SYNTHESIS OF DISTRIBUTED FUZZY HIERARCHICAL MODEL IN DECISION SUPPORT SYSTEMS IN FUZZY ENVIRONMENT*

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Abstract

As the result of the development of Fuzzy Multiple Criteria Decision Making (FMCDM) with the help of fuzzy set theory a number of innovations have been made possible. The new approach of FMCDM – Fuzzy Hierarchical Modeling - is introduced. It is shown in the article, how to use the fuzzy hierarchical model with other methods of FMCDM. Advantages of the method are described also. We propose a novel approach to overcome the inherent limitations of Hierarchical Methods by exploiting multiple distributed information repositories.

Keywords: Fuzzy multiple criteria decision making, fuzzy hierarchical modeling.

1 Distributed Fuzzy Hierarchical Model

The basis of the model is the hierarchical structure of the factors, which was received as a result of function-structured decomposition of the data domain [1, 4-6]. The meta-levels of the structure are the following: first level is the level of the global aims, the second level is the level of the rival's aims, and the third level is the level of the measures for the achievement of the global aims and rivals aims removal. The last level is the level of the concrete actions. The links in the hierarchy define the dependency of the upper level element realization from the corresponding underlying level element. Thereby, the realization of possible measures for the achievement of the aim depends on some concrete undertaken actions. This hierarchy allows evaluating the importance of all the elements of the level taking into consideration their contribution in the top levels elements realization. The hierarchical structure analysis model allows to process local factors estimations. These estimations have, as a rule, fuzzy and inconsistent nature, got from sources of different reliability (from expert with different competence level). This hierarchical model also allows to get total global consistent and reliable in the sense of theories of the fuzzy sets estimations. Thereby, each decision will be characterized by its importance taking into consideration its role in the factors structure. But, such a decision characteristic is insufficient for all-round estimation. The additional characteristics of the decision, such as its realization in economic, social, politic etc senses, must be considered. But, the hierarchical model can find the most needed decision in the current situation.

We propose a novel approach to overcome the inherent limitations of Hierarchical Methods by exploiting multiple distributed information repositories. The construction of fuzzy hierarchical model can be distributed between a number of experts. They may work to a single domain or to different domains. Also the distributed computational methods are used for making the expert estimation and for receiving the result.

2. Hierarchy Analysis Methods

The special role in the complex object analysis plays the analysis of the factors links graph's structure (the graph has the form of ordered hierarchical ranked structure). Directly influenced factors are situated on the graph's last level. The realization of these factors (as a rule they represent the concrete actions), spreading upwards on consecutively located levels of the factors hierarchical structure, will bring into the realization of all above located factors and, finally, - to the achievement of the global aims of the considered complex object development. At the moment, the strict statement of the hierarchy multilevel factors structure building problem doesn't exist. But, it is possible to indicate the principles of its practice construction. These principles are formulated in the form of six necessary conditions, which must satisfy considered hierarchical structures. It is naturally, that real hierarchical structures will satisfy these conditions only in certain measure, which depends on the used methods and algorithms of their formation.

1. Hierarchical factors structures are built on the base of the profound sense of used fac-

tors; the factors in the underlying level reveal the sense of the upper level factors, or the underlying level factors represent the events, which realization promotes the realization of upper level factors.

2. The realization of some of the factors, lying on the same level, must not influence the realization of the other factors of this level. In other words, the factors of the same level must be independent from each other.
3. Factors on the considered level directly depends only on the factors of the nearest underlying level of the hierarchy.
4. Fullness of the factors uncovering: factor on the considered hierarchy level is completely realized, if all the influencing its realization factors of the next underlying level are also realized.
5. Positive relationship between the upper level factors and underlying level factors: the realization of the underlying level factors must not provide the reduction of the realization possibility of the upper level factors.
6. Linearity of the functional links between the adjacent levels factors.

