A fixed point theorem in probabilistic metric spaces with a convex structure

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Abstract

The inequality $F_{fx,fy}(qs) \geq F_{x,y}(s)$ $(s \geq 0)$, where $q \in (0, 1)$, is generalized for multivalued mappings in many directions. Using Hausdorff distance S.B. Nadler in [7] introduced a generalization of Banach contraction principle in metric spaces. In [3] the definition of probabilistic Nadler q-contraction is given. Using some results given in [12] a fixed point theorem on spaces with a convex structure is obtained. Some fixed point theorems in such spaces are proved in [1, 2].

Keywords: multivalued mappings, coincidence point, probabilistic metric space, Menger space, triangular norm, Menger space with a convex structure.

1 Introduction

K. Menger introduced in 1942. the notion of a probabilistic metric space as a natural generalization of the notion of a metric space (M, d) in which the distance $d(p, q)$ $(p, q \in M)$ between p and q is replaced by a distribution function $F_{p,q} \in \Delta^+$. $F_{p,q}(x)$ can be interpreted as the probability that the distance between p and q is less than x. Since then the theory of probabilistic metric spaces has been developed in many directions([9]).

Since Sehgal and Bharucha-Reid proved ([10]) a fixed point theorem in Menger spaces $(S, \mathcal{F}, T_M)$ many fixed point theorem for singlevalued and multivalued mappings on Menger spaces $(S, \mathcal{F}, T)$ are obtained.

Further development of the fixed point theory in a more general Menger space $(S, \mathcal{F}, T)$ was connected with investigations of the structure of the t-norm T. This problem was very soon in some sence completely solved. V. Radu proved that if $f : S \to S$ is a probabilistic q-contraction, where $(S, \mathcal{F}, T)$ is a complete Menger space and T is a continuous t-norm, f has a fixed point if and only if T is of the H-type [8].

S.B. Nadler proved in [7] the generalization of the Banach contraction principle for multivalied mappings $f : X \to CB(X)$, where (X, d) is a metric space, by introducing the condition

$$D(fx, fy) \leq qd(x, y),$$

where D is the Hausdorff metric and $q \in (0, 1)$.

Probabilistic version of Nadler's q-contraction is given in [3].

Definition 1 *Let $(S, \mathcal{F})$ be a probabilistic metric space, M a nonempty subset of S and $f : M \to 2^S$, where 2^S is the family of all nonempty subsets of S. The mapping f is said to be a probabilistic Nadler's q-contraction, where $q \in (0, 1)$ if the following condition is satisfied:*

For every $u, v \in M$, every $x \in fu$ and every $\delta > 0$ there exists $y \in fv$ such that for every $\varepsilon > 0$

$$F_{x,y}(\varepsilon) \geq F_{u,v}(\frac{\varepsilon - \delta}{q}).$$

If f is a singlevalued mapping, then the notion of a probabilistic Nadler's q-contraction coincides with the notion of a probabilistic q-contraction by Sehgal and Bharucha-Reid [10], since the function $F_{u,v}(\cdot)$ is left-continuous.

2 Preliminaries

Let Δ^+ be the set of all distribution functions F such that $F(0) = 0$ (F is a nondecreasing, left continuous mapping from $\mathbb{R}$ into $[0,1]$ such that $\sup_{x\in\mathbb{R}} F(x) = 1$).

The ordered pair $(S,\mathcal{F})$ is said to be **a probabilistic metric space** if S is a nonempty set and $\mathcal{F} : S\times S \to \Delta^+$ ($\mathcal{F}(p,q)$ written by $F_{p,q}$ for every $(p,q)\in S\times S$) satisfies the following conditions:

1. $F_{u,v}(x) = 1$ for every $x > 0 \Rightarrow u = v$ $(u,v\in S)$.
2. $F_{u,v} = F_{v,u}$ for every $u,v\in S$.
3. $F_{u,v}(x) = 1$ and $F_{v,w}(y) = 1 \Rightarrow F_{u,w}(x+y) = 1$ for $u,v,w\in S$ and $x,y\in\mathbb{R}^+$.

