The Similarity of a Class of Adaptive Fuzzy Controllers and a Time Dependent Single Rule Controller of Takagi-Sugeno Model

Mohammad Rohmanuddin
Department of Engineering Physics
Bandung Institute of Technology
Jalan Ganesa 10, Bandung 40132
INDONESIA
e-mail: rohman@tf.itb.ac.id

Abstract

This paper presents the similarity of a class of adaptive fuzzy controllers and a time dependent single rule controller of Takagi-Sugeno (TS) model. The class of adaptive fuzzy controllers is one of iterative multi-layer structure of single input fuzzy controllers (SIFC). On the other hand, in a time dependent single rule controller of TS model, only one rule can be fired at a time. The result model is employed to simulate a control system of three-tank system.

Keywords: adaptive fuzzy controller, gradient descent, multi-input multi-output, single-input fuzzy controller, Takagi-Sugeno model, time dependent rule.

1 Introduction

In most rule based fuzzy controllers, as the number of system variables increases, the number of rules in a conventional complete rule set increases exponentially. To reduce the number of rules, the concept of hierarchical rule set was proposed [1, 2]. The similar concept has been applied by the author [3, 4]. A multi-layer structure of fuzzy control for multi-input multi-output (MIMO) systems were discussed in [5] and [8]. Wang [9] proved that the hierarchical fuzzy systems were universal approximators.

Rohmanuddin et al. [5] showed that fuzzy rule base of MIMO systems could be simplified by constructing multi-layer fuzzy controller. Unequally spaced triangular membership functions were employed as fuzzy membership function for each input variable (see Fig.1), and equally spaced fuzzy singletons were employed for those of output. It was referred to as (USES). A piecewise linear structure was produced in each input segment. It was shown that for any fuzzy controller of m-input with $(2n+1)$ fuzzy subsets for each input there are $(m+n)$ parameters of the controller to be determined.

This paper presents the similarity of a class of adaptive fuzzy controllers and a time dependent single rule controller of Takagi-Sugeno (TS) model, and it is based on the previous work discussed in [5]. It is assumed throughout this paper that if an input e yields an output u then an input $e' = -e$ yields an output $u' = -u$. The rule set is applied to simulate a fuzzy controller of liquid level of a three interconnected tanks [7].

The rest of this paper is organized as follows. Single input fuzzy controller is discussed in Section 2. Section 3 concerns with Takagi-Sugeno model. Section 4 presents the case study. The discussion is closed by some conclusions in Section 5.

2 Single Input Fuzzy Controller

The **if – then** rule of single input single output (SISO) fuzzy controller can be written as follows:

$$R_i\text{: } \textbf{if } e_1 \text{ is } E1_i \textbf{ then } u \text{ is } U_i$$

where $-n \le i \le n$, and, in this case, $E1_i$ and U_i is the input and output linguistic term, respectively. Since at any time there are only two rules with non-zero result, then using the center of gravity method the deffuzyfied output u can be expressed as

$$u = \frac{\sum_{i=I}^{I+1} \mu_i(e_1) U_i}{\sum_{i=I}^{I+1} \mu_i(e_1)} \tag{1}$$

where $-n \le I < n$, and U_i $(-n \le i \le n)$ is the center of fuzzy value of the corresponding output fuzzy set. It is clear that for any e_1 $(E1_i \le e_1 \le E1_{i+1})$

$$u = U_i + \left(\frac{e_1 - E1_i}{E1_{i+1} - E1_i} \right) (U_{i+1} - U_i) \tag{2}$$

By recalling that $\Delta E1_{i+1} = E1_{i+1} - E1_i$, $\Delta U / U_n = 1/n$ and $U_i / U_n = i/n$, a manipulation of Equation (2) will produce u as a linear function of e_1 in the segment $[E1_i, E1_{i+1}]$. That is

$$u = \frac{i}{n} + \frac{1}{n \Delta E1_{i+1}} e_1 - \frac{E1_i}{n \Delta E1_{i+1}} \tag{3}$$

In general form, for $E1_i \le |e_1| \le E1_{i+1}$, the above equation has the following expression

$$u = \mathrm{sgn}(e_1) \left(\frac{i}{n} + A_{i+1}^{(1)} | e_1 | + B_{i+1}^{(1)} \right) \tag{4}$$

where

$$A_{i+1}^{(1)} = \frac{1}{n \Delta E1_{i+1}}, \quad B_{i+1}^{(1)} = -\frac{E1_i}{n \Delta E1_{i+1}}$$

and

$$\mathrm{sgn}(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases}$$

Notice that the superscript refers to the input number

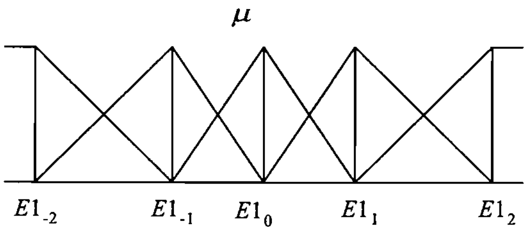

Fig. 1 Unequally spaced membership function.

to which the parameters A and B belongs. In the following, unless otherwise stated, we will only consider the case where input value is greater than or equal zero. In [5], Equation (4), representing a single input fuzzy controller (SIFC), is the basic element of the multi-layer fuzzy controller.