3 Analysis of a Hierarchy with Fuzzy Estimations

First of all, we should build the hierarchy. On the objects set $Z = \{1, 2, \ldots, N\}$ is defined the oriented graph $G_r = (Z, W)$ without cycles with the vertexes set coinciding with the objects set, and the arcs set W. The presence of the arc $(i, j) \in W$ means that the weight z_i of the object (vertex) i directly depends on the weight z_j of the object j. The graph G_r has the structure of the purposes and tasks graph of some complex system, if all the vertexes of this graph can be located on non crossing levels $V_1, \ldots, V_M$ in such a way, that the graph's arcs connect only the vertexes of the adjacent levels and these arcs lead from top to bottom, from the level V_i to the level V_{i+1}, i = 1, …, M-1; the vertexes, from which arcs don't leave, are located on the level V_M; all the vertexes, in which arcs do not enter, are located on the level V_1. The construction of the hierarchy is one of the most difficult stage because of the difficult formalization of the used objects, such as aims, rivals aims etc. After hierarchy construction, the elementary estimations should be made by experts. The elementary estimation consists on the getting for certain vertex $i \in V_m$ paired estimations $_{jk}^{(i)}$ of the arcs weights $(i, j) \in W$, $j \in \Gamma_i = \{k \mid (i, k) \in W\}$. Paired estimations show, in how many times the contribution of the object j is more than the contribution of the object k in the achievement of the object i aim; $j, k \in \Gamma_i$. These estimations can be exact ($r_{jk}^{(i)} \in R_+$ - nonnegative numbers), interval ($r_{jk}^{(i)} = [a_{jk}^{(i)}, b_{jk}^{(i)}] \subset R$ - intervals) or fuzzy numbers ($r_{jk}^{(i)} = \{(t, \mu_{jk}^{(i)}(t)) \mid t \in R_+\}$ – closed convex fuzzy sets on R_+). The last case includes the linguistic estimates and two previous cases. Thereby, we get as a result of an elementary estimation an weighted binary relation $R^{(i)} = \{((j, k), r_{jk}^{(i)}) \mid j, k \in \Gamma_i\}$ on the objects set Γ_i, which gives the intensity of the objects superiority. After getting the estimations, we must average them. In each of the elementary estimations several experts can participate, so for some pairs (j, k) of the objects $j, k \in \Gamma_i$ different experts s can assign different estimations $r_{jk}^{(i)s}$ (s – expert's number). The procedure of the expert estimation averaging consists in the determination of the mean geometric estimation.

4 Hierarchic Structure Arcs Weights Determination

The result of the pairs estimations average in the i elementary estimation – exact, interval or fuzzy relation $R_{(i)}$ – is used in the determination of the weights $y_{i,j}$, of all the arcs $(i, j) \in W$, coming out of the vertex i. The arcs weights satisfy the following condition:

$$\sum_{j \in \Gamma_i} y_{ij} = 1; \quad y_{ij} \geq 0, \quad \forall i \in \Gamma_i .$$

If there are several objects on the first level y_1, then the "zero" elementary estimation is made, it means, that the pair comparison of the objects importance coefficients must be made. As a result of the "zero" estimation, the importance coefficients of the first level objects are determined.

5. The Importance Coefficients Determination

After the elementary estimations results processing, the importance coefficients z_j of the objects $j \in V_1$ of the first level of the hierarchic structure are determined. And also the weights y_{ji} of all the arcs (i, j) ∈ W are determined (the coefficients of the relative importance of the vertex $Y_{ji}^{(s)}$ for the vertex $Y_i^{(s-1)}$ of the nearest upper level, where s – is a level number. The weights of the underlying level objects are determined by the recurrence from top to bottom recalculation of the objects weights (objects importance coefficients):

$$z_i = \sum_{j \in \Gamma_i^{-1}} y_{ji} z_j, \ i \in V_2,$$

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$$z_i = \sum_{j \in \Gamma_i^{-1}} y_{ji} z_j, \ i \in V_M$$

$$(\Gamma_i^{-1} = \{j \mid (j, i) \in W\}).$$

6 The Different Experts Estimations Consensus Analysis

The coefficients importance validity is determined by the elementary estimations results validity. In the case, then the initial pairs estimations are fuzzy or mixed, the results validity is equal to the consensus degree of the initial fuzzy relation $R^{(i)}$ and the resulting over transitive matrix, which is determined as a result of a special estimations approximation problem solution. The solution of the estimation approximation problem is made, using a modified method of Makeev and Shahnov [2-3]. In the case, then the estimations are exact or interval, the results validity is characterized by the degree of the intervals bounds changes, which are assigned by the experts.

7 Conclusion

Decomposition in Hierarchic Model is made until the level which contains factors with qualitative or quantitative scale of values. To apply the FMCDM methods the construction of every scale reflection to [0,1] is needed. It means that it necessary to create membership function that will convert every value from the scale to real number from [0,1]. The number is interpreted as preference of selected factor value for the main hierarchical goal (factor of the upper level) achievement. Zero is interpreted as index of minimum preference than One is interpreted as index of maximum preference.

On the basis of relative importance weights it is possible to construct unified scale for the scale gradations of the factors. Using FMCDM methods allows as getting the preference coefficient of the alternative, it means the preference coefficients of last level factors value collection.

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