computing* was synonymous with counting and calculating, and a science and technology that deal with the original sense of computing mathematical calculations." [3]

"Calculations" explains WIKIPEDIA as follows: "A *calculation* is a deliberate process for transforming one or more inputs into one or more results." [4]

In most cases inputs and outputs of mathematical computing are numbers: “A *number* is an abstract idea used in counting and measuring” as WIKIPEDIA explains. [5]

Computing with numbers was and is very successful in and also for the development of modern science but at the end of the 20th century Lotfi A. Zadeh proposed to compute with very different objects and in his first *AI Magazine*-article in the year 2001, *A New Direction in AI. Toward a Computational Theory of Perceptions* [6] he assumed “that progress has been, and continues to be, slow in those areas where a methodology is needed in which the objects of computation are perceptions—perceptions of time, distance, form, and other attributes of physical and mental objects.” ([6], p. 73)

Already in 1996 he had published the article *Fuzzy Logic = Computing with Words* [8] where he proposed a method for reasoning and computing with words based on the theory of fuzzy sets instead of exact computing with numbers. He argued that “the main contribution of fuzzy logic is a methodology for computing with words. No other methodology serves this purpose.” ([7], p. 103.)

“A *word* is a unit of language that carries meaning and consists of one or more morphemes which are linked more or less tightly together, and has a phonetical value.” That’s the explanation of WIKIPEDIA [8] and here is, what we reads about “meaning”: “In linguistics, *meaning* is the content carried by the words or signs exchanged by people when communicating through language. Restated, the communication of meaning is the purpose and function of language. A communicated meaning will (more or less accurately) replicate between individuals either a direct perception or some sentient derivation thereof. Meanings may take many forms, such as evoking a certain idea, or denoting a certain real-world entity.” [10]

About the concept of perceptions WIKIPEDIA says: “In psychology and the cognitive sciences, *perception* is the process of acquiring, interpreting, selecting, and organizing sensory information.” [11]

Sensory information flows in our brain. We do not understand totally how the brain can analyze these sensations that may come from our five traditional senses – sight, audition, touch, taste, and smell – or others e.g. pain, heat, cold, gravitation, etc. and we do not know how the brain transforms this information into perceptions – it seems to be an active and constructive transformation, i.e. the brain is building and interpreting the sensory inputs and perceptions are this transformation’s output. There are many examples that identical sensory inputs can result in different outputs. In these well-known examples we can perceive two perceptions that are contained in the painting and then we can swap between them.

“Humans have a remarkable capability to perform a wide variety of physical and mental tasks without any measurements and any computations. Everyday examples of such tasks are parking a car, driving in city traffic, playing golf, cooking a meal, and summarizing a story. In performing such tasks, for example, driving in city traffic, humans base whatever decisions have to be made on information that, for the most part, is perception, rather than measurement, based. The computational theory of perceptions (CTP), which is outlined in this article, is inspired by the remarkable human capability to operate on, and reason with, perception-based information.” This is a section in Zadeh’s article [6]. Here and in his 1999 article *From Computing with Numbers to Computing with Words – From Manipulation of Measurements to Manipulation of Perceptions* [11], he pointed out that 1) measurements can be represented or manipulated by numbers, 2) we are able to represent or manipulate perceptions with words.

As we saw already, fuzzy logic (FL) is a methodology for CW and now Zadeh stated “CW provides a methodology for what may be called a computational theory of perceptions (CTP).” ([11], p. 108.) Therefore we have a methodology hierarchy (fig. 1):

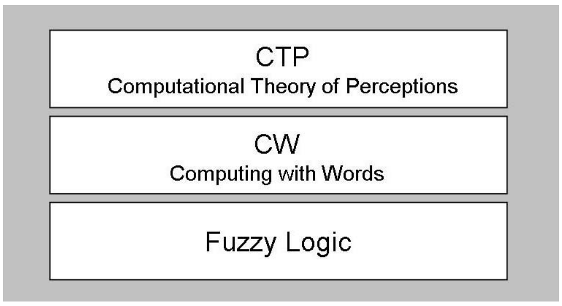

Figure 1: Zadeh’s “methodology hierarchy”.

Let’s abandon WIKIPEDIA. Let’s use Zadeh’s methodology hierarchy in philosophy of science.

