# Modelling with Temporal Fuzzy Chains

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## Abstract

The aim of this paper is to present the Temporal Fuzzy Chains (TFCs) [3] to model the dynamic systems in a linguistic manner. TFCs make use of two different concepts: the traditional method to represent the dynamic systems named state vectors [6], and the linguistic variables [8] used in fuzzy logic [7]. Thus, TFCs are qualitative and represents the ”temporal zones” using linguistic states and linguistic transitions between the linguistic states.

**Keywords:** Temporal model, linguistic model, dynamic systems, fuzzy logic.

## 1 Introduction

Dynamic systems (DS) are systems whose performance change throughout the time. A DS is described by means of a set of relevant features (input and output variables) and a set of relations among input and output variables, which represent the modifications of the output variables when the input variables are modified throughout the time. The values of the system variables at the time $t$ depend on the variables values at the times $t-1\ldots 1$.

The DSs with continuous physical magnitudes are continuous at the time, that is, at a time $t+1$ the variable value $v_{t+1}$ is *similar* to the variable value $v_t$ at the time $t$. This property is formally represented as $|v_t - v_{t-1}| < \varepsilon$ with $\varepsilon$ being a small constant. This hypothesis is supposed when we define the TFCs.

The next section recalls the definition of TFCs, the formal definition can be found in [3]. The induction algorithm is shown in section 3. In section 4 is modelled a shot put of Manuel Martínez. Finally, the conclusions and future works are exposed in section 5.

## 2 Defining the TFCs

We suggest to represent the temporal side of a DS making use of the TFCs. A TFC is formed by linguistic states and linguistic transitions. A linguistic state is defined to represent the system at a time. Between two consecutive linguistic states is established a linguistic transition that indicates the conditions necessary to enters into the next linguistic state. The change of state is described in a *linguistic way* (using linguistic labels).

Let $\Xi$ be a DS MISO with a set of $m$ real input variables $X_1, X_2 \ldots X_m$ and an output real variable $S$. The behavior of the system is given by means of a set of examples $E = \{e_1, e_2 \ldots e_n\}$ with $e_i = (x_1^i \ldots x_m^i, s^i, t_i)$ where $x_j^i \in X_j$, $s^i \in S$ and $t_i$ is the time in which occurs the example $i$.

TFCs work with linguistic variables [8]. These variables have defined an ordered set of linguistic labels over its domain named **continuous linguistic variables**, from now on *variables*. The linguistic labels (from now on labels) associated to these variables are defined before the TFC will be obtained. Thus, an **ordered set of labels** $SA_j$ is defined for each input variable $X_j$. Its structure is $SA_j = \{SA_j^1, SA_j^2 \ldots SA_j^{i_j}\}$, where $i$ is the position of $SA_j^i$ in $SA_j$ and $i_j$ is the number of linguistic labels in $SA_j$, that is $i_j = |SA_j|$. An ordered set of labels $SC$ is defined for the output variable $S$. Its structure is $SC = \{SC^1, SC^2 \ldots SC^{i_y}\}$ where $i$ is the position of $SC^i$ in $SC$ and $i_y$ is the number of linguistic labels in $SC$, that is $i_y = |SC|$.


Figure 1: Linguistic interval

Our variable takes **linguistic interval** as value. A linguistic interval (from now on interval) $LI^c_{j,p} = \{SA^p_j, SA^{p+1}_j \dots SA^{p+(c-1)}_j\}$ for a variable $X_j$ is defined as a subset of the ordered set of labels $SA_j$ that begins in the label $p$ and has $c$ labels (Figure 1). Its membership function is the sum of the membership grade of a value $a_j$ to each label belonging to the interval (Equation 1).

$$\mu_{LI^c_{j,p}}(a_j) = \sum_{SA^z_j \varepsilon LI^c_{j,p}} \mu_{SA^z_j}(a_j) \qquad (1)$$

where $z \in [p..c-1]$.