A **Menger space** (see [9]) is a ordered triple $(S,\mathcal{F},T)$, where $(S,\mathcal{F})$ is a probabilistic metric space, T is a triangular norm (abbreviated t -norm) and the following inequality holds

$$F_{u,v}(x+y) \geq T(F_{u,w}(x), F_{w,v}(y))$$

for every $u,v,w\in S$ and every $x>0, y>0$.

Recall that a mapping $T : [0,1]\times[0,1]\to[0,1]$ is called a **triangular norm** (a t-norm) if the following conditions are satisfied:
$T(a,1) = a$ for every $a\in[0,1]$; $T(a,b) = T(b,a)$ for every $a,b\in[0,1]$;

$$a\geq b,\ c\geq d \Rightarrow \ \ T(a,c)\geq T(b,d)\ \ (a,b,c,d\in[0,1]);$$

$$T(a,T(b,c)) = T(T(a,b),c)\ \ (a,b,c\in[0,1]).$$

Example 1 The following are the four basic t-norms :

(i) The **minimum** t-norm, T_M, is defined by

$$T_M(x,y) = \min(x,y),$$

(ii) The **product** t-norm, T_P, is defined by

$$T_P(x,y) = x\cdot y,$$

(iii) The **Lukasiewicz** t-norm T_L is defined by

$$T_L(x,y) = \max(x+y-1, 0),$$

(iv) The weakest t-norm, the **drastic product** T_D, is defined by

$$T_D(x,y) = \begin{cases} \min(x,y) & \text{if } \max(x,y)=1, \\ 0 & \text{otherwise.} \end{cases}$$

As regards the pointwise ordering, we have the inequalities

$$T_D < T_L < T_P < T_M.$$

The $(\epsilon,\lambda)-$ topology in S is introduced by the family of neighbourhoods $\mathcal{U} = \{U_v(\epsilon,\lambda)\}_{(v,\epsilon,\lambda)\in S\times\mathbb{R}_+\times(0,1)}$, where

$$U_v(\epsilon,\lambda) = \{u;\ F_{u,v}(\epsilon) > 1-\lambda\}.$$

If a t-norm T is such that $\sup_{x<1} T(x,x) = 1$, then S is in the (ϵ,λ) topology a metrizable topological space.

Each t-norm T can be extended (see [6]) (by associativity) in a unique way to an n-ary operation taking for $(x_1,\ldots,x_n)\in[0,1]^n$ the values

$$\mathbf{T}^0_{i=1}x_i = 1,\ \mathbf{T}^n_{i=1}x_i = T(\mathbf{T}^{n-1}_{i=1}x_i, x_n).$$

We can extend T to a countable infinitary operation taking for any sequence $(x_n)_{n\in\mathbb{N}}$ from $[0,1]$ the values

$$\mathbf{T}^\infty_{i=1}x_i = \lim_{n\to\infty}\mathbf{T}^n_{i=1}x_i.$$

Limit of right side exists since the sequence $(\mathbf{T}^n_{i=1}x_i)_{n\in\mathbb{N}}$ is nonincreasing and bounded from below.

In the fixed point theory it is of interest to investigate the classes of t-norms T and sequences $(x_n)_{n\in\mathbb{N}}$ from the interval [0, 1] such that $\lim_{n\to\infty} x_n = 1$, and

$$\lim_{n\to\infty}\mathbf{T}^\infty_{i=n}x_i = \mathbf{T}^\infty_{i=1}x_{n+i} = 1.$$

In [5] it is proved that for the Dombi, Aczél-Alsina and Sugeno-Weber family of t-norms exists the sequence $(x_n)_{n\in\mathbb{N}}$ from (0, 1] such that the last condition is satisfied.

