Self-Contradiction Degrees in Intuitionistic Fuzzy Sets

E. Castiñeira*, **S. Cubillo, C. Torres, V. Zarzosa**
Dept. Matemática Aplicada
Univ. Politécnica de Madrid
28660 Boadilla del Monte
ecastineira@fi.upm.es

Abstract

This paper is an attempt to model to what extent an intuitionistic fuzzy set is self-contradictory, both in the case of self-contradiction regarding a strong intuitionistic negation, and without depending on a specific negation. For this purpose, firstly, a geometrical study on self-contradiction regarding a negation is considered; afterwards some functions to measure degrees of self-contradictory depending on a negation are defined. Finally, the degree of self-contradiction independently of any negation is dealt with other functions, and in both cases some properties are found.

Keywords: Intuitionistic fuzzy sets, intuitionistic fuzzy generators and fuzzy negations, degrees of contradiction.

1 Introduction

1.1 The intuitionistic fuzzy sets, as it is well known, were introduced by K. T. Atanassov in 1983 as follows:

Definition 1.1. ([1]) *An intuitionistic fuzzy set (IFS) A, in the universe $X \neq \emptyset$, is a set given as $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ such that, for all $x \in X$, $\mu_A(x) + \nu_A(x) \leq 1$, and where $\mu_A : X \to [0,1]$, $\nu_A : X \to [0,1]$ are called functions of membership and non-membership, respectively.*

This set could be considered as a L-fuzzy set as defined by Goguen ([7]) being, in this case, $L = \{(\alpha_1, \alpha_2) \in [0,1]^2 : \alpha_1 + \alpha_2 \leq 1\}$, with the partial order $\leq_L$ defined as follows: given $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$, $\boldsymbol{\beta} = (\beta_1, \beta_2) \in L$,

$$\boldsymbol{\alpha} \leq_L \boldsymbol{\beta} \iff \alpha_1 \leq \beta_1 \ \& \ \alpha_2 \geq \beta_2 .$$

$(L, \leq_L)$ is a complete lattice with smallest element, $\mathbf{0_L} = (0,1)$, and greatest element, $\mathbf{1_L} = (1,0)$.

So, an IFS A is a L-fuzzy set whose L-membership function $\chi^A \in L^X = \{\chi : X \to L\}$ is defined for each $x \in X$ as $\chi^A(x) = (\mu_A(x), \nu_A(x))$. Let us denote the set of all intuitionistic fuzzy sets on X as $\mathcal{IF}(X)$.

Furthermore, recall that a decreasing function $\mathcal{N} : L \to L$ is an intuitionistic fuzzy negation (IFN) if $\mathcal{N}(\mathbf{0_L}) = \mathbf{1_L}$ and $\mathcal{N}(\mathbf{1_L}) = \mathbf{0_L}$ hold. Moreover, $\mathcal{N}$ is a strong IFN if the equality $\mathcal{N}(\mathcal{N}(\boldsymbol{\alpha})) = \boldsymbol{\alpha}$ holds for all $\boldsymbol{\alpha} \in L$. Bustince *et al.* in [2] introduced the intuitionistic fuzzy generators, which can be used to build intuitionistic fuzzy negations, and Deschrijver *et al.* in [6] focus on this problem and proved that any strong IFN $\mathcal{N}$ is characterized by a strong negation $N : [0,1] \to [0,1]$ by means of the formula $\mathcal{N}(\alpha_1, \alpha_2) = (N(1-\alpha_2), 1 - N(\alpha_1))$, for all $(\alpha_1, \alpha_2) \in L$. Regarding strong fuzzy negations, they were characterized by Trillas in [8]. He showed that N is a strong negation if and only if there exists an order automorphism in the unit interval, $g : [0,1] \to [0,1]$, such that $N(\alpha) = g^{-1}(1 - g(\alpha))$, for all $\alpha \in [0,1]$. So, a strong IFN $\mathcal{N}$ is also determined by an order automorphism g in $[0,1]$.

