Fuzzy Model Based Sliding Mode Control of a Linear Precision Motion Control System

M.J.Knight
DMME, University of Plymouth
Drake Circus, Plymouth. Devon.
mjknight@plymouth.ac.uk

R.Sutton
DMME, University of Plymouth
Drake Circus, Plymouth. Devon.
rsutton@plymouth.ac.uk

D.F.L Jenkins
DCEE, University of Plymouth
Drake Circus, Plymouth. Devon.
dfljenkins@plymouth.ac.uk

Abstract

A method of sliding mode control based on a fuzzy model identified through input output data is presented. In this approach the advantages of the sliding mode control technique are maintained, however parametric uncertainty and unmatched disturbance are acknowledged as limiting factors of controller performance and their effects are sought to be minimised. Controller performance is compared with an equivalent conventional sliding mode controller.

Keywords: Fuzzy model identification, Adaptive sliding mode control, motor control.

Introduction

Precision motion control has relevance in many technological areas, it has found application in medical, electronic manufacture and mechanical disciplines to name but a few. As the requirement for progressively more accurate position control continues to grow, the performance requirements placed on the positioning device become ever more stringent. In systems where friction is present, it is a notoriously difficult phenomenon to measure, in addition to the parametric uncertainties associated with the controller and device load. It is therefore necessary to employ methods of control that will perform in a prescribed manner despite system uncertainty.

One of the earliest approaches to control of uncertain systems was sliding mode control (SMC) or variable structure control (VSC), first introduced to western researchers by the seminal works of Utkin [10] and Itkis [5]. The central feature of SMC is the sliding mode, in which the dynamic motion of the controlled system is constrained to remain within a prescribed subspace of the full state space. The sliding mode is achieved by ensuring that the prescribed manifold within the state space is made attractive to the system [5]. Once the manifold is reached, the system is forced to remain on it thereafter. When on the manifold, i.e. during the sliding motion, the system dynamics are equivalent to a system of lower order, which is insensitive to both parametric uncertainty and unknown disturbances that satisfy the matching condition.

One of the principle drawbacks of sliding mode is that it in general only applies to systems that satisfy the matching condition [11]. Secondly and most significantly, the control law is discontinuous across the sliding manifold, this leads to a phenomenon termed 'control chatter' in practical systems. Chatter involves high frequency control switching and may lead to excitation of previously neglected high frequency system dynamics. Smoothing techniques such as boundary layer normalisation have been employed in order to negate the effects of control chatter, however such an approach leads to a controller that can only guarantee tracking accuracy to within the ε-vicinity of the demand [3], where ε is the radius of the boundary layer. A compromise must therefore be sought between desired tracking accuracy and controller bandwidth.

The apparent similarities between the sliding mode and fuzzy controllers was illustrated in [8], this has subsequently motivated considerable research effort in combining the two topologies in a manner that serves to reduce the mentioned drawbacks of the sliding mode. The most common approach to this has been to replace the continuous switching function of the boundary layer with an equivalent fuzzy switching function. However, as pointed out in [7], the fuzzy rule base commonly serves as a mimic of the original switching function and the advantages of such an approach are therefore unclear. Others, e.g.[4], have used a fuzzy rule base in making the sliding manifold adaptive, so as to minimise the reaching phase, good results have been reported. Babuška [1] has demonstrated the ability of the affine Takagi-Sugeno consequent to locally model a system through rule extraction from cluster data obtained within the regression space. [9] subsequently uses such fuzzy models in order to extract locally linear state space models of the system and demonstrate model based control of both single input-single output (SISO) and multi input, single output (MISO) systems.

In this work, the parametric uncertainty and disturbances that the system is subject to are recognised as the root cause of the high gain feedback requirement and control chatter. It follows that if these uncertainties can be reduced then enhanced controller performance may be achieved as will be shown.

Fuzzy Modelling

Fuzzy identification is a term used that has come to represent the use of fuzzy logic in the modelling and representation of a system. Since fuzzy models may be viewed as general function approximators, they are readily applied to the nonlinear regression problem. The approach adopted within this work is to decompose the model into a static nonlinear regression. The problem of model identification is then decomposed into two separate problems, the first is selection of the regression structure, the second, the selection of the fuzzy model form, for example, the required number of membership functions and membership crispness.

The desired regression may be expressed in the form

$$\hat{y}(t|\theta) = f(\varphi(t), \theta) \tag{1}$$

It has been shown in [1] that the regression surface within the product space may be represented as a series of local approximations.

Through use of a subspace clustering algorithm such as the Gustafson-Kessel algorithm, it is possible to derive local approximations to this regression surface. Further, through use of the eigenvalues of the cluster covariance matrix given by

$$F_i = \frac{\sum_{k=1}^{N} (\mu_{i,k})^m (z_k - v_i)(z_k - v_i)^T}{\sum_{k=1}^{N} (\mu_{i,k})^m} \tag{2}$$

it is possible to interpret these local models and subsequently derive a fuzzy rule to represent this local approximation. In repeating this process for each data cluster, a global model of the system may be generated.