A set of $m$ intervals defined on $m$ variables is an **ordered set of *m* intervals** for each one of the input variable, and is represented as $SLI_m = \{LI^{c_1}_{1,p_1}, LI^{c_2}_{2,p_2} \dots LI^{c_m}_{m,p_m}\}$. The membership function of a $SLI_m$ is calculated applying a t-norm to the membership grade of the intervals in the $SLI_m$ (Equation 2).

$$\mu_{SLI_m}(e_i) = *(\mu_{LI^{c_j}_{j,p_j}}(x_j)) \qquad (2)$$

where $e_i = (x^i_1 \dots x^i_m, s^i, t_i)$ is an example belonging to the set $E$, $j \varepsilon [1..m]$ and * is a t-norm.

$SLI_m$ is used to represent linguistically the range of values of the $m$ input variables.

Finally, a **linguistic state** $i$ (from now on state) is defined as a tuple $est_i =< A^i_m, SE_i >$ where $A^i_m$ is an ordered set of $m$ intervals of the state $i$ corresponding to the $m$ input variables of the DS, and $SE_i$ is the output label of the state $i$ corresponding to the output variable of the DS.

A **linguistic transition** $i$ (from now on transition) is a tuple $trans_i =< T^i_m, ST_i >$ where $T^i_m$ is an ordered set of $m$ intervals of the transition $i$ corresponding to the $m$ input variables of the DS, and $ST_i$ is the output label of the transition $i$ corresponding to the output variable of the DS.

A **TFC** is a tuple $CHAIN =< EST, TRANS >$ where $EST = \{est_1 \dots est_{ns}\}$ is an ordered set of $ns$ states, and $TRANS = \{trans_1 \dots trans_{ns-1}\}$ is an ordered set of $ns-1$ transitions. Transition $i$ reflects the conditions to change from $est_i$ to $est_{i+1}$.

In order to reproduce the DSs with TFCs, we offer an inference method in [3]. The inference algorithm needs a set of examples $E$ as input and is based on the definition of a state $est_{cur}$ named *current state*. $est_{cur}$ indicates the state in which the DS is, and allows to calculate the output at this time. The inference method begin selecting $est_1$ as the first current state. Next $\mu_{A^{cur}_m}(e_i)$ of $e_i$ to the state $est_{cur}$ and the membership function $\mu_{T^{cur}_m}(e_i)$ of $e_i$ to the transition $trans_{cur}$ are calculated. If $\mu_{A^{cur}_m}(e_i)$ is greater than $\mu_{T^{cur}_m}(e_i)$ then the obtained output $s$ is $SE_{cur}$ (corresponding to $est_{cur}$) and there isn't *state change*. In other case, if $\mu_{A^{cur}_m}(e_i)$ is less than or equal to $\mu_{T^{cur}_m}(e_i)$, then there is a *state change*: the obtained output $s$ is $ST_{cur}$ (corresponding to $trans_{cur}$) and the new current state is the next in the TFC, i.e., $est_{cur+1}$. This process is repeated for each example in $E$.

## 3 Inducing the TFCs

In this section we show briefly the suggested algorithm to induce TFCs [5]. Firstly, some necessary concepts are shown.

**Definition 3.1** *Let* $SA_j = \{SA^1_j, SA^2_j \dots SA^{i_j}_j\}$ *be an ordered set of labels defined on* $X_j$ *and a set of examples* $E_{SA_j}$. *A* ***simplified interval*** $LI^{u-p+1}_{j,p} = \{SA^p_j, SA^{p+1}_j \dots SA^u_j\}$ *depending on* $E_{SA_j}$ *is an interval which verifies that: Its first label* $SA^p_j$ *is the first one that verifies the equation 3; and its last label* $SA^u_j$ *is the last one that verifies the equation 4.*

$$\exists e_i \varepsilon E_{SA_j} / \mu_{SA^f_j}(x^i_j) > 0 \qquad (3)$$

$$\exists e_i \varepsilon E_{SA_j} / \mu_{SA^l_j}(x^i_j) > 0 \qquad (4)$$

where $x^i_j$ is the real value in the position $j$ in the example $e_i$.

In short, a simplified interval $LI^{u-p+1}_{j,p}$ is an interval where its first label is the first one of $SA_j$ that has some example of $E_{SA_j}$ with membership grade greater than zero, and its last label is the last one of $SA_j$ that has some example of $E_{SA_j}$ with membership grade greater than zero.