2 Philosophy of Science

Philosophy of science is the branch in philosophy that reflects the basis of science, their assumptions and implications, their methods and results, their theories and experiments. There are various scientific disciplines and therefore it is understood that we can distinguish between the philosophy of astronomy and physics, chemistry, and other empirical sciences, and we can also be interested in philosophies of social sciences and the humanities. However, these differing philosophies of scientific disciplines arose in differing historical periods and the earliest philosophical reflections on modern science started with theories and experiments in mechanics in the 17th century. Two main views in philosophy of science arose in about the same period: The philosophical view of *rationalism* came to fundamental, logical and theoretical investigations using logics and mathematics to formulate axioms and laws whereas the view of *empiricism* was an is to have experiments to find or prove or refute natural laws. In both directions – from experimental results to theoretical laws or from theoretical laws to experimental proves or refutations – scientists have to bridge the gap that separates theory and practice in science.

From the empiricist point of view the source of our knowledge is sense experience. The English philosopher John Locke (1632-1704) used the analogy of the mind of a newborn as a "tabula rasa" that will be written by the sensual perceptions the baby has later. In Locke's opinion this perceptions provide information about the physical world. Locke's view is called "material empiricism" whereas the so called idealistic empiricism was hold by George Berkeley (1684-1753) and David Hume (1711-1776), an Irish and a Scottish philosopher: there exists no material world, only the perceptions are real.

This epistemological dispute is of great interest for historians of science but it is ongoing till this day and therefore it is of great interest for today's philosophers of science, too. Searching a bridge over the gap between rationalism and empiricism is a slow-burning stove in the history of philosophy of science.

Two trends in obtaining systematic rational reconstructions of empirical theories can be found in the philosophy of science in the latter half of the 20th century: the *Carnap approach* and the *Suppes approach* (named after the German-US-American philosopher Rudolf Carnap (1891-1970) and the US-American philosopher Patrick Suppes (born 1922)). In both approaches, the first step consists of an axiomatization that intends to determine the mathematical structure of the theory in question. The difference of these views can be found in the way this task is performed. Rudolf Carnap was firmly convinced that only formal languages can provide the suitable tools to achieve the desired precision. Consequently the Carnap approach says that a theory has to be axiomatized within a formal language.

On the other hand, the Suppes approach uses informal logic and informal set theory. Thus, in this approach, one is able to axiomatize physical theories in a precise way without recourse to formal languages. This approach traces back to the proposal of Suppes in the 1950s to include the axiomatization of empirical theories of science in the metamathematical program of the French group "Bourbaki" [13]. The Suppes approach is the basis of what is called today the *structuralist view* in philosophy of science.

In the 1970s, the US-American physicist and philosopher Joseph D. Sneed developed informal semantics meant to consider not only mathematical aspects, but also application subjects of scientific theories in this framework, based on this method. In his book *The Logical Structure of Mathematical Physics* [14], Sneed presents this view as stating that all empirical claims of physical theories have the form "x is an S" where "is an S" is a set-theoretical predicate (e.g., "x is a classical particle mechanics"). Every physical system that fulfills this predicate is called a model of the theory. To give concrete examples, the class M of a theory's models is characterized by empirical laws that consist of conditions governing the connection of the components of physical systems. Therefore, we have models of a scientific theory, and by removing their empirical laws, we get the class M_p of so-called potential models of the theory. Potential models of an empirical theory consist of theoretical terms, i.e., observables with values that can be measured in accordance with the theory. This connection between theory and empiricism is the basis of the philosophical "problem of theoretical terms".

If we remove the theoretical terms of a theory in its potential models, we get structures that are to be treated on a purely empirical layer; we call the class M_{pp} of these structures of a scientific theory its "partial potential models." Finally, every physical theory has a class I of intended systems (or applications) and, of course, different intended systems of a theory may partially overlap. This means that there is a class C of constraints that produces cross connections between the overlapping intended systems. In brief, this structuralist view of scientific theories regards the core K of a theory as a quadruple $K = \langle M_p, M_{pp}, M, C\rangle$. This core can be supplemented by the class I of intended applications of the theory $T = \langle K, I\rangle$. To make it clear that this concept reflects both sides of scientific theories, these classes of K and I are shown in figure 3. Thus we notice that M_{pp} and I are entities of an empirical layer whereas M_p and M_{pp} are structures in a theoretical layer oft the schema. In the next section we will extend this structuralist view of theories by fuzzy sets and fuzzy relations to represent perceptions as important components in the interpretation of scientific theories.