1.2 Trillas *et al.* introduced and studied the concept of contradictory set in [9] and [10] in the framework of fuzzy sets. These papers formalise the idea that a set is self-contradictory

*Corresponding author

(or contradictory to be short) if it violates the principle of not contradiction in the following sense: the statement "If x is P, then x is not P" holds with some degree of truth. So, they establish that the fuzzy set associated with the predicate P, and determined by μ_P, is contradictory if "$\mu_P(x) \to \mu_{\neg P}(x)$ for all x" representing the implication "$\to$" by means of the reticular inequality $\leq$ of $[0,1]$, that is, μ_P is self-contradictory regarding a strong negation N, or N-self-contradictory, if $\mu_P \leq N \circ \mu_P$. The condition $\mu \leq N \circ \mu$ is equivalent to $\operatorname{Sup}_x(\mu(x)) \leq \alpha_N$, where α_N is the fixed point of N; nevertheless, the extent to which this condition holds, that is, how contradictory μ is, is a matter for consideration, since μ can behave quite differently regarding this characteristic. For example, if the fuzzy set determined by μ verifies that $\operatorname{Sup}_x(\mu(x)) = \alpha_N$, then a minimal variation in this supreme could produce a non-N-contradictory fuzzy set. But, if $\operatorname{Sup}_x(\mu(x))$ is much smaller than α_N, then small changes in this supreme do not modify the contradictoriness of the disturbed μ. The need to speak not only of contradiction but also of degrees of contradiction was later raised in [3] and [4], where a function was considered for the purpose of determining (or measuring) this degree.

The study of contradiction in the framework of intuitionistic fuzzy sets was initiated in [5]. Similarly to fuzzy case, an IFS $A \in \mathcal{IF}(X)$ is said to be a self-contradictory set with respect to some strong IFN, $\mathcal{N}$, if $\chi^A(x) \leq_L (\mathcal{N} \circ \chi^A)(x)$ for all $x \in X$, where χ^A is the L-membership function of A. Since it is interesting to know not only if a set is contradictory, but also the extend to which this property holds, in this work, we deal with the problem of measuring the contradiction in the case of IFS.

2 Measuring $\mathcal{N}$-self-contradiction in $\mathcal{IF}(X)$

In this section we analyse firstly the regions of L in which the contradictory sets for a given negation are located. The purpose of this study is to find some relation suggesting the way to measure how contradictory an IFS is. Secondly, we will propose some measures to determine the searched degrees of contradiction.

2.1 Regions of $\mathcal{N}$-contradiction

In [3], it is proved that, given $A \in \mathcal{IF}(X)$, with $\chi^A = (\mu_A, \nu_A) \in L^X$, and $\mathcal{N}$ a strong IFN, associated with the strong negation N, then

(i) A is $\mathcal{N}$-contradictory $\Leftrightarrow N(\mu_A(x)) + \nu_A(x) \geq 1$ for all $x \in X$.

(ii) A is $\mathcal{N}$-contradictory $\Leftrightarrow g(\mu_A(x)) + g(1 - \nu_A(x)) \leq 1$ for all $x \in X$, provided g is the generator of N.

Above inequalities, that are equivalent, determine a region free of contradiction in L and other one where the contradictory sets must remain. Let us see those regions for some particular negations and afterwards in the general case.

(a) $\mathcal{N}_s$-contradiction with standard negation $\mathcal{N}_s(\alpha_1, \alpha_2) = (\alpha_2, \alpha_1)$

If we consider the standard negation, $\mathcal{N}_s$, that is given by $N = 1 - id$, where $g = id$; then the above statements become: A is $\mathcal{N}$-contradictory if and only if $1 - \mu_A(x) + \nu_A(x) \geq 1$, or $\nu_A(x) \geq \mu_A(x)\ \ \forall x \in X$.

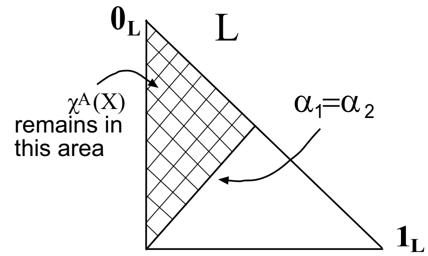

Figure 1: $\mathcal{N}_s$-contradiction area.

So, A is $\mathcal{N}$-contradictory if and only if $\chi^A(X) = \{\chi^A(x) : x \in X\} \subset \{(\alpha_1, \alpha_2) \in L : \alpha_1 \leq \alpha_2\}$; therefore, the image of X under χ^A, that we also call *range of A*, should be inside of the region showed in figure 1, and line $\alpha_1 = \alpha_2$ is the boundary between the contradictory and non-contradictory regions.