**Definition 3.2** *Let $LI^{c_{j_1}}_{j,p_{j_1}}, LI^{c_{j_2}}_{j,p_{j_2}} \dots LI^{c_{j_n}}_{j,p_{j_n}}$ be n intervals defined on $X_j$, its **union** is defined as another interval $LI^{u-p+1}_{j,p}$ where the first label $SA^p_j$ is the smallest label of $LI^{c_{j_1}}_{j,p_{j_1}}, LI^{c_{j_2}}_{j,p_{j_2}} \dots LI^{c_{j_n}}_{j,p_{j_n}}$; and the last label $SA^u_j$ is the greatest label of $LI^{c_{j_1}}_{j,p_{j_1}}, LI^{c_{j_2}}_{j,p_{j_2}} \dots LI^{c_{j_n}}_{j,p_{j_n}}$.*

**Definition 3.3** *Let $SLI^1_m, SLI^2_m \dots SLI^n_m$ be n ordered sets of m intervals, its **union** is defined as another $SLI_m$, where each interval $LI^{c_j}_{j,p_j}$ of $SLI_m$ is obtained making the union of n intervals $LI^{c_{j_1}}_{j,p_{j_1}}, LI^{c_{j_2}}_{j,p_{j_2}} \dots LI^{c_{j_n}}_{j,p_{j_n}}$ of $SLI^1_m, SLI^2_m \dots SLI^n_m$.*

**Definition 3.4** *Let $est_1, est_2 \dots est_n$ be n states and $E_{A^1_m}, E_{A^2_m} \dots E_{A^n_m}$ n sets of examples associated to the n ordered sets of intervals, the **total union of the** n **states** is another state $est_{union}$ where:*

1. *$A^{union}_m$ is obtained by making the union of the n intervals $A^1_m, A^2_m \dots A^n_m$ of the states $est_1, est_2 \dots est_n$ (definition 3.3).*
2. *The output label is calculated using the equation:*

$$max_{SC^w} \mu_{SC^w}(v) \tag{5}$$

*The set of examples needs in this equation is $E_{union} = \bigcup E_{A^1_m}, E_{A^2_m} \dots E_{A^n_m}$.*

The total union of the states is used when a state that represents the successive examples from $e_a$ to $e_b$ has the same output label than the state that represents the examples from $e_{b+1}$ to $e_c$.

**Definition 3.5** *Let $LI^{c1_j}_{j,p1_j}$ and $LI^{c2_j}_{j,p2_j}$ be two intervals with its labels defined in $SA_j$, **the difference of** $LI^{c2_j}_{j,p2_j}$ **respect to** $LI^{c1_j}_{j,p1_j}$ is the following set of labels:*

$$LI^{c2_j}_{j,p2_j} - LI^{c1_j}_{j,p1_j} = \{SA^i_j \in LI^{c2_j}_{j,c2_j} \setminus SA^i_j \text{ not} \in LI^{1_j}_{j,p1_j}\}$$

In short, the difference between two intervals $LI^{c2_j}_{j,p2_j}$ and $LI^{c1_j}_{j,p1_j}$ is a set of labels compounded by the labels that belong to $LI^{c2_j}_{j,p2_j}$ and do not belong to $LI^{c1_j}_{j,c1_j}$.

**Definition 3.6** *Let $SLI^1_m = \{LI^{c1_1}_{1,p1_1}, LI^{c1_2}_{2,p1_2}, \dots, LI^{c1_m}_{m,p1_m}\}$ and $SLI^2_m = \{LI^{c2_1}_{1,p2_1}, LI^{c2_2}_{2,p2_2}, \dots, LI^{c2_m}_{m,p2_m}\}$ be two ordered sets of m labels with the labels of each variable defined on $SA_1, SA_2 \dots SA_m$, the **difference between** $SLI^2_m$ **respect to** $SLI^1_m$ is the set of sets of labels formed by:*

$$SLI^2_m - SLI^1_m = \{LI^{c2_1}_{1,p2_1} - LI^{c1_1}_{1,p1_1}, LI^{c2_2}_{2,p2_2} - LI^{c1_2}_{2,p1_2} \dots LI^{c2_m}_{m,c2_m} - LI^{c1_m}_{m,p1_m}\}$$

The difference between two $SLI_m$ is used to know when the state change can be detected between two successive states.