Definition 2 *: Let $(S,\mathcal{F},T)$ be a Menger space, $\emptyset\neq M\subset S$, $f : M\to M$ i $A : M\to 2^M$ (the family of nonempty subsets of M). The mapping A is f-strongly demicompact if for every sequence $\{x_n\}_{n\in\mathbb{N}}$ from M, such that $\lim_{n\to\infty} F_{fx_n,y_n}(\epsilon) = 1$, for some sequence $\{y_n\}_{n\in\mathbb{N}}$, $y_n\in Ax_n$, $n\in\mathbb{N}$ and every $\epsilon>0$, there exists a convergent subsequence $\{fx_{n_k}\}_{k\in\mathbb{N}}$.*

A mapping $A : M\to 2^M$ is weakly commuting with $f : M\to M$ if for every $x\in M$

$$f(Ax)\subset A(fx).$$

In [12] the following theorem is proved.

Theorem 1 *: Let $(S,\mathcal{F},T)$ be a complete Menger space, $\sup_{a<1}T(a,a) = 1$, A is a nonempty and closed subset of S, $f : A\to A$ a continuous mapping, $L, L_1 : A\to 2_c^{f(A)}$ closed, multivalued mappings such* *that the following condition is satisfied:*

For every $u, v \in A$, $x \in Lu$ *and* $\delta > 0$, *there exists* $y \in L_1 v$ *such that*

$$F_{x,y}(\varepsilon) \geq F_{fu,fv}(\frac{\varepsilon - \delta}{q}), \text{ for all } \varepsilon > 0, \text{ where } q \in (0, 1).$$

If L *and* L_1 *are weakly commuting with* f *and (i) or (ii) are satisfied, then there exists* $x \in A$ *such that* $fx \in Lx \cap L_1 x$ *where*

(i) L *or* L_1 *is* f*-strongly demicompact.*

(ii) There exists $x_0, x_1 \in A$, $fx_1 \in Lx_0$ *and* $\mu \in (0, 1)$ *such that t-norm* T *satisfies the following condition*

$$\lim_{n\to\infty} \mathbf{T}_{i=n}^{\infty} F_{fx_0, fx_1}(\frac{1}{\mu^i}) = 1.$$

3 Main results

W. Takahashi introduced in [11] the notion of a metric space with a convex structure. This class of metric spaces includes normed linear spaces and metric spaces of the hiperbolic type.

Let us recall that a metric space (S, d) has a convex structure in the sense of Takahashi, if there exists a mapping $W : S \times S \times [0, 1] \to S$ such that for every $(u, x, y, \delta) \in S \times S \times S \times [0, 1]$

$$d(u, W(x, y, \delta)) \leq \delta d(u, x) + (1 - \delta) d(u, y).$$

This definition can be generalized in Menger spaces $(S, \mathcal{F}, T)$. The notion of convex structure in probabilistic metric spaces, as well as Definition 3 and Definition 4 belong to O. Hadžić [4].

A mapping $W : S \times S \times [0, 1] \to S$ is said to be a convex structure on S if for every $(x, y) \in S \times S$

$W(x, y, 0) = y$, $W(x, y, 1) = x$, and for every $\delta \in (0, 1)$, $u \in S$, $\varepsilon > 0$

$$F_{u,W(x,y,\delta)}(2\varepsilon) \geq T(F_{u,x}(\frac{\varepsilon}{\delta}), F_{u,y}(\frac{\varepsilon}{1-\delta})).$$

It is easy to see that every metric space (S, d) with a convex structure W can be considered as a Menger space $(S, \mathcal{F}, T_M)$ with the same function W.