(b) $\mathcal{N}_g$-contradiction being $\mathcal{N}_g$ the strong IFN associated with a Sugeno's negation

The order automorphism $g(\alpha) = \frac{\ln(1+\alpha)}{\ln 2}$ determines the strong Sugeno's negation $N_g(\alpha) = \frac{1-\alpha}{1+\alpha}$. In this case, the set $A \in \mathcal{IF}(X)$ is $\mathcal{N}_g$-contradictory if and only if

$$\frac{1-\mu_A(x)}{1+\mu_A(x)} + \nu_A(x) \geq 1 \ \forall x \in X$$

So, A is $\mathcal{N}_g$-contradictory if and only if

$$\begin{aligned} \chi^A(X) &\subset \{(\alpha_1, \alpha_2) \in L : \alpha_1 + \alpha_1\alpha_2 - 2\alpha_1 \geq 0\} \\ &= \left\{(\alpha_1, \alpha_2) \in L : \alpha_2 \geq \frac{2\alpha_1}{1+\alpha_1}\right\} \end{aligned}$$

(c) $\mathcal{N}_r$-contradiction with $\mathcal{N}_r$ determined by $g_r(\alpha) = \alpha^r$, $r > 0$

Let us consider the family of strong negations $\{N_r\}_{r>0}$, where for each $r > 0$ the automorphism determining N_r is $g_r(\alpha) = \alpha^r$. This family collects as particular case the negation given in (a), and for each $r > 0$ is $N_r(\alpha) = (1-\alpha^r)^{1/r}$ with fixed point $\alpha_{N_r} = \frac{1}{2^{1/r}}$.

$A \in \mathcal{IF}(X)$ is $\mathcal{N}_r$-contradictory, where $\mathcal{N}_r$ is the IFN associated with N_r (or with g_r), if and only if

$$\chi^A(X) \subset \{(\alpha_1, \alpha_2) \in L : \alpha_1^r + (1-\alpha_2)^r \leq 1\}$$

For each $r > 0$ the curve $\alpha_1^r + (1-\alpha_2)^r = 1$ is the boundary which delimit the contradiction region, and if an IFS has some L-value under that curve, then it is not $\mathcal{N}_r$-contradictory.

In particular, A is $\mathcal{N}_2$-contradictory if and only if the image of X under χ^A is inside or on the circumference with centre $\mathbf{1_L}$ and radius 1 (fig. 2).

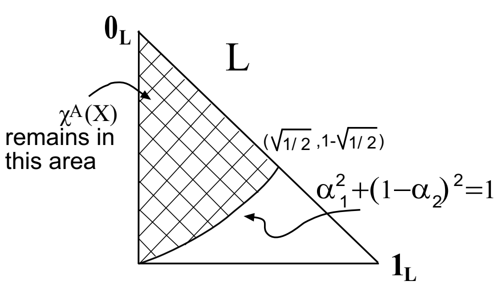

Figure 2: $\mathcal{N}_2$-contradiction area with $\mathcal{N}_2$ determined by $g_2(\alpha) = \alpha^2$.

Let us note that the boundary curve of the contradiction region, with equation $\alpha_1^2 + (1-\alpha_2)^2 = 1$, intersects the line with equation $\alpha_1 + \alpha_2 = 1$ at the point $(\alpha_{N_g}, 1-\alpha_{N_g}) = (1/\sqrt{2}, 1-1/\sqrt{2})$, where α_{N_g} is the fixed point of N_g.

Let us note that the more r increases , the more the curves $\alpha_1^r + (1-\alpha_2)^r = 1$ come closer to axis α_1 (except for $\alpha_1 = 1$); to be precise, the family of functions $\{1-(1-\alpha_1^r)^{1/r}\}_{r>0}$ pointwise converges, when $r \to \infty$, to the null function for all $\alpha_1 \in [0,1)$ and to 1 at $\alpha_1 = 1$; therefore, the not $\mathcal{N}_r$-contradiction region decreases. Furthermore, when $r \to 0$, the family of functions $\{(1-(1-\alpha_2)^r)^{1/r}\}_{r>0}$ converges for all $\alpha_2 \in [0,1)$ to null function, and for $\alpha_2 = 1$ converge to 1; that is, the more r decreases, the more the curves delimiting the contradiction region come closer to axis α_2 (except for $\alpha_2 = 1$), and then the not $\mathcal{N}_r$-contradiction region spreads when r decreases.