**Algorithm 1** Direct Induction Algorithm of TFCs

```
EST ← θ
TRANS ← θ
N_state ← 2
E_{A^cur_m} ← {e_1}
A^cur_m ← Simplify(SA_1 ... SA_m)
SE_cur ← CalculateConsequent(E_{A^cur_m})
for i = 2 to |E| do
   E_{A^{N_state}_m} ← {e_i}
   A^{N_state}_m ← Simplify(SA_1 ... SA_m)
   SE_{N_state} ← CalculateConsequent(E_{A^{N_state}_m})
   if SE_{N_state} = SE_cur or A^cur_m − A^{N_state−2}_m = θ then
      est_cur ← TotalUnion(est_cur, est_{N_state})
      E_{A^cur_m} ← Union(E_{A^cur_m}, E_{A^{N_state}_m})
   else
      EST ← EST + est_cur
      if N_state − 2 > 0 then
         T^{N_state−2}_m ← A^cur_m
         ST_{N_state−2} ← CL(SE_{N_state−2}, SE_cur)
         TRANS ← TRANS + trans_{N_state−2}
      end if
      est_cur ← est_{N_state}
      E_{A^cur_m} ← E_{A^{N_state}_m}
      N_state ← N_state + 1
   end if
end for
EST ← EST + est_{N_cur}
T^{N_state−2}_m ← A^cur_m
ST_{N_state−2} ← CL(SE_{N_state−2}, SE_cur)
TRANS ← TRANS + trans_{N_state−2}
```

**Definition 3.7** *Let $SA_j = \{SA^1_j, SA^2_j \dots SA^{i_j}_j\}$ be an ordered set of labels and two labels $SA^p_j$ and $SA^u_j$ of $SA_j$, the label $SA^{\frac{p+u}{2}}_j$ is named the **central label**. The exponent $\frac{p+u}{2}$ is rounded by excess if $SA^p_j < SA^u_j$, and down if $SA^p_j > SA^u_j$.*

We make use of the central label concept for calculating the output associated to each transition between two states $i$ and $i+1$. Thus, when $SE_i$ and $SE_{i+1}$ are consecutive the output label $ST_i$ of the transition $i$ is $SE_{i+1}$, in other case $ST_i$ will be the label in the middle of $SE_i$ and $SE_{i+1}$. The central label between $SE_i$ and $SE_{i+1}$ is represented as $CL(SE_i, SE_{i+1})$ in the algorithm.

To finish this section, the algorithm 1 is shown. This algorithm is used to induce the TFCs and is named *Direct Induction Algorithm of TFCs*. Its inputs are a set of examples $E$, the ordered sets of labels $SA_1 \ldots SA_m$ for the $m$ input variables and the ordered set of labels $SC$ for the output variable.

In the algorithm 1, $EST$ and $TRANS$ are the ordered set of states and transitions respectively; $est_{cur}$ is the current state, that is, the state that is going to be inserted as the last state of the ordered set of states $EST$; and $N_{state}$ is the number of the next state to $est_{cur}$ and is used to detect when $est_{cur}$ is completely constructed (with all its examples in $E_{A_m^{cur}}$).

Firstly, $EST$, $TRANS$ and $N_{state}$ are initialized to $\theta$, $\theta$ and 2 respectively, and the first current state $est_{cur}$ is created: each interval $LI_{j,p_j}^{c_j}$ of the $A_m^{cur}$ of $est_{cur}$ is initialized to the simplification of the ordered set of labels $SA_j$ using as set of examples associated $E_{A_m^{cur}} = \{e_1\}$ (definition 3.1). The simplification of the ordered sets of labels $SA_1, SA_2 \ldots SA_m$ depending on the set of examples $E_{SA_j}$ is a set of $m$ intervals $SLI_m$, i.e., $A_m^{cur}$. The output label of the first current state $est_{cur}$ is calculated using the equation 5.

$$\text{where w=1..}i_y \text{ and } v = \frac{\sum_{i=1}^{|E_{SA_j}|} s^i}{|E_{SA_j}|}$$

where $|E_{SA_j}|$ is the number of examples in $E_{SA_j}$, and $s^i$ is the output real value in the example $e_i$ which belongs to $E_{SA_j}$.