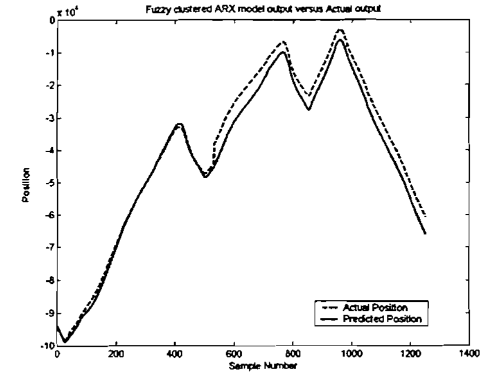

Figure 1: Typical model result compared to system output.

Previous work has considered the accuracy of this approach in comparison to neural networks and 'white box' models, results have demonstrated that this local approach to modelling can provide superior results [6].

Model Extraction

It has been shown in [9] that once the Takagi Sugeno model has been derived, local linear state space models can be calculated according to the following,

$$y_l(k+1) = \frac{\sum_{i=1}^{K_l} \mu_{li}(x_l(k)) \cdot y_{li}(k+1)}{\sum_{i=1}^{K_l} \mu_{li}(x_l(k))} \tag{3}$$

$$y_{li}(k+1) = (\varsigma_{li} y(k) + \eta_{li} u(k) + \theta_{li}) \tag{4}$$

where

$$\zeta_l^* = \frac{\sum_{i=1}^{K_l} \mu_{li}(x_l(k)) \cdot \zeta_{li}}{\sum_{i=1}^{K_l} \mu_{li}(x_l(k))} \tag{5}$$

$$\eta_l^* = \frac{\sum_{i=1}^{K_l} \mu_{li}(x_l(k)) \cdot \eta_{li}}{\sum_{i=1}^{K_l} \mu_{li}(x_l(k))} \tag{6}$$

and

$$\theta_l^* = \frac{\sum_{i=1}^{K_l} \mu_{li}(x_l(k)) \cdot \theta_{li}}{\sum_{i=1}^{K_l} \mu_{li}(x_l(k))} \tag{7}$$

In the case here, previous inputs are not considered and the A, B and C matrices of the model are thus simplified, the matrices are given

$$A = \begin{bmatrix} \zeta_{1,1}^* & \zeta_{1,2}^* & \cdots & \zeta_{1,\alpha_1}^* \\ 1 & 0 & \cdots & 0 \\ \zeta_{2,1}^* & \zeta_{2,2}^* & \cdots & \zeta_{2,\alpha_1}^* \\ 0 & \vdots & \ddots & \vdots \\ \zeta_{n_0,1}^* & \zeta_{n_0,2}^* & \cdots & \zeta_{n_0,\alpha_1}^* \end{bmatrix} \tag{8}$$

$$B = \begin{bmatrix} \eta^*_{1,1} & \eta^*_{1,2} & \cdots & \eta^*_{1,n_i} \\ 0 & \cdots & \cdots & 0 \\ \eta^*_{2,1} & \eta^*_{2,2} & \cdots & \eta^*_{2,n_i} \\ \vdots & \vdots & \ddots & \vdots \\ \eta^*_{n_0,1} & \eta^*_{n_0,2} & \cdots & \eta^*_{n_0,n_i} \end{bmatrix} \tag{9}$$

$$C = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & 1 \end{bmatrix} \tag{10}$$

Results

The principle of the proposed controller is illustrated in Figure 2. Essentially, enhanced information about the controlled system may be extracted from the fuzzy model. This information may then be employed in the design of the controller gains in order to achieve optimality of the controller pole locations. In this manner the proposed controller may be described as a Fuzzy Adaptive Sliding Mode Controller (FASMC).

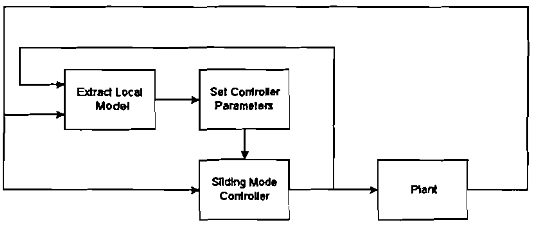

Figure 2: Principle of FASMC

The controller was compared to a benchmark sliding mode controller with integral action [2]. Both controllers were designed to provide critical damping at a natural frequency of 22rad/s. The controllers were tested over a sample period of 70 seconds. Results are illustrated in figure 3.

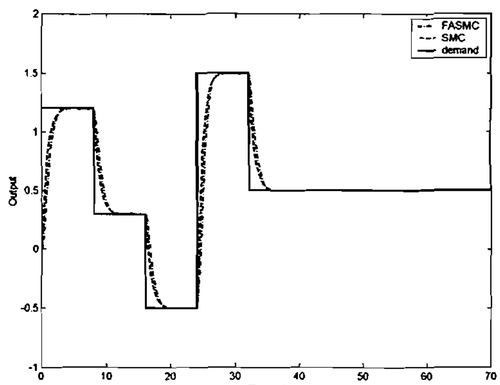

Figure 3: System outputs over 70 seconds

It can be seen that in terms of system response that there is little to differentiate the two.