It is well-known that every probabilistic normed space is a probabilistic metric space with a convex structure $W(x, y, \delta) = \delta x + (1 - \delta) y$ $(x, y \in S)$ since for every $\varepsilon > 0$ and $\delta \in [0, 1]$ we have

$$\begin{aligned} F_{u,W(x,y,\delta)} &= F_{u-\delta x-(1-\delta y)}(2\varepsilon) \\ &= F_{\delta u+(1-\delta u)-\delta x-(1-\delta)y}(2\varepsilon) \\ &= F_{\delta(u-x)+(1-\delta)(u-y)}(2\varepsilon) \\ &\geq T(F_{\delta(u-x)(\varepsilon}, F_{(1-\delta)(u-y)}(\varepsilon)) \\ &= T(F_{u-x}(\frac{\varepsilon}{\delta}), F_{u-y}(\frac{\varepsilon}{1-\delta}))). \end{aligned}$$

In this paper we shall suppose that a convex structure W on a Menger space $(S, \mathcal{F}, T)$ satisfies the condition

$$F_{W(x,z,\delta),W(y,z,\delta)}(\varepsilon\delta) \geq F_{x,y}(\varepsilon) \tag{1}$$

for every $(x, y, z) \in S \times S \times S$, every $\varepsilon > 0$ and $\delta \in (0, 1)$.

For the next theorem we shall need the following definitions.

Definition 3 *Let* $(S, \mathcal{F}, T)$ *be Menger space with a convex structure* W. *A nonempty subset* M *of* S *is called* W*-starshaped if there exists* $x_0 \in M$ *such that the set* $\{W(x, x_0, \lambda) : x \in M, \lambda \in (0, 1)\} \subset M$. *The point* x_0 *is said to be the star-centre of* M.

Clearly, every convex set is a starshaped set and the inverse is not true.

Definition 4 *Let* $(S, \mathcal{F}, T)$ *be a Menger space with a convex structure* W *and* M *a nonempty subset of* S. *A mapping* $f : M \to S$ *is said to be* (W, x_0)*-convex if for each* $(x, \lambda) \in M \times [0, 1]$

$$W(fx, x_0, \lambda) = f(W(x, x_0, \lambda)).$$

Lemma 1 *Let* $(S, \mathcal{F}, T)$ *be a Menger space,* M *a nonempty subset of* S *which is* W*-starshaped with the star-centre in* x_0, $f : M \to S$ *is the mapping which is* (W, x_0)*-convex. Then the* $f(M)$ *is the* (W, x_0)*-convex.*

Proof: Let $u \in f(M)$. Then there exists $x \in M$ so that $u = f(x)$. Let us prove that for every $\lambda \in [0, 1]$,

$$z = W(u, x_0, \lambda) \in f(M).$$

As $u = f(x)$ it follows

$$z = W(f(x), x_0, \lambda).$$

Since f is (W, x_0)-convex it follows

$$z = f(W(x, x_0, \lambda)).$$

From the condition we have that the set M is W-starshaped, and it follows that $W(x, x_0, \lambda) \subset M$, i.e. $z \in f(M)$.

Lemma 2 *Let $(S, \mathcal{F}, T)$ be a Menger space, T a t-norm such that $\sup_{x<1} T(x, x) = 1$, M a nonempty subset of S, $f : M \to S$ a continuous mapping, $L : M \to C(S)$ (where $C(S)$ is the family of all nonempty and closed subset of S) and the following inequality is satisfied:*

For every $u, v \in M$, every $x \in Lu$ and every $\delta > 0$, there exists $y \in Lv$ such that

$$F_{x,y}(\varepsilon) \geq F_{fu,fv}(\frac{\varepsilon - \delta}{q}) \qquad (2)$$

for every $\varepsilon > 0$. Then the mapping L is closed.