On the other hand, if $0 < r < s$ the curve $\alpha_1^s + (1-\alpha_2)^s = 1$ is under the curve $\alpha_1^r + (1-\alpha_2)^r = 1$ (in figure 3 some of them are showed), and, if $A \in \mathcal{IF}(X)$ is $\mathcal{N}_r$-contradictory, it is $\mathcal{N}_s$-contradictory for all $s > r$. Indeed, if $r < s$ it is $\alpha_1^r > \alpha_1^s$ for all $\alpha_1 \in (0,1)$, and, as $g_{\frac{1}{r}}$ is increasing and $1/s < 1/r$, it is $(1-\alpha_1^r)^{1/r} < (1-\alpha_1^s)^{1/r} < (1-\alpha_1^s)^{1/s}$, from which it follows that the coordinate α_2 of the curve related to s is below that the one related to r.

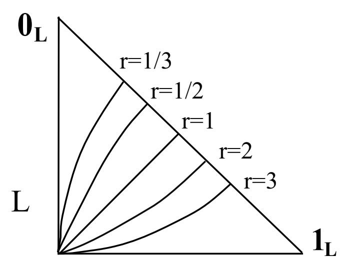

Figure 3: Curves $\alpha_1^r + (1-\alpha_2)^r = 1$.

Finally, let us observe that the aforementioned family of curves almost "cover" the lattice L with the exception of the axis without the origin, that is:

$$\begin{aligned} &\cup_{r>0} \{(\alpha_1, \alpha_2) \in L : \alpha_1^r + (1-\alpha_2)^r = 1\} = \\ &\quad L \setminus \{(\alpha_1, \alpha_2) \neq (0,0) : \alpha_1 = 0 \text{ or } \alpha_2 = 0\}. \end{aligned}$$

(d) General case of $\mathcal{N}$-contradiction

If $\mathcal{N}$ is a strong IFN associated with the strong negation N, a set $A \in \mathcal{IF}(X)$ is $\mathcal{N}$-contradictory if and only if

$$\chi^A(X) \subset \{(\alpha_1, \alpha_2) \in L : N(\alpha_1) + \alpha_2 \geq 1\},$$

and the boundary curve delimiting the non-contradiction region, $N(\alpha_1)+\alpha_2 = 1$, and that we name $\mathcal{N}$*-boundary curve* (or, to simplify, boundary curve, if it is not misleading), verifies the following properties:

1) It is increasing at the variable α_1.

2) Its range contains the point (0,0).

3) The intersection of $N(\alpha_1) + \alpha_2 = 1$ and $\alpha_1 + \alpha_2 = 1$ is the point $(\alpha_N, 1 - \alpha_N)$, being α_N the fixed point of N.

2.2 Degrees of $\mathcal{N}$-contradiction

As we noted in the introduction, it is important to measure how much contradictory a set is, and not only in the fuzzy case, but also in the intuitionistic one. In fact, the IFS with a constant null L-value, $\chi^{\mathbf{0_L}}(x) = \mathbf{0_L}$ for all $x \in X$, is $\mathcal{N}$-contradictory for any IFN $\mathcal{N}$, and $A \in \mathcal{IF}(X)$ taking all its L-values on the contradiction boundary curve, $N(\alpha_1) + \alpha_2 = 1$ (where N is the strong negation associated with $\mathcal{N}$), is also $\mathcal{N}$-contradictory. Nevertheless, small disturbances in the L-values of A on the boundary curve will return a new set, very similar to A, but not $\mathcal{N}$-contradictory, whereas small disturbances will never change the contradictoriness of $\mathbf{0_L}$. So, it seems quite suitable to assign the value 0 as the degree of $\mathcal{N}$-contradiction of A, and also, of any set taking L-values on the area underneath the boundary curve. Analogously, it seems appropriate to assign positive degree to a set whose range is above the boundary curve and it will be as much higher as the range is farther away from the curve. Taking in account these comments, we will define different functions that could be used to determine the contradiction degrees.