In brief, the selected label for the output of the state is the one that has the maximum grade of membership to the medium value of the output values of the examples in $E_{SA_j}$. Others possibilities are given in [7].

Next, the loop **for** is used to examine from $e_2$ to the last example $e_n$ in $E$. For each example $e_i$ is calculated the state $est_{N_{state}}$ where: $A_m^{N_{state}}$ is assigned to the simplification of the ordered sets of labels $SA_1, SA_2 \ldots SA_m$ using $E_{A_m^{N_{state}}} = \{e_i\}$ as set of examples associated; and the output label $SE_{N_{state}}$ is calculated by using of the equation 5.

If the output label of $est_{N_{state}}$ is equal to the output label $est_{cur}$ or if $A_m^{cur} - A_m^{N_{state}-2} = \theta$ means that both states (that represent set of examples consecutive at time) have the same output or the change can't be detected (because $A_m^{cur}$ is contained in $A_m^{N_{state}-2}$) respectively, thus, $est_{cur}$ is assigned to the total union of the states $est_{cur}$ and $est_{N_{state}}$ (definition 3.4) because represent the same output. In other case, $est_{cur}$ is added to the ordered set $EST$, and if $est_{cur}$ isn't the first state then the transition $trans_{N_{state}-2}$ is created, i.e., the previous transition to the last inserted state $est_{cur}$ where: $T_m^{N_{state}-2}$ is assigned to $A_m^{cur}$; and $ST_{N_{state}-2}$ is the central label between the output label of the states $SE_{state-2}$ and $SE_{cur}$ (definition 3.7), that is, the central label between the two last states of the TFC.

Next, the new transition $trans_{N_{state}-2}$ is added to $TRANS$. Finally, $est_{cur}$ takes the value of $est_{N_{state}}$, the set $E_{A_m^{cur}}$ is now $E_{A_m^{N_{state}}}$ and $N_{state}$ is incremented in 1.

When the loop **for** is finished the last state and transition are added to $EST$ and $TRANS$.

## 4 TFC of a shot put

Algorithm 1 is evaluated using a shot put of the Spanish athlete Manuel Martinez, the current world champion of shot put. The set of examples $E$ was captured during the thesis [2]. This data corresponds to a shot put of 19.43 meters. For more information about the capture process see [1, 2].

The input variables of $E$ are: pelvis-scapular angle ($PSA$), Elbow angle ($EA$), right-left axis ($RLA$) and backward-forward-axis ($BFA$). The output variable is the height of the weight ($H$).

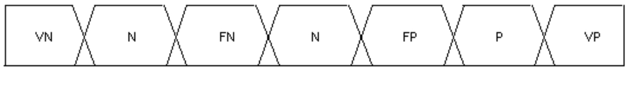


Figure 2: Sequence of 7 labels

A sequence of 7 labels with domain equally spaced is used for all variables. The structure of this sets is shown in Figure 2 with $VN$ being Very Negative, $N$: Negative, $FN$: Few Negative, $NR$: Norm, $FP$: Few Positive, $P$: Positive and $VP$: Very Positive.