Proof:

Let $(x_n)_{n\in\mathbb{N}}$ be a sequence from M such that $\lim_{n\to\infty} x_n = x$ and let $y_n \in Lx_n$, for every $n \in \mathbb{N}$ such that $\lim_{n\to\infty} y_n = y$. Let us prove that $y \in Lx$.
Since Lx is closed we shall prove that $y \in \overline{L}x$. Let $\varepsilon > 0$ and $\lambda \in (0, 1)$ be given. It remains to be proved that there exists $b \in Lx$ such that $b \in U_y(\varepsilon, \lambda)$, i.e. $F_{b,y}(\varepsilon) > 1 - \lambda$. Using condition 2, where $u = x_n$, $v = x$ and $\delta = \frac{3\varepsilon}{4}$ from $y_n \in Lx_n$ it follows that there exists $b_n \in Lx$ such that

$$F_{y_n,b_n}(\varepsilon) \geq F_{fx_n,fx}(\frac{\varepsilon}{4q}).$$

So,

$$\begin{aligned} F_{y,b_n}(\varepsilon) &\geq T(F_{y,y_n}(\frac{\varepsilon}{2}), F_{y_n,b_n}(\frac{\varepsilon}{2})) \\ &\geq T(F_{y,y_n}(\frac{\varepsilon}{2}), F_{fx_n,fx}(\frac{\varepsilon}{4q})). \end{aligned}$$

From the continuity of the mapping f it follows $\lim_{n\to\infty} fx_n = fx$, i.e.

$\lim_{n\to\infty} F_{fx_n,fx}(\varepsilon) = 1$. We also have $\lim_{n\to\infty} F_{y,y_n}(\varepsilon) = 1$, for every $\varepsilon > 0$. From the condition $\sup_{x<1} T(x, x) = 1$, it follows that for a given $\lambda \in (0, 1)$ there exists $\eta \in (0, 1)$ such that $T(\eta, \eta) > 1 - \lambda$, so there exists $n_0(\eta)$ such that for every $n \geq n_0(\eta)$

$$F_{y,y_n}(\tfrac{\varepsilon}{2}) > \eta \text{ i } F_{fx_n,fx}(\tfrac{\varepsilon}{4q}) > \eta.$$

Hence,

$$\begin{aligned} F_{y,b_n}(\varepsilon) &\geq T(\eta, \eta) \\ &> 1 - \lambda, \end{aligned}$$

i.e. $b_{n_0} \in U_y(\varepsilon, \lambda) \cap Lx$.

Theorem 2 *Let $(S, \mathcal{F}, T)$ be a complete Menger space with a convex structure W and a continuous t-norm T and M is nonempty, closed and W-starshaped subset of S with the star-centre x_0. Let $f : M \to M$ be a continuous , (W, x_0)-convex mapping and $L : M \to 2_c^{f(M)}$ such that $\overline{L(M)}$ is compact set and the following condition is satisfied:*

For every $u, v \in M$, $x \in Lu$ and $\delta > 0$ there exists $y \in Lv$ such that

$$F_{x,y}(\varepsilon) \geq F_{fu,fv}(\varepsilon - \delta), \text{ for every } \varepsilon > 0. \qquad (3)$$

If L is weakly comuting with f, then there exists $x \in M$ such that $fx \in Lx$.

Proof: Let $(k_n)_{n\in\mathbb{N}}$ be a sequence from the interval $(0, 1)$ such that $\lim_{n\to\infty} k_n = 1$. For every $n \in \mathbb{N}$, and every $x \in M$ let $L_n x = W(Lx, x_0, k_n)$ i.e.

$$L_n x = \bigcup_{z\in Lx} W(z, x_0, k_n), \quad n \in \mathbb{N}, \quad x \in M.$$