Definition 2.1. *Let $A \in \mathcal{IF}(X)$ be an IFS determined by $\chi^A = (\mu_A, \nu_A) \in L^X$; then*

i) $\mathcal{C}_1^{\mathcal{N}}(A) = \text{Max}\left(0, \operatorname*{Inf}_{x\in X}(N(\mu_A(x)) + \nu_A(x) - 1)\right)$

is the $\mathcal{N}$-contradiction degree of A according to the strong negation N associated with $\mathcal{N}$.

ii) $\mathcal{C}_2^{\mathcal{N}}(A)=\text{Max}\left(0,1-\operatorname*{Sup}_{x\in X}(g(\mu_A(x)) + g(1-\nu_A(x))\right)$

is the $\mathcal{N}$-contradiction degree of A according to the automorphism g determining $\mathcal{N}$.

iii) The contradiction degree according to the distance to the contradictory boundary curve is $\mathcal{C}_3^{\mathcal{N}}(A) = 0$, provided A is non-$\mathcal{N}$-contradictory, and in other case

$$\mathcal{C}_3^{\mathcal{N}}(A) = \frac{d(\chi^A(X), \mathcal{L}_{\mathcal{N}})}{d(\mathbf{0_L}, \mathcal{L}_{\mathcal{N}})},$$

where d is the euclidean distance, and

$$\mathcal{L}_{\mathcal{N}} = \{\boldsymbol{\alpha} = (\alpha_1, \alpha_2) \in L : N(\alpha_1) + \alpha_2 = 1\}$$

is the boundary curve; so,

$d(\chi^A(X), \mathcal{L}_{\mathcal{N}})=\text{Inf}\{d(\chi^A(x), \boldsymbol{\alpha}): x \in X, \boldsymbol{\alpha}\in \mathcal{L}_{\mathcal{N}}\}$ *and*

$d(\mathbf{0_L}, \mathcal{L}_{\mathcal{N}}) = \text{Inf}\,\{d(\mathbf{0_L}, \boldsymbol{\alpha}) : \boldsymbol{\alpha} \in \mathcal{L}_{\mathcal{N}}\}.$

Remark

The three above functions take their values in $[0, 1]$. The function $\mathcal{C}_1^{\mathcal{N}}$ is motivated by the characterization of the contradiction (i) of 2.1, whereas $\mathcal{C}_2^{\mathcal{N}}$ is originated by (ii). Although both characterizations are equivalent, $\mathcal{C}_1^{\mathcal{N}}$ and $\mathcal{C}_2^{\mathcal{N}}$ do not match up, as it is showed in the next example (2.3). Besides, $\mathcal{C}_3^{\mathcal{N}}$ represents a relative distance: the euclidean distance between the range of a IFS and the boundary curve, relative to the distance between the "most contradictory" set and the same curve, whereas $\mathcal{C}_1^{\mathcal{N}}$ represents the infimum of the distances between the ordinates of the L-values of the IFS and those of the boundary curve (see the figure 4). Regarding $\mathcal{C}_2^{\mathcal{N}}$, it is possible to find some geometrical interpretations in some particular cases.

Proposition 2.2. *Let $\mathcal{N}_s$ be the standard IFN, then for all $A \in \mathcal{IF}(X)$ degrees of contradiction of A by means of the formula in definition 2.1 verify that*

$$\mathcal{C}_1^{\mathcal{N}_s}(A) = \mathcal{C}_2^{\mathcal{N}_s}(A) = \mathcal{C}_3^{\mathcal{N}_s}(A).$$

Nevertheless, in general, the three measures are different, as the following example shows.

Example 2.3. Let $A \in \mathcal{IF}([0, 1])$ with L-membership function $\chi^A(x) = (x/4, 1 - x/2)$, and let us consider the strong IFN $\mathcal{N}$ determined by the fuzzy negation $N(x) = \sqrt{1-x^2}$, with $g(x) = x^2$. Then:

$$\mathcal{C}_1^{\mathcal{N}}(A) = \text{Max}\left(0, \inf_{x\in[0,1]}\left(\sqrt{1-(\tfrac{x}{4})^2} - \tfrac{x}{2}\right)\right) = \tfrac{\sqrt{15}-2}{4},$$

$$\mathcal{C}_2^{\mathcal{N}}(A) = \text{Max}\left(0, 1 - \sup_{x\in[0,1]}\left(\left(\tfrac{x}{4}\right)^2 + \left(\tfrac{x}{2}\right)^2\right)\right) = \tfrac{11}{16}.$$