3 Single Input Takagi-Sugeno Model

The general form of the Takagi-Sugeno model for SISO system is described by the rule set:

R_i: **if** e_1 is $E1_i$ **then** $u_i = a_0^i + a_1^i e_1$

Notice that, if for the equation in the consequent part of the above rule we set $a_1^i = 0$, we will obtain the same result as that in Section 2. That is fuzzy rule set with fuzzy singleton in the consequent part. For any $e_1 = E1_i$ $(i = -n, \ldots, n)$, according to the corresponding rule, which spans from $e_1 = E1_{i-1}$ to $e_1 = E1_{i+1}$, we will have $u_i = U_i$. Since for every input segment there are two active rules, then the crisp output of the TS fuzzy controller is

$$u = \mu_i(e_1) U_i + \mu_{i+1}(e_1) U_{i+1} \tag{5}$$

In order to make the control system to be adaptive we can always set $E1_1$ (and $E1_{-1}$) at any time such that, the input e_1 always belongs to the varying input segment $[E1_{-1}, E1_1]$. With this setting we will have an adaptive system with varying universe of discourse. By setting $E1_1$ this way we will have a modified fuzzy controller which can be expressed as

R_i: **if** e_1 is $E1_1$ **then** $u_i = a_1^i e_1$

With this varying universe of discourse of the input variable, and the limitation that e_1 belongs to the varying segment $[E1_{-1}, E1_1]$, two important points can be concluded:

(1) The (varying) segment $[E1_{-1}, E1_1]$ becomes the effective (varying) universe of discourse.
(2) The above rule becomes the only rule of the fuzzy controller, which is called the time dependent single rule of TS model.

4 Case Study

A simulation of two-input two-output control system is presented as a case study [7]. The plant consists of three tanks denoted by T_1, T_2, and T_3 with cross section S_T (m^2). They are connected serially with each other by cylindrical pipes with nominal outflow S_P (m^2), as shown in Fig. 2. At T_2 a single valve, with the same nominal outflow S_P (m^2), is located. Two pumps are filling T_1 and T_2 with flow-rate U_1 and U_2 (m^3/det.), where $U_{1\max}$ and $U_{2\max}$ are 0.160×10^{-3} and 0.150×10^{-3} m^3/det, respectively. The liquid delivered by the two pumps can be manipulated. The level variation of the tanks, dx_i (m), are assumed to be proportional to the total flow of liquid from and to them. Hence, the system has three degrees of freedom and can be modeled by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \frac{1}{S_T} \begin{bmatrix} -Q_{13} \\ -Q_{23} - Q_{20} \\ -Q_{13} + Q_{23} \end{bmatrix} + \begin{bmatrix} U_1 \\ U_2 \\ 0 \end{bmatrix} \tag{6}$$