From the Lemma 1 it follows that $f(M)$ is (W, x_0)-convex. We shall prove that $L_n x \subset f(M)$, i.e. that for every $z \in L_n x$ it follows $z \in f(M)$. Since $z \in L_n x = W(Lx, x_0, k_n)$, there exists $u \in Lx$ such that $z = W(u, x_0, k_n)$. Since $Lx \subset f(M)$ it follows that $u \in f(M)$, so $W(u, x_0, k_n) \subset f(M)$ i.e. $z \in f(M)$. It means that $L_n x \subset f(M)$.
From the condition (1) it follows that the mapping W

is continuous in respect to the first variable, and since Lx is closed it follows that Lx is compact (as a subset of $L(M)$) such that $W(Lx, x_0, k_n)$ is closed for every $n \in \mathbb{N}$. It follows that $L_n x$ is closed for every $n \in \mathbb{N}$ and $x \in M$.
Next, we shall prove that for every $u, v \in M$ and every $x \in L_n u$ and $\delta > 0$ there exists $y \in L_n v$ such that

$$F_{x,y}(\varepsilon) \geq F_{fu,fv}(\frac{\varepsilon - \delta}{k_n}), \qquad \varepsilon > 0.$$

Let $u, v \in M$, $\delta > 0$ and $x \in L_n u = W(Lu, x_0, k_n)$. Then, there exists $z \in Lu$ such that $x = W(z, x_0, k_n)$ and from (3) there exists $y' \in Lv$ such that

$$F_{z,y'}(\varepsilon) \geq F_{fu,fv}(\varepsilon - \frac{\delta}{k_n}).$$

Let $y = W(y', x_0, k_n) \in L_n v$. Then

$$\begin{aligned} F_{x,y}(\varepsilon) &= F_{W(z,x_0,k_n),\,W(y',x_0,k_n)}(\frac{\varepsilon}{k_n}k_n) \\ &\geq F_{z,y'}(\frac{\epsilon}{k_n}) \\ &\geq F_{fu,fv}(\frac{\varepsilon - \delta}{k_n}). \end{aligned}$$

Let us prove that L_m is a f-strongly demicompact. Suppose that $(x_n)_{n\in\mathbb{N}}$ is a sequence in M such that for every $\varepsilon > 0$

$$\lim_{n\to\infty} F_{fx_n,y_n}(\varepsilon) = 1$$

for some sequence $(y_n)_{n\in\mathbb{N}}$ $y_n \in L_m x_n$.
Since $L_n(M) = W(L(M), x_0, k_n)$, $n \in \mathbb{N}$ is relatively compact, from $y_n \in L_m x_n$ it follows that $(y_n)_{n\in\mathbb{N}}$ has a convergent subsequence $(y_{n_k})_{k\in\mathbb{N}}$ and let $\lim\limits_{k\to\infty} y_{n_k} = z$. Then we have

$$F_{fx_{n_k},z}(\varepsilon) \geq T(F_{fx_{n_k},y_{n_k}}(\frac{\varepsilon}{2}), F_{y_{n_k},z}(\frac{\varepsilon}{2}))$$

i.e. $\lim\limits_{k\to\infty} fx_{n_k} = z$, which means that the mapping L_m is f-strongly demicompact.
We are going to prove that the mapping L_n is weakly comuting with f, i.e. that for every $x \in M$

$$f(L_n x) \subset L_n(fx) = W(L(fx), x_0, k_n).$$

Let $u \in L_n x = W(Lx, x_0, k_n)$. Then there exists $z \in Lx$ such that $u = W(z, x_0, k_n)$, so

$$\begin{aligned} f(u) &= f(W(z, x_0, k_n)) \\ &= W(fz, x_0, k_n) \\ &\subset W(f(Lx), x_0, k_n). \end{aligned}$$

From the condition that the mapping L is weakly commuting with f it follows $W(f(Lx), x_0, k_n) = W(L(fx), x_0, k_n)$, i.e. $f(u) \in L_n(fx)$.
Hence, all the conditions of the Theorem 1 are satisfied and according to it, for every $n \in \mathbb{N}$, there exists $x_n \in M$ such that

$$fx_n \in L_n x_n.$$

Since $L_n x_n = \bigcup\limits_{z\in Lx_n} W(z, x_0, k_n)$ there exists $z_n \in Lx_n$ such that $fx_n = W(z_n, x_0, k_n)$. Then, for every $n \in \mathbb{N}$