And, as A is $\mathcal{N}$-contradictory, then

$$\mathcal{C}_3^{\mathcal{N}}(A) = \frac{d(\chi^A(X), \mathcal{L}_{\mathcal{N}})}{d(\mathbf{0_L}, \mathcal{L}_{\mathcal{N}})} = 1 - \frac{\sqrt{5}}{4}. \qquad \triangleleft$$

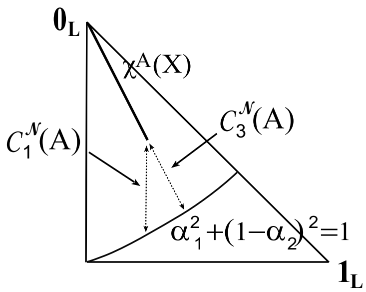

Figure 4: Geometrical interpretation of different contradiction degrees

The following properties of above measures of $\mathcal{N}$-self-contradiction can be proved.

Proposition 2.4. *For $i = 1, 2, 3$, the function $\mathcal{C}_i^{\mathcal{N}} : \mathcal{IF}(X) \to [0,1]$ given for each $A \in \mathcal{IF}(X)$ as in the definition 2.1 verifies:*

(i) If $\mathbf{0_L}$ denotes the IFS such that $\chi^{\mathbf{0_L}}(x) = \mathbf{0_L}$ for all $x \in X$, then $\mathcal{C}_i^{\mathcal{N}}(\mathbf{0_L}) = 1$.

(ii) $\mathcal{C}_i^{\mathcal{N}}$ is anti-monotonic with respect to the orders $\leq_L$ in L and the usual one of $\mathbb{R}$: If $A, B \in \mathcal{IF}(X)$ with $\chi^A \leq_L \chi^B$ (that is, $\chi^A(x) \leq_L \chi^B(x)$ for all $x \in X$), then $\mathcal{C}_i^{\mathcal{N}}(B) \leq \mathcal{C}_i^{\mathcal{N}}(A)$.

(iii) If $A \in \mathcal{IF}(X)$ verifies that $\inf_{x\in X} \nu_A(x) = 0$, then $\mathcal{C}_i^{\mathcal{N}}(A) = 0$.

3 Measuring Self-contradiction in $\mathcal{IF}(X)$

The previous section establishes the contradiction of an IFS related to a chosen negation. We now address contradiction more generally, without depending on a specific IFN. In [5] an IFS $A \in \mathcal{IF}(X)$ was defined self-contradictory (or contradictory to be short) if it was $\mathcal{N}$-self-contradictory regarding some strong IFN $\mathcal{N}$, and the following result was proved.

Proposition 3.1. ([5]) *Let $A \in \mathcal{IF}(X)$ be, with L-membership function $\chi^A = (\mu_A, \nu_A) \in L^X$, the following holds:*
(i) If A is self-contradictory, then $\sup_{x\in X} \mu_A(x) < 1$.
(ii) If $\inf_{x\in X} \nu_A(x) > 0$, then A is self-contradictory.

With the purpose of measuring how much contradictory an IFS is, and taking in account the preceding proposition, we will define some functions; nevertheless, before that a corollary is given.

Corollary 3.2. *If $A \in \mathcal{IF}(X)$, with membership function $\mu_A \in [0,1]^X$, is contradictory, then* $\sup_{x\in X}(\mu_A(x) - \nu_A(x)) < 1$.

An important aspect to take into account, and that could clarify the definition of these functions measuring the contradiction degree, is collected by the following result.

Proposition 3.3. *If $A \in \mathcal{IF}(X)$, with L-membership function $\chi^A = (\mu_A, \nu_A) \in L^X$, is self-contradictory then for all $\{x_n\}_{n\in\mathbb{N}} \subset X$ such that $\lim_{n\to\infty} \nu_A(x_n) = 0$, then $\lim_{n\to\infty} \mu_A(x_n) = 0$ holds.*

Definition 3.4. *Let $A \in \mathcal{IF}(X)$ be the set determined by $\chi^A = (\mu^A, \nu^A) \in L^X$; the following contradiction degrees of A are proposed:*

i) $\mathcal{C}_1(A) = \inf_{x\in X} \nu_A(x)$.