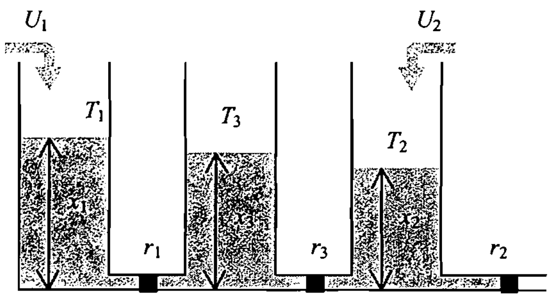

Fig. 2 The three-tank system

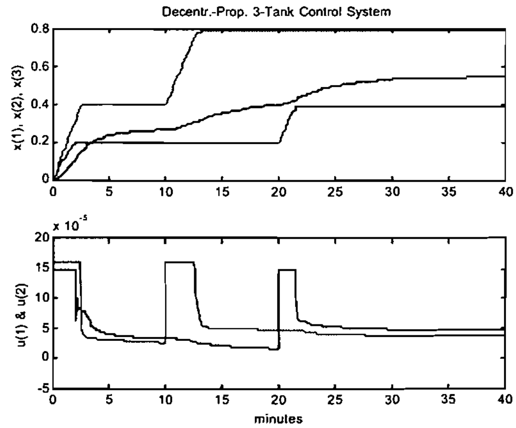

Fig. 3 Decentralized proportional fuzzy control system

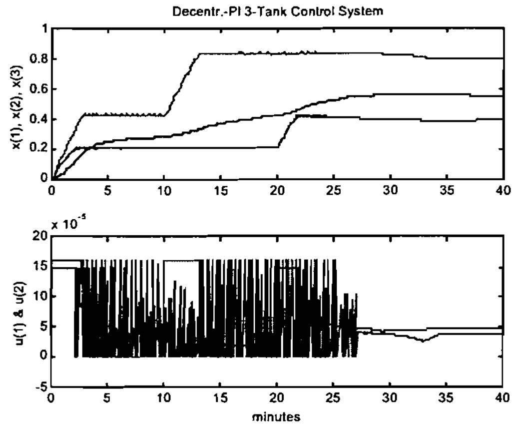

Fig. 4 Decentralized PI fuzzy control system

where $Q_{ij}(x_1, x_2, x_3)$ is the flow rate from T_i to T_j, and is provided by the law of Torricelli

$$Q_{ij}(x_1, x_2, x_3) = r_{ij}\,\text{sgn}(x_i - x_j)\sqrt{2g|x_i - x_j|} \tag{7}$$

for i,j=1,2,3, where g = 9.81 m/sec^2 is the gravity constant and r_{ij} (m^2) is the corresponding outflow coefficient and is proportional to S_P. Relation (6) can be rewritten in the following state-space form

$$\begin{aligned} \dot{\mathbf{x}} &= f(\mathbf{x}) + G(\mathbf{x}) \cdot \mathbf{u} \\ \mathbf{y} &= h(\mathbf{x}) \end{aligned} \tag{8}$$

where

$$f(\mathbf{x}) = \frac{1}{S_T} \begin{bmatrix} -r_1\,\text{sgn}(x_1 - x_3)\sqrt{2g|x_1 - x_3|} \\ -r_3\,\text{sgn}(x_2 - x_3)\sqrt{2g|x_2 - x_3|} - r_2\,\text{sgn}(x_2)\sqrt{2g|x_2|} \\ -r_1\,\text{sgn}(x_1 - x_3)\sqrt{2g|x_1 - x_3|} + r_3\,\text{sgn}(x_2 - x_3)\sqrt{2g|x_2 - x_3|} \end{bmatrix} \tag{9}$$

$$G(\mathbf{x}) = \frac{1}{S_T} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \text{ and } h(\mathbf{x}) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{10}$$

The control objective for the system is the regulation of the levels x_1 and x_2 to pre-specified values. Five types of fuzzy controller have been simulated, three of which are presented in this paper, i.e.:

(1) Decentralized Adaptive Proportional Fuzzy Controllers.
(2) Decentralized Adaptive PI-Fuzzy Controllers.
(3) Multivariable Controller Based on Adaptive Fuzzy Inverse Plant Model.

The parameters to be learned are basically input gain constants, each of which is equivalent to the corresponding universe of discourse. Each simulation start from the zero value parameters. At each time steps

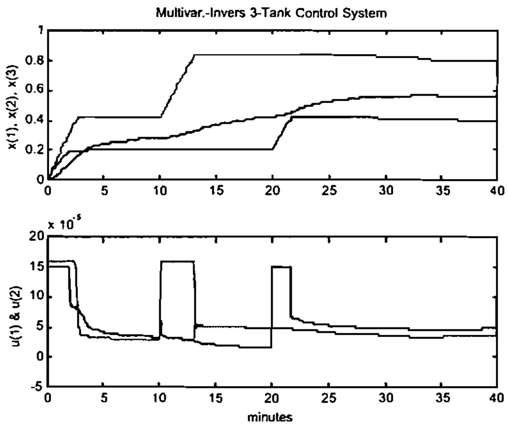

Fig. 5 Multivariable fuzzy control system based on fuzzy inverse model

the parameters are improved based on the squared of errors using the gradient descent method. The controller is operated according to the following rules:

- When the error is greater than 5% of the set point, the corresponding pump must be operated in the maximum flow-rate.
- The corresponding fuzzy controller is operated only when the error is less than or equal to 5% of the set point.

From the simulation results, the type-1 and the type-3 produced the best results, as shown in Figs 3 – 5. Although there was a small offset in the output level, the pumps worked in a normal way. On the other hand, compared to the formers, the decentralized adaptive PI-fuzzy controllers, did not show a better performance. The pumps worked in the on-off way, which should be avoided, and the offset in the output level was created during a relatively long period of transient time.

5 Conclusions

The similarity of a class of adaptive fuzzy controllers and a time dependent single rule controller of Takagi-Sugeno model has been presented in this paper. The similarity consists of two main properties: their universe of discourse are varying in time, and each of them has the controller's output which is time varying proportional to it's input. From the case study, it was shown that the proposed concept gave good results.

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