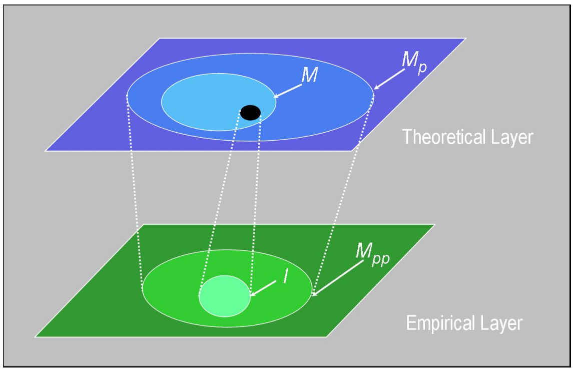

Figure 2: Empirical and theoretical structure layers to consider scientific theories.

3 Fuzzy Structuralism

Our modification of this approach pertains to the empirical layer in figure 2. Now, we will distinguish between real systems and phenomena at the one hand and perceptions of these entities at the other hand. Thus we introduce a lower layer – the real layer – and we rename our former empirical layer into "fuzzy layer" because the partial potential models and intended systems are not real systems because they have got a minimal structure by the scientist's observation. They are perception-based systems and therefore we have to distinguish them from real systems and phenomena that have no structure before someone imposes it tom them.

The layer of perceptions lies between the layer of real systems and phenomena and the layer of theoretical structures. According to Zadeh's Computational Theory of Perceptions (CTP) we represent perceptions in this intermediate layer as fuzzy sets. Whereas measurements are crisp perceptions are fuzzy, and because of the resolutions of our sense organs (e.g. aligning discrimination of the eye) perceptions are also granular, as Zadeh wrote in the *AI Magazine* in 2001: „perceptions, in general, are both fuzzy and granular or, for short *f*-granular [6]. Figure 3 shows Zadehs confrontation of crisp (C) and fuzzy (F) granulation of a linguistic variable.

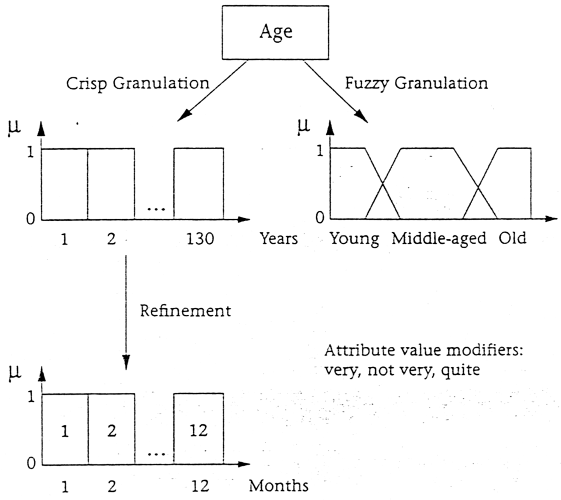

Figure 3: F- and C-Granulation of *Age*. [11].

When Zadeh established the CTP on the basis of Computing with Words (CW) that in turn bases on his theory of fuzzy sets and systems, he believed devoutly that these methodologies will get a certain place in science: "In coming years, computing with words and perceptions is likely to emerge as an important direction in science and technology" [7]. To take Zadeh at his word we establish his methodologies of fuzzy sets, computing with words and perceptions in our structuralist approach in philosophy of science. As aforementioned we introduce a fuzzy-layer of perceptions between the empirical layer of real systems and phenomena and the theoretical layer where we have the structures of models and potential models. Thus the relationship from real systems and theoretical structures has two parts: fuzzification and defuzzification.

- *Fuzzification: From Phenomena to Perceptions:*

Measurements are crisp and perceptions are fuzzy and granular. To represent perceptions we use fuzzy sets, e. g. A^F, B^F, C^F, ... It is also possible that a scientist perceives not only single but interlinked phenomena, e.g. two entities move similarly or inversly, or something is faster or slower than the other, or it is more bright or dark, or it smells analogue etc. Such relationships can be charakterized by fuzzy-relations f^F, g^F, h^F, ….