The obtained TFC is shown in Figure 3. Formally the obtained TFC is represented as a tuple $CHAIN =<$

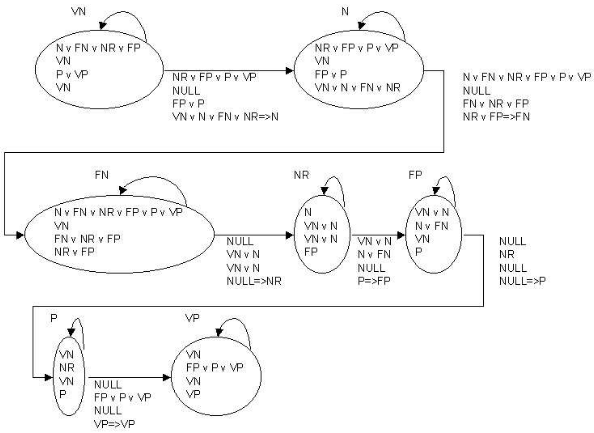


Figure 3: TFC of shot put

$EST, TRANS >$ where $EST =< est_1, est_2, \ldots est_7 >$ and $TRANS =< trans_1, trans_2, trans_3 \ldots trans_6 >$. Figure 4 shows the temporal zone of each rule, each rule covers a zone of the output graphic.

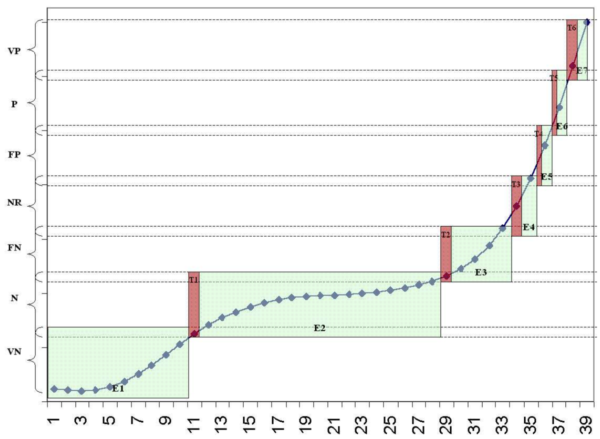


Figure 4: Temporal zone for each state

Finally, a inference process is made by using the inference algorithm, the set of example $E$ is used as input. The obtained error is 0.0662°. Figure 4 shows graphically the comparison between the real output and the obtained output. The obtained line is very similar to the real line and the error is small.

## 5 Conclusions

TFCs are a new method to represent the DSs. TFCs are qualitative making use of linguistic labels defined a priori, and represent the performance of the DS. Thus, TFCs are a good approach to model the DS. The

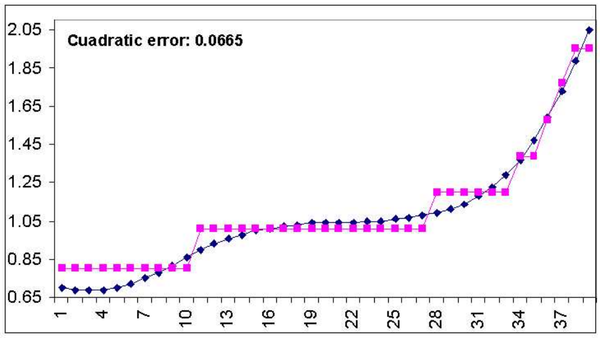


Figure 5: Inference using $E$

temporality is represented in the order of the states and transitions, that is an important difference with the fuzzy models. The obtained output is improved using the central label in the transitions [5]. The central label represents the temporal zones in which the output evolution is faster than the evolution of the input variables. This is a problem in the fuzzy models that no consider the time.

The temporality is not representing in traditional fuzzy models, only with a variable for the time. Thus, to solve this, the TFCs are ordered, so, the time and the phase of the dynamic system evolution is indicated in the order of the states and transitions. Each variable in the time is modelled using a set of linguistic labels (named intervals), thus, the movement of the dynamic system is represented linguistically. In the environment of this work, TFCs can be used to create trainers for athletes. These trainers indicates, linguistically, the instant when the athlete don't make the correct movement and how is corrected this movement.

We will also study the relation between TFCs and traditional systems of fuzzy rules and the relation between TFCs and *Temporal Fuzzy Models* [4]. We will design a new induction algorithm that uses more than one set of examples as input. We will develop an algorithm to study the best number of labels and its domain for each input and output variables. Finally, we will work in two different aspects: (1) A method to covert the intervals to expressions like ”quick increment”, ”quick decrement”, etc (2) Adding to the TFC the ”time linguistic variable” to represent the ”duration of the state”, ”time per label”, etc.

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