$$\begin{aligned} F_{fx_n,z_n}(\varepsilon) &= F_{z_n,W(z_n,x_0,k_n)}(\varepsilon) \\ &\geq T(F_{z_n,z_n}(\frac{\varepsilon}{2k_n}), F_{z_n,x_0}(\frac{\varepsilon}{2(1-k_n)})) \\ &= T(1, F_{z_n,x_0}(\frac{\varepsilon}{2(1-k_n)})) \\ &= F_{z_n,x_0}(\frac{\varepsilon}{2(1-k_n)}). \end{aligned}$$

Since $\lim\limits_{n\to\infty} \frac{\varepsilon}{2(1-k_n)} = \infty$, it follows that $\lim\limits_{n\to\infty} F_{z_n,x_0}(\frac{\varepsilon}{2(1-k_n)}) = 1$ because $z_n \in L(M)$ which is probabilistically bounded. Then we have $\lim\limits_{n\to\infty} F_{fx_n,z_n}(\varepsilon) = 1$. Since $z_n \in Lx_n$ and $\overline{L(M)}$ is compact it follows that there exists convergent subsequence $(z_{n_k})_{k\in\mathbb{N}}$ of the sequence $(z_n)_{n\in\mathbb{N}}$. From $\lim\limits_{n\to\infty} fx_{n_k} = z$ it follows $\lim\limits_{n\to\infty} z_{n_k} = z$.
It remains to be proved that $fz \in Lz$. From $\lim\limits_{n\to\infty} z_{n_k} = z$ and from the continuity of the mapping f it follows $\lim\limits_{n\to\infty} fz_{n_k} = fz$. Since $z_{n_k} \in Lx_{n_k}$ it follows that

$$fz_{n_k} \in f(Lx_{n_k}) \subset L(fx_{n_k}).$$

From the Lemma 2 it follows that the mapping L is closed i.e.

$$fz \in Lz.$$

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References

[1] Shih-sen Chang, Yeol Je Cho, Shin Min Kang, Probabilistic metric spaces and nonlinear operator theory, Sichuan University Press, 1994.

[2] O. Hadžić, A fixed point theorem in Menger spaces, Publ. Inst. Math. Beograd T 20 (1979), 107–112.

[3] O. Hadžić, Fixed point theorems for multivalued mappings in probabilistic metric spaces, Mat. Vesnik 3 (16)(31), (1979), 125–133.

[4] O. Hadžić, Fixed point theory in probabilistic metric spaces, Serbian academy of science and arts branch in Novi Sad, University of Novi Sad, Institute of Mathematics, 1995.

[5] O. Hadžić, E. Pap, Fixed Point Theory in Probabilistic Metric Spaces, Theory in Probabilistic Metric Spaces, Kluwer Academic Publishers, Dordrecht, 2001.

[6] E.P. Klement, R. Mesiar, E. Pap, Triangular Norms, Kluwer Academic Publishers, Trends in Logic 8, Dordrecht (2000a).

[7] S.B. Nadler, Multivalued contraction mappings, Pacific J. Math. 30 (1969), 475–478.

[8] V. Radu, On the contraction principle in Menger spaces, Seminarul the theoria probabilitatilor siaplicatii, University of Timisoara, Faculty of Natural Science, 1983.

[9] B. Schweizer, A. Sklar, Probabilistic metric spaces, Elsevier North-Holland, New York, 1983.

[10] V.M. Sehgal, A.T. Baharucha-Reid Fixed points of contraction mappings on probabilistic metric spaces, Math. Syst. Theory 6 (1972), 97–102.

[11] W. Takahashi, A convexity in metric spaces and nonexpansive mappings I, Kodai Math. rep. 22 (1970), 142–149.

[12] T. Žikić, Multivalued probabilistic q-contraction, Journal of Electrical Engineering, Vol. 53 No.12/s, (2002), 13–16.