- *Defuzzification: From Perceptions to Models:*

"Measure what is measurable and make measurable what is not so." is a sentence imputed to Galileo. In modern scientific theories this is the way to come from perceptions to measurements resp. quantities to be measured. We interpret this transfer as a defuzzification from perceptions represented by fuzzy sets A^F, B^F, C^F, … and relations between perceptions represented by fuzzy relations f^F, g^F, h^F, … to ordinary (crisp) sets A^C, B^C, C^C, … and relations f^C, g^C, h^C, … These sets and relations are basic entities to construct (potential) models of a scientific theory in the theoretical layer.

- *Theoretization: From Phenomena to Perceptions*

The composition of fuzzification and defuzzification yields the operation of a relationship T that can be named as *theoretization*, because it transfers phenomena and systems in the real (or empirical) layer into structures in the theoretical layer (fig. 4).

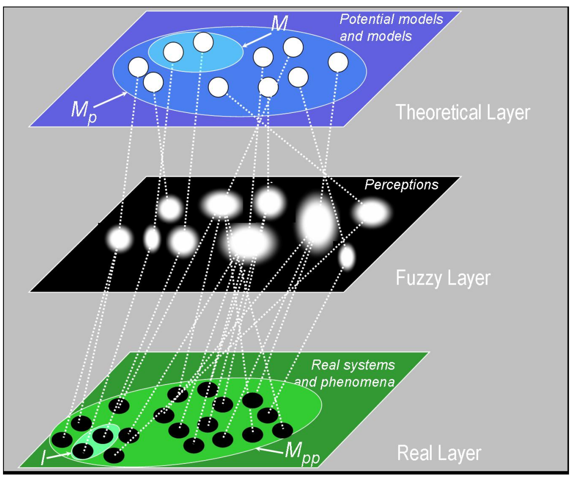

Figure 4: Empirical, fuzzy, and theoretical layer of crisp and fuzzy structures in scientific research.

In the structuralist view of theories the concept of theoretization is defined as an intertheoretic relation, i. e. a set theoretical relation between two theories T and T'. This theoretization relation exists if T' results from T by adding new theoretical terms und introduction of new laws that connect the former theoretical terms of theory T with this new theoretical terms in theory T'.

Sucessive adding of new theoretical terms establishes a hierarchy of theories and a comparative concept of theoriticity. In this manner the space-time theory arose from the euclidean geometry by adding the term „time" to the term „length", and from classical space-time-theory we get classical kinematics by adding the term „velocity". Classical kinematics turns to classical (Newtonian) mechanics by additional introduction of the terms „force" and „mass".

- The old theory T is covered with a new theoretical layer by the new theory T'.
- T-theoretical terms are not T'-theoretical but T'-non-theoretical terms and reciprocally they may not be some of the T-non-theoretical terms. The old theory must not change by the new theory in any way!
- In this hierarchy it holds that the higher in the hierarchy the more theoretical terms exist and the lower layers are characterized by the non-theoretical basic of the theory.

What happens in the lowest layer of this hierarchy? – Here exists a theory T with theoretical terms and relations but it is not a theoretization of another theory. This theory T covers phenomena and intended systems initially with theoretical terms. This is an initial theoretization because the T-theoretical terms are the only theoretical terms in this situation. They have been derived directly as measurements from observed phenomena. This derivation has got the name theoretization in the last section and it is a serial connection of fuzzification and defuzzification.

4 Fuzzy Structuralist Views on Scientific Disciplines

In this section we present two examples of new scientific theories in the 20th and in the 19th century in physics and in biology: quantum mechanics and evolutionism. Both theories have been established because previous theories did not provide the appropriate tool to explain new phenomena that have been observed in experiments or in nature.

4.1 The Case of Quantum Mechanics

The scientific revolution of quantum mechanics led to a new mathematical conceptual basis in physics concerning subatomic systems. Whereas in classical theories, the state of a system is represented by observables (e. g., position and momentum in the case of Newton's particle mechanics) that relate to human perception, states in quantum mechanics are not.

In classical physics we can observe these classical variables in experiments with classical systems. But quantum mechanical systems are not particles. The German physicist Werner Heisenberg (1901-1976), the Danish physicist Niels Bohr (1885-1962), and others introduced new theoretical systems (quanta) into the new quantum mechanics theory that differ significantly from those of classical physics. To determine the state of a quantum is much more difficult than to determine that of classical systems, as we cannot measure sharp values for all required variables simultaneously. This is the meaning of Heisenberg's uncertainty principle. We can experiment with a quantum in order to measure a position value, and we can also experiment with a quantum in order to measure a momentum value. However, we cannot conduct both experiments simultaneously and thus are not able to get both values for the same point in time respectively. Nonetheless, we can predict these values as outcomes of experiments at this point in time. Since predictions are targeted on future events, we cannot give them the logical values "true" or "false," but must assign them probabilities.