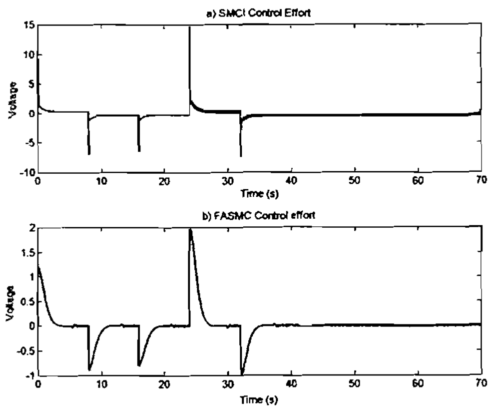

Figure 4: System control efforts

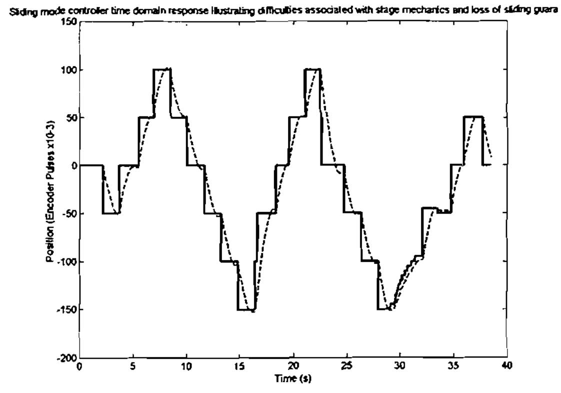

Figure 5: Real time sliding mode controller response

However, consideration of the corresponding control effort (figure 4) shows that that the high gain associated with the SMCI is not apparent with the FASMC. In addition it is worthwhile to note that the ε-vicinity of the FASMC was 6 times smaller than the corresponding SMCI.

Implementation of the SMCI shows good agreement with the simulation results. However, as system load torque varies with position, which is manifested as unmatched disturbance, it can clearly be seen how the controller performance is subject to these effects and how it is left to the integral action states to bring the system back to zero steady state error. This is due entirely to the poor representation of the system by the model.

In a second test, an unmatched disturbance was introduced to the system and the fuzzy model re-trained to incorporate the uncertainty. Figure 6 illustrates the effect of the disturbance on the SMCI, and it can be seen that the disturbance significantly effects transient performance. Since the disturbance is constant the effects are not as profound as they might be. The FASMC on the other hand appears to recover the system to the steady state in almost the same time as the system without disturbance (figure 6). The initial change in direction is caused by the lack of initial control effort from the controller.

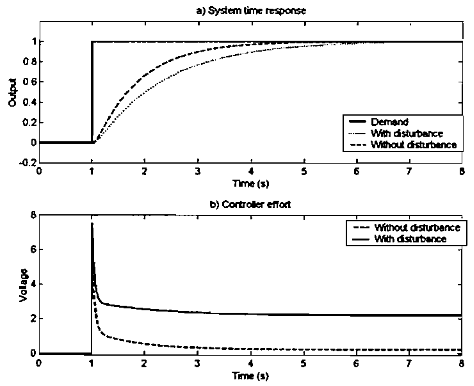

Figure 6: SMCI response to unmatched disturbance

From a practical perspective, it is unlikely that the control system would be subjected to a test of this severity. However, since the load torque has been recognised as unmatched, and since this is known to vary with both position and time [6], similar effects can be observed by simply moving the carriage to a different location and repeating the step change in demand (figure 5).

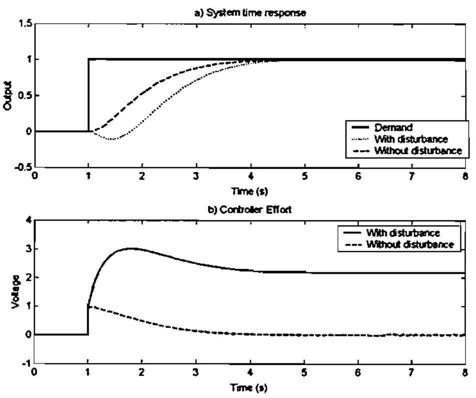

Figure 7: FASMC response to unmatched disturbance

Conclusion

The proposed algorithm has demonstrated how the synergy of traditional control structures and fuzzy logic can be used in order to produce an improved controller. Practical implementation of the sliding mode with integral action controller has demonstrated the ability of this controller to reject unmatched disturbance, as originally discussed in [2], and has also served to validate simulation results obtained. Before the apparent advantages of this controller can be confirmed, further cautious research is required. However, the initial results are suggestive of a controller that demonstrates reduced controller gain, reduced sensitivity to unmatched disturbance and an improved guarantee of final tracking accuracy. Work on the practical implementation of the FASMCI continues.

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