ii) $\mathcal{C}_2(A) = 0$ if there exists $\{x_n\}_{n\in\mathbb{N}} \subset X$ such that $\lim_{n\to\infty} \nu_A(x_n) = 0$, and, in other case

$$\mathcal{C}_2(A) = \inf_{x\in X} \frac{1 - \mu_A(x) + \nu_A(x)}{2}.$$

Example 3.5. Let $A, B \in \mathcal{IF}([0,1])$ be the sets determined by $\chi^A(x) = (1/4, 1/4)$ and $\chi^B(x) = (3/4, 1/4)$ for all $x \in [0,1]$, respectively. Then $\mathcal{C}_1(A) = \mathcal{C}_1(B) = 1/4$; however, $\mathcal{C}_2(A) = 1/2$ and $\mathcal{C}_2(B) = 1/4$. How could we explain these results? As both sets are contradictory, it is obvious that the two measures should be positive for them. The first one measures how much each set needs to stop being self-contradictory, as that is just what is missing to "touch" the axis α_1. But, how to interpret that the degree for $\mathcal{C}_2$ is greater for A? The answer is that the set A is $\mathcal{N}$-contradictory for the same negations $\mathcal{N}$ that B and, furthermore, for a lot more of them, that is, there are more negations $\mathcal{N}$ that make A contradictory than that make B contradictory. In this sense, the measure $\mathcal{C}_2$ provides more information than $\mathcal{C}_1$ about contradictoriness. $\triangleleft$

Remark

On the one hand, it is evident that the function $\mathcal{C}_1$ measures the euclidean distance from the range of a contradictory set $A \in \mathcal{IF}(X)$ to the axis α_1 (that we denote $\mathcal{L}_1$):

$$\mathcal{C}_1(A) = d(\chi^A(X), \mathcal{L}_1) = \frac{d(\chi^A(X), \mathcal{L}_1)}{d(\mathbf{0_L}, \mathcal{L}_1)}.$$

On the other hand,

$$\mathcal{C}_2(A) = \frac{d_1(\chi^A(X), \mathbf{1_L})}{2} = \frac{d_1(\chi^A(X), \mathbf{1_L})}{d_1(\mathbf{0_L}, \mathbf{1_L})},$$

that is, the function $\mathcal{C}_2$ measures the reticular distance between the range of A and $\mathbf{1_L}$, relative to the reticular distance from $\mathbf{0_L}$ to $\mathbf{1_L}$ (let us remind that $d_1(\boldsymbol{\alpha}, \boldsymbol{\beta}) = |\alpha_1 - \beta_1| + |\alpha_2 - \beta_2|$). These geometrical interpretations of the measures $\mathcal{C}_1$ y $\mathcal{C}_2$ suggest another way to measure the contradiction degree.

Definition 3.6. *The function $\mathcal{C}_3 : \mathcal{IF}(X) \to [0,1]$ is defined for each $A \in \mathcal{IF}(X)$, with L-membership function $\chi^A = (\mu_A, \nu_A)$, as follows:*

$$\mathcal{C}_3(A) = \begin{cases} 0 & \text{if } \exists \{x_n\}_{n\in\mathbb{N}} \subset X / \lim_{n\to\infty} \nu_A(x_n) = 0, \\ \dfrac{d(\chi^A(X), \mathbf{1_L})}{d(\mathbf{0_L}, \mathbf{1_L})} & \text{in other case.} \end{cases}$$

In a similar way to proposition 2.1 the following result can be proved.

Proposition 3.7. *For each $i = 1, 2, 3$, the above defined functions $\mathcal{C}_i : \mathcal{IF}(X) \to [0,1]$ verify:*

(i) $\mathcal{C}_i(\mathbf{0_L}) = 1$.

(ii) $\mathcal{C}_i$ is anti-monotonic respect to the orders $\leq_L$ into L and the usual of $\mathbb{R}$: If $A, B \in \mathcal{IF}(X)$ such that $\chi^A \leq_L \chi^B$, then $\mathcal{C}_i(B) \leq \mathcal{C}_i(A)$.

(iii) If $A \in \mathcal{IF}(X)$ verifies $\operatorname{Inf}_{x\in X} \nu_A(x) = 0$, then $\mathcal{C}_i(A) = 0$.

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