To determine the state of a quantum, we have to modify the classical concept: analogous to the state of classical systems, the state of a quantum mechanical system consists of all the probability distributions of all the system's properties that are formally possible in this physical theory. In classical physics the probability distributions for the observables position and momentum and all the other formally possible properties are marginal probability distributions of the unique probability distribution of the system's state. In quantum mechanics, however, there is no probability distribution that would be the meeting point of all probability distributions of all observables: in accordance with the uncertainty principle, not all of them are compatible.

In 1926, the German physicist Max Born (1882-1970) proposed an interpretation of the non-classical peculiarity of quantum mechanics, namely that the quantum mechanical wave function is a "probability-amplitude" [14]. This means that the absolute square of its value equals the probability of its having a certain position or a certain momentum if we measure the position or momentum respectively. Thus, the absolute square of the quantum mechanical state function equals the probability density function of its having a certain position or a certain momentum in the position or momentum representation of the wave function respectively. But there is no joint probability for the event in which both variables have a certain value, as there is no classical probability space (no Boolean algebra) that comprises these events.

In 1932, the Hungarian mathematician John von Neumann published the Mathematical Foundations of Quantum Mechanics [15], in which he defined the quantum mechanical probability amplitude as a one-dimensional subspace of an abstract Hilbert space, which is defined as the state function of a quantum. There are varying representations of a quantum's state, e.g., the "position picture" and the "momentum picture." These representations are complementary, which means that a subatomic system cannot be presented in both classical pictures at the same time.

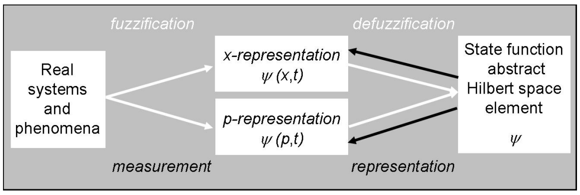

Figure 5: Quantum mechanical *theoretization* from real systems/ phenomena to a Hilbert space vector.

Figure 5 illustrates this case example of the fuzzy structuralist view in philosophy of science: A physician experiments with real subatomic systems or phenomena. He measures or predicts probability distributions of position- or momentum values. With these observable values he represents the quantum in the position or momentum picture with probability amplitudes $\psi(x,t)$ or $\psi(p,t)$ – we interpreted this process as *fuzzification* because quanta don't have position and momentum. The abstraction process from these classical pictures to get an abstract Hilbert space element ψ that representing the quantum theoretical *state* of the quantum, we interpret as *defuzzification*. The composition of these both processes is the quantum mechanical theoretization.

4.2 The Case of Evolutionary Biology

In the last third of the 20th century and in our first years of the 21st century biology is the leading scientific discipline. One of the most important researchers in this time and area was the evolutionary biologists and one of the architects of the synthetic theory of evolution, the German-Us-American Ernst Mayr (1904-2005) who was also an important historian and philosopher of biology. *What is biology?* was the title of his last book, and therein he distinguished between two rather different fields, mechanistic or functional biology and historical biology. "Functional biology deals with the physiology of all activities of living organisms, particularly with all cellular processes, including those of the genome. These functional processes ultimately can be explained purely mechanistically by chemistry and physics.

The other branch of biology is historical biology. A knowledge of history is not needed for the explanation of a purely functional process. However, it is indispensable for the explanation of all aspects of the living world that involve the dimension of historical time – in other words, as we now know, all aspects dealing with evolution. This field is evolutionary biology." ([16], p. 24)

Philosophy of science in the 20th century was based on exact sciences and especially on the new theories in physics (theory of relativity and quantum mechanics). Most philosophers of science in that century did have their background in physics but not in biology. There were only some scientists – and Mayr was one of them since the 1970s – who argued that we need a philosophy of modern biology that is different from philosophy of exact sciences and particularly Mayr emphasized this difference. E. g. in physics it is important to discover new facts or natural laws but in biology it is more important to develop new concepts and to round out concepts in being.

One reason for the missing of a philosophy of biology is that the basic principles of physics are simply not applicable to animate systems and another reason is that biology potentially bases on self-contained principles that are inapplicable to inanimate systems. The discovery of this difference in the basics of physics at the one hand and biology at the other hand was a fundamental intellectual revolution that began with Charles Darwin's *Origin of Species* in 1859. Thereupon modern biology emerged as an autonomous scientific discipline and a restructuring of the philosophy of science was prepared. To establish a philosophy of modern biology it was necessary 1) to eliminate and replace the principles of exact sciences by principles pertinent to biology and 2) to add new basic biological principles. Mayr "found that biology, even enough it is a genuine science, has certain characteristics not found in other sciences." ([16], p. 4) Among others he referred to the following:

… *unsharp separation of classes of phenomena*:
"The seemingly endless variety of phenomena, it was said, actually consisted of a limited number of natural kinds (essences or types), each forming a class. The members of each class were thought to be identical, constant, and sharply separated from the members of any other essence. Therefore variation was nonessential and accidental. […] Typological thinking, therefore, is unable to accommodate variation ad has given rise to a misleading conception of human race. Caucasians, Africans, Asians, and Inuits are types for a typologist that differ conspicuously from other human ethnic groups and are sharply separated from them. This mode of thinking leads to racism. Darwin completely rejected typological thinking and instead used an entirely different concept, now called *population thinking*" […]. ([16], p. 27)

… *variation or chance events*:
"One of the consequences of the acceptance of deterministic Newtonian laws was that it left no room for variation or chance events. […] The refutation of strict determinism and of the possibility of absolute prediction freed the way for the study of variation and of chance phenomena, so important in biology." ([16], p. 27)

… missing strict regularities:
"The philosophers of logical positivism, and indeed all philosophers with a background in physics and mathematics, base their theories on natural laws and such theories are therefore usually strictly deterministic. In biology there are also regularities, but various authors […] severely questions whether these are the same as the natural laws of the physical sciences. There is no consensus yet in the answer to this controversy. Laws certainly play a rather small role in theory construction in biology." ([16], p. 28)

In the times before the last decades in the 20th century these characteristics have got probabilistic formulations, but we think that this is the wrong way to get fruitful solutions in the philosophy of biology. Physics concern the inanimate world with many indistinguishable objects and therefore it can be meaningful to argue with probabilities but: "In a biopopulation, by contrast, every individual is unique, while the statistical mean value of a population is an abstraction." ([16], p. 29) Because biological systems are high complex Mayr concluded: "Population thinking and populations are not laws but concepts. It is one of the most fundamental differences between biology and the so called exact sciences that in biology theories usually are based on concepts while in the physical sciences they are based on natural laws. Examples of concepts that became important bases of theories in various branches of biology are territory, female choice, sexual selection, resource, and geographic isolation. And even though, through appropriate rewording, some of these concepts can be phrased as laws, they are something entirely different from the Newtonian natural laws. ([16], p. 30)

Obviously we can consider these concepts with our fuzzy glasses and may be this is a good way to get interesting results in philosophy of biology. This means that the difference between the exact sciences and the life sciences is manifest in the missing exact structures of biological theories in the theoretical layer of sciences. Until today there is no part of defuzzification of a biological theoretization process and therefore no exact theory of evolution (fig. 6).

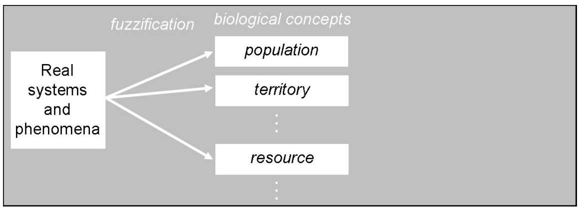

Figure 6: Fuzzification to evolutionary concepts.

5 Conclusion

The Computational Theory of Perceptions is an appropriate methodology to represent what happens in scientific research to bridge the gap between empirical observations and the abstract construction of theoretical structures. In the structuralist view of theories there is an empirical layer of real phenomena and systems that have some minimal structure and a theoretical layer of potential models and models that are full structured entities. But there is no representation of the scientist's perceptions. The modified view on the structuralist approach that is presented in this paper is scientific work in progress. This fuzzy structuralist view may open up a fruitful way to understand scientific research.

Acknowledgement

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